HW8：一维单原子链　第j个格波于n个格点引起 μ_{nj}

$$\mu_{nj} = a_j \sin(\omega_j t + naq_j + \delta_j)$$　δ_j - 相因子　计算的平均振幅

格波 $Ae^{i(\omega t - naq)}$

$$\mu_n = \sum_j \mu_{nj} = \sum_j a_j \sin(\omega_j t + naq_j + \delta_j)$$

$$\therefore \overline{\mu_n^2} = \sum_j \overline{\mu_{nj}^2} + \sum_{i \neq j} \overline{\mu_{ni}} \cdot \overline{\mu_{nj}^*}$$

$$\approx \sum_j \overline{\mu_{nj}^2}$$

由于 $\overline{\mu_{nj}^2} = \frac{1}{T_0}\int_0^{T_0} a_j^2 \sin^2(\omega_j t + naq_j + \delta_j)\, dt = \frac{1}{2}a_j^2$

而动能的对时间平均：

$$\overline{T_{nj}} = \frac{1}{T_0}\int_0^{T_0} dt \int_0^{L} dx \left(\frac{1}{2}\frac{m}{a}\left(\frac{d\mu_{nj}}{dt}\right)^2 \right)$$

$$= \frac{1}{4}\rho \omega_j^2 L a_j^2 = \frac{1}{2}kT$$

$$\therefore \overline{\mu_{nj}^2} = \frac{kT}{\rho L \omega_j^2} \Rightarrow \overline{\mu_j^2} = \frac{kT}{\rho L}\sum_j \omega_j^{-2}$$