Extra Credit 3 Answer

Due: May 23, 2022 **Points:** 20

Remember, you must *justify all your answers*.

1. (*20 points*) Consider a memory in which contiguous segments $S_1, \ldots, S_n$ are placed strictly in their order of creation from one end of the store to the other. That is, when segment S_{n+1} is being created, it is placed immediately after segment S_n even though some of the segments $S_1, \ldots, S_n$ may have already been deleted. When the boundary between segments (in use or deleted) and the hole reaches the other end of the store, the segments in use are compacted. Let s and t denote the average length and lifetime of a segment (measured in words and memory references). Let f denote the fraction of the memory which is unused under equilibrium conditions. Show that the fraction of time F spent on compacting is constrained by $F \geq \frac{1-f}{1+kf}$, where $k = \frac{t}{2s} - 1$.

   Hint: Find the average speed at which the boundary crosses the memory and assume that copying of a single word requires at least two memory references.

   Answer: The fraction of time spent compacting F is simply the ratio of that time to the total time. Consider a memory of m words after a compaction. The memory occupied by segments is $(1-f)m$; the hole, which trails the area of occupied memory, is fm.

   As segments are created and deleted, the boundary line between the hole at the end of memory and the segments moves towards the end of memory. When that boundary reaches the end, compaction occurs. When does this happen?

   In equilibrium conditions, segments are added and deleted at a constant rate. Hence, every t references, the boundary moves s words closer to the end of memory. So, it reaches the end of memory after $\frac{fmt}{s}$ references.

   Again, in equilibrium conditions, $(1-f)m$ words of memory are being used; hence compaction, which requires 2 references per word, takes at least $2(1-f)m$ references (there may be slightly more, due to overhead). Hence the fraction of time spent compacting is:

$$\begin{aligned}
F &\geq \frac{2(1-f)m}{2(1-f)m + \frac{fmt}{s}} \\
&\geq \frac{2s(1-f)m}{2s(1-f)m + fmt} \\
&\geq \frac{2s(1-f)}{2s(1-f) + ft} \\
&\geq \frac{1-f}{1-f+\frac{ft}{2s}} \\
&\geq \frac{1-f}{1+f(\frac{t}{2s}-1)} \\
&\geq \frac{1-f}{1+kf}
\end{aligned}$$

   where $k = \frac{t}{2s} - 1$, which is the fraction given in the problem.