

# Parsing

Aditya Thakur

# Top-down parsing

- Constructs parse tree for input string starting from the root
- Finds the leftmost derivation for an input string

Context-free grammar for arithmetic expressions

$$\begin{aligned}E &\rightarrow T E' \\E' &\rightarrow +T E' \mid \varepsilon \\T &\rightarrow F T' \\T' &\rightarrow * F T' \mid \varepsilon \\F &\rightarrow (E) \mid id\end{aligned}$$

Can we derive the string  $id + id * id$ ?

$$E \rightarrow T E'$$

$$E' \rightarrow +T E' \mid \varepsilon$$

$$T \rightarrow F T'$$

$$T' \rightarrow * F T' \mid \varepsilon$$

$$F \rightarrow (E) \mid id$$

$$E \rightarrow T E' \qquad E$$

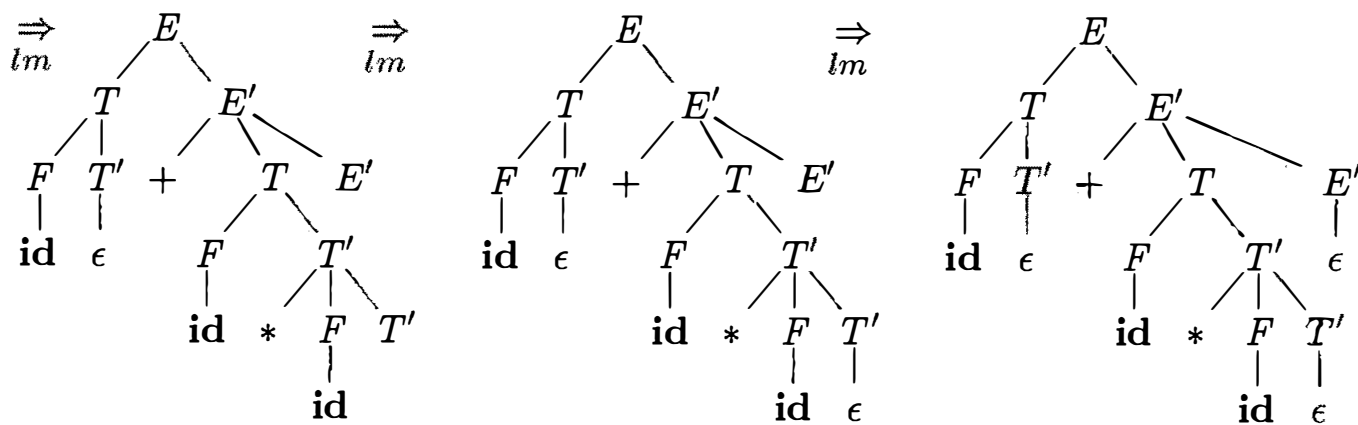
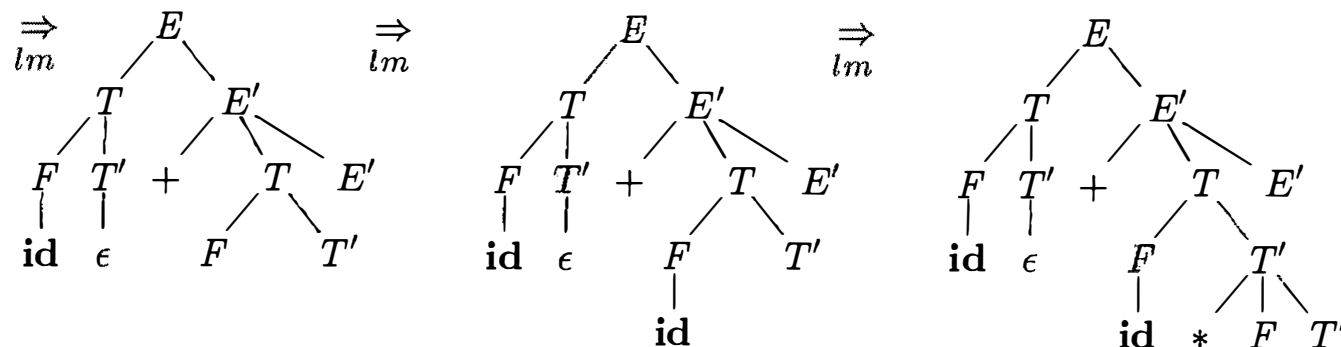
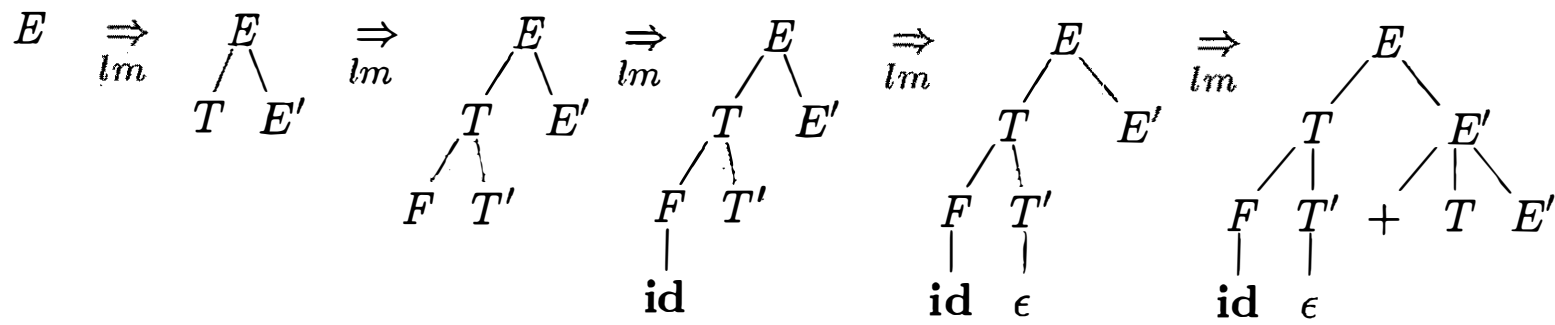
$$E' \rightarrow +T E' \mid \varepsilon$$

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$$\begin{aligned}
E &\rightarrow T E' \\
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T &\rightarrow F T' \\
T' &\rightarrow * F T' \mid \varepsilon \\
F &\rightarrow (E) \mid id
\end{aligned}$$



# Top-down parsing

- Constructs parse tree for input string starting from the root
- Finds the leftmost derivation for an input string
- Two types:
  - Recursive descent parsing
  - Predictive parsing

# Recursive-descent parsing

- Consists of a procedure for each nonterminal
- Execution begins with the procedure for the start symbol

```
void A() {  
1)    Choose an  $A$ -production,  $A \rightarrow X_1X_2 \cdots X_k$ ;    Nondeterministic, might require backtracking  
2)    for (  $i = 1$  to  $k$  ) {  
3)        if (  $X_i$  is a nonterminal )  
4)            call procedure  $X_i()$ ;  
5)        else if (  $X_i$  equals the current input symbol  $a$  )  
6)            advance the input to the next symbol;  
7)        else /* an error has occurred */;  
    }  
}
```

Typical procedure for nonterminal  $A$



# Predictive parser

- Top-down parser that can correctly choose  $A$ -production by looking at at the next  $k$  symbols in the input
- No backtracking during parsing

# LL(1) parser

- Predictive parser that only looks at the next input symbol
- Derives the leftmost derivation

# LL(1) parser

- Constructs parsing table using FIRST and FOLLOW sets of the grammar

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
$E'$		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow \text{id}$			$F \rightarrow (E)$		

# FIRST sets

- $FIRST(\alpha)$  is the set of terminals that begin strings derived from the string of grammar symbols  $\alpha$

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- $FIRST(\alpha)$  is the set of terminals that begin strings derived from the string of grammar symbols  $\alpha$
- If  $X$  is a terminal,  $FIRST(X) = \{X\}$
- If  $X$  is a nonterminal and  $X \rightarrow Y_1 Y_2 \dots Y_k$  is a production, then everything in  $FIRST(Y_1)$  is in  $FIRST(X)$ .  
If  $Y_1$  derives  $\varepsilon$ , then add  $FIRST(Y_2)$  to  $FIRST(X)$ , and so on.  
If  $\varepsilon$  is in all  $FIRST(Y_j)$  then add  $\varepsilon$  to  $FIRST(X)$
- If  $X \rightarrow \varepsilon$  is a production, add  $\varepsilon$  to  $FIRST(X)$ .

# FIRST sets

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$$E \rightarrow T E'$$

$$E' \rightarrow +T E' \mid \varepsilon$$

$$T \rightarrow F T'$$

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	First
E	
E'	
T	
T'	
F	

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E	
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	First
E	
E'	
T	( id
T'	
F	( id

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	First
E	( id
E'	
T	( id
T'	
F	( id

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	First
E	( id
E'	+ $\varepsilon$
T	( id
T'	
F	( id

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- $FIRST(\alpha)$  is the set of terminals that begin strings derived from the string of grammar symbols  $\alpha$

$$E \rightarrow T E'$$

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$$T \rightarrow F T'$$

$$T' \rightarrow * F T' \mid \varepsilon$$

$$F \rightarrow (E) \mid id$$

	First
E	( id
E'	+ $\varepsilon$
T	( id
T'	* $\varepsilon$
F	( id

# FOLLOW

- $FOLLOW(A)$  is the set of terminals that can appear immediately to the right of a nonterminal  $A$  in a derivation

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- $FOLLOW(A)$  is the set of terminals that can appear immediately to the right of a nonterminal  $A$  in a derivation
- $\$$  is in  $FOLLOW(S)$  where  $S$  is the start symbol and  $\$$  is an endmarker
- If there is a production  $A \rightarrow \alpha B \beta$ , then  $FIRST(\beta) \setminus \varepsilon$  is in  $FOLLOW(B)$
- If there is a production  $A \rightarrow \alpha B$  or  $A \rightarrow \alpha B \beta$ , where  $FIRST(\beta)$  contains  $\varepsilon$ , then everything in  $FOLLOW(A)$  is in  $FOLLOW(B)$ .

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$$T \rightarrow F T'$$

$$T' \rightarrow * F T' \mid \varepsilon$$

$$F \rightarrow (E) \mid id$$

	FOLLOW
E	) \$
E'	) \$
T	+ ) \$
T'	+ ) \$
F	+ * ) \$

# Parsing table

- Parsing table  $M: N \times T \rightarrow R$  where  $N$  is the set of nonterminals,  $T$  is the set of terminals,  $R$  is the set of production rules

For each production  $A \rightarrow \alpha$ :

- For each terminal  $a$  in  $FIRST(\alpha)$ ,  $M[A, a] = A \rightarrow \alpha$
- If  $\varepsilon \in FIRST(\alpha)$  and  $b \in FOLLOW(A)$ ,  $M[A, b] = A \rightarrow \alpha$



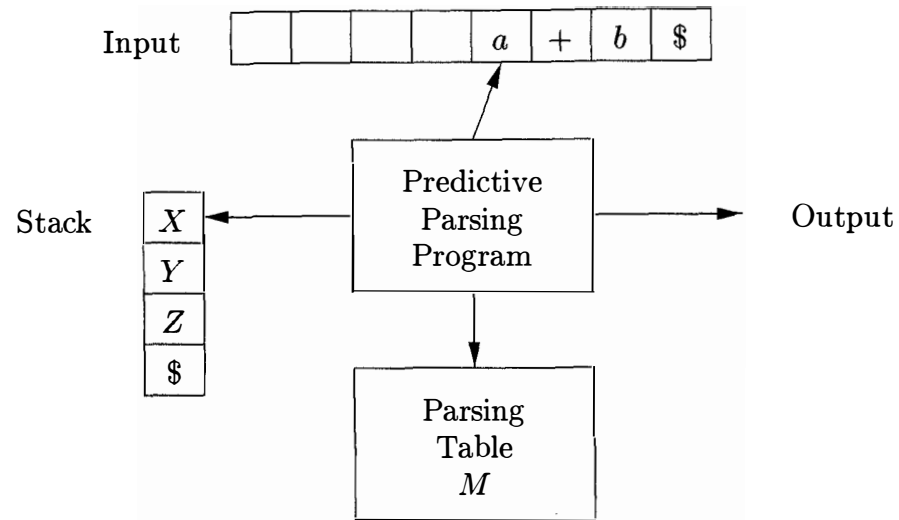
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 F &\rightarrow (E) \mid id
 \end{aligned}$$

	First
E	( id
E'	+ ε
T	( id
T'	* ε
F	( id

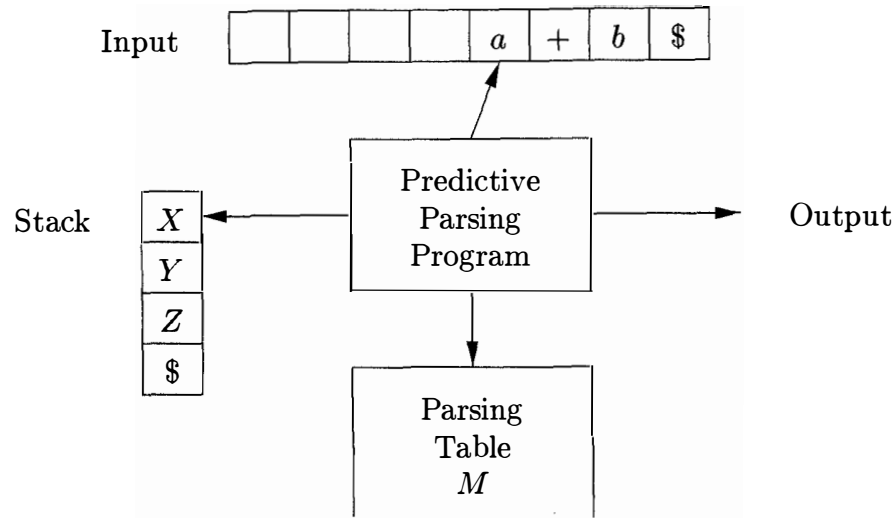
	FOLLOW
E	) \$
E'	) \$
T	+ ) \$
T'	+ ) \$
F	+ * ) \$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
<i>E</i>	<i>E</i> → <i>TE'</i>	<i>E'</i> → + <i>TE'</i>		<i>E</i> → <i>TE'</i>	<i>E'</i> → ε	<i>E'</i> → ε
<i>E'</i>						
<i>T</i>	<i>T</i> → <i>FT'</i>	<i>T'</i> → ε	<i>T'</i> → * <i>FT'</i>	<i>T</i> → <i>FT'</i>	<i>T'</i> → ε	<i>T'</i> → ε
<i>T'</i>						
<i>F</i>	<i>F</i> → id			<i>F</i> → ( <i>E</i> )		

# Table-driven predictive parser



# Table-driven predictive parser



**INPUT:** A string  $w$  and a parsing table  $M$  for grammar  $G$ .

**OUTPUT:** If  $w$  is in  $L(G)$ , a leftmost derivation of  $w$ ; otherwise, an error indication.

```
set  $ip$  to point to the first symbol of  $w$ ;  
set  $X$  to the top stack symbol;  
while (  $X \neq \$$  ) { /* stack is not empty */  
    if (  $X$  is  $a$  ) pop the stack and advance  $ip$ ;  
    else if (  $X$  is a terminal )  $error()$ ;  
    else if (  $M[X, a]$  is an error entry )  $error()$ ;  
    else if (  $M[X, a] = X \rightarrow Y_1 Y_2 \cdots Y_k$  ) {  
        output the production  $X \rightarrow Y_1 Y_2 \cdots Y_k$ ;  
        pop the stack;  
        push  $Y_k, Y_{k-1}, \dots, Y_1$  onto the stack, with  $Y_1$  on top;  
    }  
    set  $X$  to the top stack symbol;  
}
```

Deriving the string  $id + id * id$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
$E'$		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

MATCHED	STACK	INPUT	ACTION
---------	-------	-------	--------

Moves made by the parser

$E\$ \quad id + id * id\$$

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$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

MATCHED	STACK	INPUT	ACTION
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$

Moves made by the parser

Deriving the string  $id + id * id$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
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$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
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$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

MATCHED	STACK	INPUT	ACTION
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$

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$F$	$F \rightarrow id$			$F \rightarrow (E)$		

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	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$
	$id T'E'\$$	$id + id * id\$$	output $F \rightarrow id$

Moves made by the parser

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	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$
	$id\ T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
$id$	$T'E'\$$	$+ id * id\$$	match $id$

Moves made by the parser



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	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
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	$id\ T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
$id$	$T'E'\$$	$+ id * id\$$	match $id$
$id$	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$

Moves made by the parser

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	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$
	$id\ T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
$id$	$T'E'\$$	$+ id * id\$$	match $id$
$id$	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
$id$	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$

Moves made by the parser

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$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

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	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
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	$id\ T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
$id$	$T'E'\$$	$+ id * id\$$	match $id$
$id$	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
$id$	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$
$id +$	$TE'\$$	$id * id\$$	match $+$

Moves made by the parser

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$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
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	$id\ T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
$id$	$T'E'\$$	$+ id * id\$$	match $id$
$id$	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
$id$	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$
$id +$	$TE'\$$	$id * id\$$	match $+$
$id +$	$FT'E'\$$	$id * id\$$	output $T \rightarrow FT'$

Moves made by the parser

Deriving the string  $id + id * id$

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$id$	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
$id$	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$
$id +$	$TE'\$$	$id * id\$$	match $+$
$id +$	$FT'E'\$$	$id * id\$$	output $T \rightarrow FT'$
$id +$	$id\ T'E'\$$	$id * id\$$	output $F \rightarrow id$
$id + id$	$T'E'\$$	$* id\$$	match $id$

Moves made by the parser

Deriving the string  $id + id * id$

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$id +$	$TE'\$$	$id * id\$$	match $+$
$id +$	$FT'E'\$$	$id * id\$$	output $T \rightarrow FT'$
$id +$	$id\ T'E'\$$	$id * id\$$	output $F \rightarrow id$
$id + id$	$T'E'\$$	$* id\$$	match $id$
$id + id$	$* FT'E'\$$	$* id\$$	output $T' \rightarrow * FT'$

Moves made by the parser

Deriving the string  $id + id * id$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
$E'$		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

MATCHED	STACK	INPUT	ACTION
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$
	$id\ T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
$id$	$T'E'\$$	$+ id * id\$$	match $id$
$id$	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
$id$	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$
$id +$	$TE'\$$	$id * id\$$	match $+$
$id +$	$FT'E'\$$	$id * id\$$	output $T \rightarrow FT'$
$id +$	$id\ T'E'\$$	$id * id\$$	output $F \rightarrow id$
$id + id$	$T'E'\$$	$* id\$$	match $id$
$id + id$	$* FT'E'\$$	$* id\$$	output $T' \rightarrow * FT'$
$id + id *$	$FT'E'\$$	$id\$$	match $*$

Moves made by the parser



Deriving the string  $id + id * id$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
$E'$		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

MATCHED	STACK	INPUT	ACTION
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$
	$id\ T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
$id$	$T'E'\$$	$+ id * id\$$	match $id$
$id$	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
$id$	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$
$id +$	$TE'\$$	$id * id\$$	match $+$
$id +$	$FT'E'\$$	$id * id\$$	output $T \rightarrow FT'$
$id +$	$id\ T'E'\$$	$id * id\$$	output $F \rightarrow id$
$id + id$	$T'E'\$$	$* id\$$	match $id$
$id + id$	$* FT'E'\$$	$* id\$$	output $T' \rightarrow * FT'$
$id + id *$	$FT'E'\$$	$id\$$	match $*$
$id + id *$	$id\ T'E'\$$	$id\$$	output $F \rightarrow id$

Moves made by the parser

Deriving the string  $id + id * id$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
$E'$		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

MATCHED	STACK	INPUT	ACTION
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$
	$id T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
id	$T'E'\$$	$+ id * id\$$	match id
id	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
id	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$
id +	$TE'\$$	$id * id\$$	match +
id +	$FT'E'\$$	$id * id\$$	output $T \rightarrow FT'$
id +	$id T'E'\$$	$id * id\$$	output $F \rightarrow id$
id + id	$T'E'\$$	$* id\$$	match id
id + id	$* FT'E'\$$	$* id\$$	output $T' \rightarrow * FT'$
id + id *	$FT'E'\$$	$id\$$	match *
id + id *	$id T'E'\$$	$id\$$	output $F \rightarrow id$
id + id * id	$T'E'\$$	$\$$	match id

Moves made by the parser

Deriving the string  $id + id * id$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
$E'$		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

MATCHED	STACK	INPUT	ACTION
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$
	$id T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
id	$T'E'\$$	$+ id * id\$$	match id
id	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
id	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$
id +	$TE'\$$	$id * id\$$	match +
id +	$FT'E'\$$	$id * id\$$	output $T \rightarrow FT'$
id +	$id T'E'\$$	$id * id\$$	output $F \rightarrow id$
id + id	$T'E'\$$	$* id\$$	match id
id + id	$* FT'E'\$$	$* id\$$	output $T' \rightarrow * FT'$
id + id *	$FT'E'\$$	$id\$$	match *
id + id *	$id T'E'\$$	$id\$$	output $F \rightarrow id$
id + id * id	$T'E'\$$	$\$$	match id
id + id * id	$E'\$$	$\$$	output $T' \rightarrow \epsilon$

Moves made by the parser

Deriving the string  $id + id * id$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
$E$	$E \rightarrow TE'$			$E \rightarrow TE'$		
$E'$		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow FT'$			$T \rightarrow FT'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow id$			$F \rightarrow (E)$		

Parsing table

MATCHED	STACK	INPUT	ACTION
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	output $T \rightarrow FT'$
	$id\ T'E'\$$	$id + id * id\$$	output $F \rightarrow id$
$id$	$T'E'\$$	$+ id * id\$$	match $id$
$id$	$E'\$$	$+ id * id\$$	output $T' \rightarrow \epsilon$
$id$	$+ TE'\$$	$+ id * id\$$	output $E' \rightarrow + TE'$
$id +$	$TE'\$$	$id * id\$$	match $+$
$id +$	$FT'E'\$$	$id * id\$$	output $T \rightarrow FT'$
$id +$	$id\ T'E'\$$	$id * id\$$	output $F \rightarrow id$
$id + id$	$T'E'\$$	$* id\$$	match $id$
$id + id$	$* FT'E'\$$	$* id\$$	output $T' \rightarrow * FT'$
$id + id *$	$FT'E'\$$	$id\$$	match $*$
$id + id *$	$id\ T'E'\$$	$id\$$	output $F \rightarrow id$
$id + id * id$	$T'E'\$$	$\$$	match $id$
$id + id * id$	$E'\$$	$\$$	output $T' \rightarrow \epsilon$
$id + id * id$	$\$$	$\$$	output $E' \rightarrow \epsilon$

Moves made by the parser

# Not all grammars are LL(1)

Grammar

$$\begin{aligned} S &\rightarrow iEtSS' \mid a \\ S' &\rightarrow eS \mid \epsilon \\ E &\rightarrow b \end{aligned}$$

Parsing table

NON - TERMINAL	INPUT SYMBOL					
	<i>a</i>	<i>b</i>	<i>e</i>	<i>i</i>	<i>t</i>	\$
<i>S</i>	$S \rightarrow a$			$S \rightarrow iEtSS'$		
<i>S'</i>			$S' \rightarrow \epsilon$ $S' \rightarrow eS$			$S' \rightarrow \epsilon$
<i>E</i>		$E \rightarrow b$				

Multiple rules possible in the same entry in the parsing table, because of ambiguity.

All LL(1) grammars are unambiguous.

## **Optional reading**

Aho, A. V., Lam, M. S., Sethi, R., & Ullman, J. D. Compilers: principles, Techniques and tools.

Q: What type of parser is used in the Go compiler?