Left recursion

- A grammar is *left-recursive* if there exists a nonterminal A such that there is a derivation $A \Rightarrow^{+} A\,\alpha$ for some string α

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid id
\end{aligned}
$$

We cannot compute FIRST sets for left-recursive grammars.

Left recursion

- A left-recursive grammar can be transformed to eliminate left recursion

$$
\begin{aligned}
E &\to E + T \mid T \\
T &\to T * F \mid F \\
F &\to (E) \mid id
\end{aligned}
$$

$$
\begin{aligned}
E &\to T\,E' \\
E' &\to +T\,E' \mid \varepsilon \\
T &\to F\,T' \\
T' &\to *\,F\,T' \mid \varepsilon \\
F &\to (E) \mid id
\end{aligned}
$$

Eliminating immediate left recursion

$$A \rightarrow A\alpha \mid \beta$$

$$\begin{aligned} A &\rightarrow \beta A' \\ A' &\rightarrow \alpha A' \mid \varepsilon \end{aligned}$$

Eliminating immediate left recursion

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid id
\end{aligned}
$$

$$\Rightarrow$$

$$
\begin{aligned}
E &\rightarrow T\,E' \\
E' &\rightarrow +T\,E' \mid \varepsilon \\
T &\rightarrow F\,T' \\
T' &\rightarrow *\,F\,T' \mid \varepsilon \\
F &\rightarrow (E) \mid id
\end{aligned}
$$

Eliminating immediate left recursion

In general…

$$A \to A\alpha_1 \mid A\alpha_2 \mid \cdots \mid A\alpha_m \mid \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n$$

$$\begin{aligned} A &\to \beta_1 A' \mid \beta_2 A' \mid \cdots \mid \beta_n A' \\ A' &\to \alpha_1 A' \mid \alpha_2 A' \mid \cdots \mid \alpha_m A' \mid \epsilon \end{aligned}$$

Indirect left recursion

$$S \rightarrow A\ a \mid b$$
$$A \rightarrow A\ c \mid S\ d \mid \epsilon$$

$$S \rightarrow A\ a \mid b$$
$$A \rightarrow b\ d\ A' \mid A'$$
$$A' \rightarrow c\ A' \mid a\ d\ A' \mid \epsilon$$

Eliminating left recursion

```
1)  arrange the nonterminals in some order A_1, A_2, ..., A_n.
2)  for ( each i from 1 to n ) {
3)          for ( each j from 1 to i - 1 ) {
4)                  replace each production of the form A_i → A_j γ by the
                        productions A_i → δ_1 γ | δ_2 γ | ··· | δ_k γ, where
                        A_j → δ_1 | δ_2 | ··· | δ_k are all current A_j-productions
5)          }
6)          eliminate the immediate left recursion among the A_i-productions
7)  }
```

Eliminating left recursion

$$S \rightarrow A\ a \mid b$$
$$A \rightarrow A\ c \mid S\ d \mid \epsilon$$

$$S \rightarrow A\ a \mid b$$
$$A \rightarrow A\ c \mid A\ a\ d \mid b\ d \mid \epsilon$$

$$S \rightarrow A\ a \mid b$$
$$A \rightarrow b\ d\ A' \mid A'$$
$$A' \rightarrow c\ A' \mid a\ d\ A' \mid \epsilon$$