

correspondence

Understanding the Dynamics of COVID-19

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To the Editor

Understanding the dynamics of an epidemic such as that caused by coronavirus disease 2019 (COVID-19) requires some mathematics and some data. For example, in a population of N persons let Y symbolize the number of infected persons and X symbolize the number of susceptible persons, then the change in the number of infected persons per unit time is given as:¹

$$dY/dt = c * p * Y * X/N - v * Y \quad (1)$$

Here, c symbolizes the rate of contact, p the transmission probability, and v the recovery rate (ie, $1 -$ the case fatality rate). By letting $X = N - Y$, the equation can be rewritten in the form:

$$dY/dt = a * Y - b * Y^2 \quad (2)$$

with $a = c * p - v$ and with $b = (a + v)/N$. The solution to this differential equation is 2:

$$Y(t) = \frac{a * Y(0)}{b * Y(0) + (a - b * Y(0)) * \exp(-a * (t - t_0))} \quad (3)$$

with $Y(0)$ denoting the number infected at time zero and $Y(t)$ the number infected at time t .

In what follows I illustrate this approach with data reported by Jondavid Klipp (jondavid@laboratoryeconomics.ccsen.com) regarding the cruise ship Diamond Princess, which took on a single infected passenger from Hong Kong. Subsequently, all on board were tested for COVID-19, and 712 of the 3,711 persons on board tested positive after being forced to remain on board for a month. Ten died. Thus, values for N , Y , X , t , and v were known. Using a value of approximately 1.3 for the product $c * p$, the prediction for Y at 30 days was 712, which equaled that observed, and **Figure 1** shows a plot of predicted values of Y over a course of 50 days (the point is for the time of 30 days when the passengers left the ship).

The logic used in equation 1 clearly implies that the product $c * p$ relates closely to how fast a population becomes infected, and this in turn depends on a number of factors including patient ages, comorbidities, geographical population densities and, of course, the virus. The value of approximately 1.3 for $c * p$ in the Diamond Princess population may be high for COVID-19, because the geography of this population was so restricted and because many passengers were older. Only follow-up studies as the epidemic matures will provide additional estimates of $c * p$. The Diamond Princess data also suggest that many cases are asymptomatic. Thus, the widely reported numbers of new cases could omit cases who were asymptomatic and therefore not tested. This in turn could yield higher case fatality rates and therefore smaller estimates of the response rate, v , than are realistic. Regardless, the success for the logistic growth model applied here to COVID-19 suggests that for many populations the number of infected may eventually reach a limit.

Robin Vollmer, MD

Department of Pathology
Durham Veterans Affairs Hospital
Durham, NC.

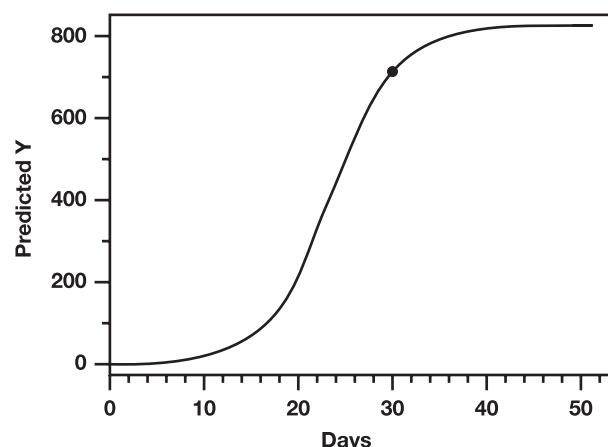


Figure 1 Plot of the predicted number of infected persons vs days on board the Diamond Princess. The predictions come from equation 3.

References

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