Extra Credit 1

Due: April 11, 2022 **Points:** 20

Remember, you must *justify all your answers*.

1. The classical batch processing system completely ignores the cost of increased waiting time for users. Consider a single batch characterized by the following parameters:

   - M average mounting time
   - T average service time per job
   - N number of jobs
   - S unit price of service time
   - W unit price of waiting time per user

   Show that the optimal batch size minimizing the cost of service time and waiting time per user within a single batch is

   $$N_{opt} = \sqrt{\frac{MS}{TW}}$$

   Answer: The time to execute a batch is $B = M + NT$. The cost of using the processor for this time while having N users wait is

   $$C = B(S + NW) = MS + (MW + ST)N + TWN^2$$

   So, the cost of having one user wait is

   $$C_{one} = \frac{C}{N} = \frac{MS}{N} + MW + ST + TWN$$

   Recall that C_{one} is either a maximum or minimum when N is chosen so that the first derivative with respect to N is 0:

   $$\frac{dC_{one}}{dN} = TW - MSN^{-2} = 0$$

   Solving for N gives the required value.

   To show this is indeed a minimum and not a maximum, note that

   $$\frac{d^2C_{one}}{dN^2} = 2MSN^{-3} > 0$$

   because $N > 0$ always (if the system has no users, it is not running, and having a negative number of users makes no sense).