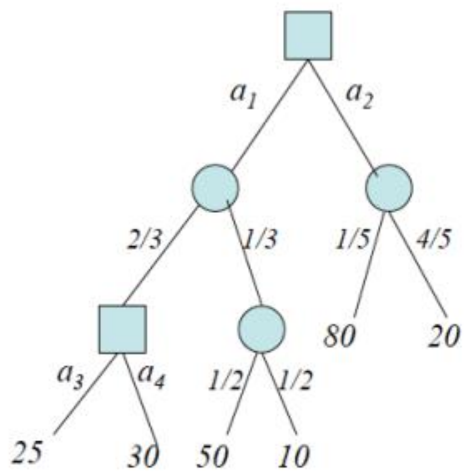
PROJECT 1

Platon Karageorgis,

p3180068

Exercise 1

Consider the following decision tree:



i. If the preferences of the decider are described by the Bayesian criterion, find the normalized utility function π(x).
ii. Find again the normalized function π(x), if we know that it is a 2nd degree polynomial with a, b, c ∈ R as its coefficients and that π(30) = 0.5.
iii. Analyze the decision tree based on the utility function $u(x) = x^2$. Which is the suggested strategy? Update the decision tree accordingly.

Solution

i. Since the decider is choosing based on the Bayesian criterion, we know from theory that the goal is the maximization of the average income. Therefore, we arrange the leaf values in an increasing order in the following way:

10 20 25 30 50 80

And we set π(10) = 0 and π(80) = 1 that help us build these equations:

- 0 = 10 * α + β
- 1 = 80 * α + β

Solving the system, we get that $\alpha = \frac{1}{70}$ and $\beta = -\frac{1}{7}$ and we conclude:

$$\pi(x) = \frac{1}{70} * x - \frac{1}{7}$$

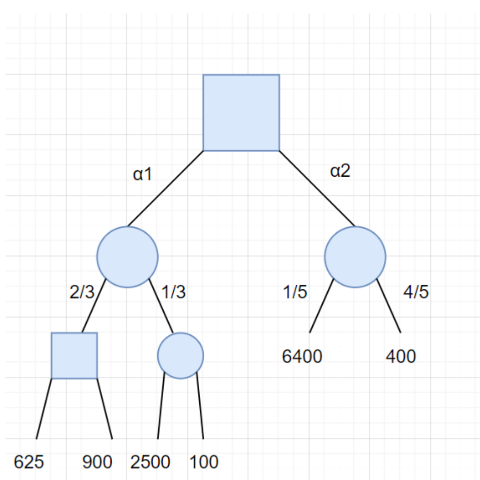
ii. This case is very similar to the previous one, the only difference is that we need to solve a 3x3 system this time. Using the information that π(30) = 0.5 we once again build our equations:

- 0.5 = 900*α + 30*β + γ
- 0 = 100*α + 10*β + γ
- 1 = 6400*α + 80*β + γ

Solving the system, we get that $\alpha = -\frac{3}{14000}$, $\beta = \frac{47}{1400}$ and $\gamma = -\frac{11}{35}$ and once again we conclude:

$$\pi(x) = -\frac{3}{14000} * x^2 + \frac{47}{1400} * x - \frac{11}{35}$$

iii. We will start by attaching the updated tree:



Subsequently, we calculate which is the selected strategy:

- $\alpha 1 = 30^2 * \frac{2}{3} + \frac{1}{3} * \left(\frac{1}{2} * 50^2 + \frac{1}{2} * 10^2\right) = 600 + \frac{1}{3} * (1250 + 50) = 600 + 433 = 1033$
- $\alpha 2 = 80^2 * \frac{1}{5} + 20^2 * \frac{4}{5} = 1600$

So, since the average income of α2 is bigger than α1 we can conclude that the decider will choose α2.

Exercise 2

Find the Arrow-Pratt coefficient τ(x) for the following functions. Assume that x belongs in every case in (0, +oo).

1. $u1(x) = \ln(3 * x + 1)$
2. $u2(x) = 6 * x + 3$
3. $u3(x) = \sqrt{2 * x + 5}$
4. $u4(x) = x^4 + 4 * x$
5. $u5(x) = x^5$

Solution

For each case we will calculate the first and the second derivative. The result will be fed directly to the formula of Arrow-Pratt. Then after calculating every τi(x) we will arrange them based on how fast they approximate 0.

1. $u1'(x) = \frac{3}{3*x+1}$ and $u1''(x) = -\frac{9}{(3x+1)^2}$ which result to:

   $\tau 1(x) = \frac{3}{3*x+1}$

2. $u2'(x) = 6$ and $u2''(x) = 0$ which result to:

   $\tau 2(x) = 0$

3. $u3'(x) = \frac{1}{\sqrt{2*x+5}}$ and $u3''(x) = -\frac{1}{(2*x+5)^{3/2}}$ which result to:

$$\tau3(x) = \frac{1}{2*x+5}$$

4. $u4'(x) = 4*x^3 + 4$ and $u4''(x) = 12*x^2$ which result to:

$$\tau4(x) = \frac{-12*x^2}{4*x^3+4}$$

5. $u5'(x) = 5*x^4$ and $u5''(x) = 20*x^3$ which result to:

$$\tau5(x) = -\frac{4}{x}$$

Now that we have calculated every coefficient, we first separate them according to the strategy they seem to follow. The first and the third case are conservative strategies since τ(x) has a positive sign. Similarly, the fourth and the fifth case are risky strategies since τ(x) has a negative sign. Lastly, the second case is classified as a maximization of the average income since τ(x)=0 which means that the decider is neither risky nor conservative.

Conservative Strategy	**Bayesian Criterion**	**Risky Strategy**
u1	u2	u4
u3	-	u6

Finally, we arrange the coefficients:

u3 <= u1 < u2 < u4 < u5

The first and the third coefficient are considered to be almost identical since they both converge to 0 with the same speed. Asymptotically, 2x+5 and 3x+1 are indifferent. They are followed by the second which is neutral and finally the fourth and the fifth are the most risky behaviors. The fifth is riskier than the fourth since it converges to 0 slower than the fourth.

Exercise 3

An investor has an initial capital K and is thinking of investing the entire amount in stocks. After careful studying, he decides that the rate of the stocks can be modelled like a continuous random variable r.

i. Assume that the utility function of the investor is u(x) = √ x and that the rate r is described by the uniform distribution in [a, b] for some parameters a, b. Calculate the average utility if the investment is completed, in terms of the parameters K, a and b.
If a = -0.05 and b = 0.1, will the investment be granted by the investor?

ii. Now, assume that the utility function of the investor is u(x) = x and the rate r is described by a distribution in [a, b] with a probability density function f(r). If the mean value of r is μ = E[r] and without knowing anything about f, calculate the average utility in terms of K and μ. Comment on how the investor is guided by the parameter μ.

Solution

i. We need to calculate E[u(x)] which is the expected value of u(x).

E[u(x)] = $\int_a^b \frac{1}{b-a} * \sqrt{(1+r) * K}\, dr$

E[u(x)] = $[\frac{\sqrt{K} * (1+r)^{\frac{3}{2}} * 2}{(b-a) * 3}]_a^b$

E[u(x)] = $\frac{\sqrt{K} * (1+b)^{\frac{3}{2}} * 2}{(b-a) * 3}$ - $\frac{\sqrt{K} * (1+a)^{\frac{3}{2}} * 2}{(b-a) * 3}$

Moving on if we substitute for a = -0.05 and b = 0.1 we get:

E[u(x)] = $\frac{\sqrt{K} * (1+0.1)^{\frac{3}{2}} * 2}{(b-a) * 3}$ - $\frac{\sqrt{K} * (1-0.05)^{\frac{3}{2}} * 2}{(b-a) * 3}$

E[u(x)] = $2 * \sqrt{K} * \frac{1.1^{3/2} - 0.95^{3/2}}{0.45}$

E[u(x)] = $2 * \sqrt{K} * \frac{1.153 - 0.925}{0.45}$

E[u(x)] = $\sqrt{K} * \frac{0.456}{0.45}$

Which is a bit bigger than $\sqrt{K}$ and this means that the investment will be granted.

i. In this case we once again need to calculate E[u(x)] but this time the calculation will be parametric.

E[u(x)] = $\int_a^b (1+r) * K * f(r)\, dr$

E[u(x)] = K * ($\int_a^b f(r)\, dr$ + $\int_a^b r * f(r)\, dr$) (1)

From the way the exercise is written we can deduce that:

- $\int_a^b r * f(r)\, dr$ = μ (2)

Also, we know that:

- $\int_a^b f(r)\, dr$ = 1 (3)

So (1) with the additions of (2) and (3) will be:

$$E(u(x)) = K * (1 + \mu)$$

By this result we can say that the decider will invest if and only if μ is greater than 0. In any other case there is no profit according to this utility function and therefore there will not be an investment.

Exercise 4

In New Orleans, after the Katrina hurricane many insurance companies are offering a variety of insurance programs for utter property disaster from a flood, hurricane, fire etc. Assume that a candidate client who possesses a house of value W. There is the possibility that the client buys a similar program that has the following properties:

- The client chooses up to which amount he wants to insure the house, i.e., they can choose to insure for q euros, where q<=W (which is company policy).

- If the client buys this program that covers them q euros, then they will have to pay to the company z*q euros where $0 < z < 1$ (with z being << 1).
- In an utter property disaster scenario happens within the contract time window, the company pays the client q euros. If it does not happen, then the client will just have to pay z*q euros.

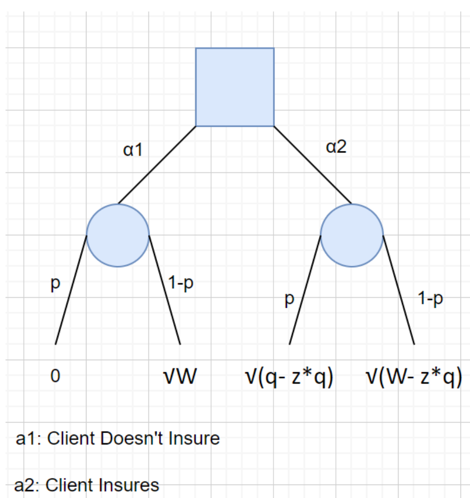
Taking account of the location of the house and the frequency of the hurricanes, the tornadoes and all the other phenomenons that inflict New Orleans from time to time, there is a possibility p, where $0 < p < 1$, that the clients house is utterly destroyed within the time window covered by the program. Assume that the utility function of the client is u(x) = √ x, with x e [0, +00 and x also includes the value of the house, meaning that in case the client is not secured and no disaster happens, his utility will be equal to u(W).

i. Which is the best possible amount q>0 for the client? For now you don't need to check if q<=W, but you need to prove that q is less than W/z. Perform a parametric analysis.

ii. If W = 1,000,000 and p = 0.01 and z = 0.03 which is the best possible q for the client where q e (0,W]? Is it in the clients' interests to get the program for this amount q?

<u>Solution</u>

i. First, we need to design a decision tree that fits the above description:



From this tree we create an equation between the t

We need to find the optimal q that maximizes the following relation:

- $p * \sqrt{q - z * q} + (1 - p) * \sqrt{W - z * q} - (1 - p) * \sqrt{W} = 0$

This equation is extracted from the decision tree, and it is essentially the relation between the average expected income from each case, which is sign the insurance contract or don't. Subsequently, we define a function f:

- $f(q) = \ p * \sqrt{q - z * q} + (1 - p) * \sqrt{W - z * q} - (1 - p) * \sqrt{W}, q \ e \ (0, W]$

The goal is to find the q that maximizes this function. To do that, we first calculate its derivative:

- $f'(q) = \frac{p*(1-z)}{2*\sqrt{q-z*q}} - \frac{z*(1-p)}{2*\sqrt{W-z*q}}$

And we solve:

$$f'(q) = \ 0 \quad \leftrightarrow \quad \frac{p*(1-z)}{2*\sqrt{q-z*q}} - \frac{z*(1-p)}{2*\sqrt{W-z*q}} = 0 \quad \leftrightarrow$$

$$p * (1 - z) * \sqrt{W - z * q} = z * (1 - p) * \sqrt{q - z * q} \quad \leftrightarrow$$

$$p^2 * (1 - z)^2 * (W - z * q) = z^2 * (1 - p)^2 * (q - z * q) \quad \leftrightarrow$$

$$p^2 * (z^2 - 2 * z + 1) * (W - z * q) = z^2 * (p^2 - 2 * p + 1) * (q - z * q) \quad \leftrightarrow$$

$$(p^2 * z^2 - 2 * p^2 * z + p^2) * (W - z * q) \\ = (z^2 * p^2 - 2 * p * z^2 + z^2) * (q - z * q) \quad \leftrightarrow$$

$$p^2 * z^2 * W - 2 * p^2 * z * W + 2 * p^2 * z^2 * q - p^2 * z^3 * q + \ p^2 * W + \ p^2 * z * q = \\ p^2 * z^2 * q - p^2 * z^3 * q - 2 * p * \ z^2 * q + 2 * p * z^3 * q + z^2 * q - \ z^3 * q \quad \leftrightarrow$$

$$p^2 * z^2 * W - 2 * p^2 * z * W + \ p^2 * W = -p^2 * z^2 * q + p^2 * z * q - 2 * p * \ z^2 * q + \\ 2 * p * z^3 * q + z^2 * q - \ z^3 * q \quad \leftrightarrow$$

$$p^2 * z^2 * W - 2 * p^2 * z * W + \ p^2 * W \\ = q * (-p^2 * z^2 + p^2 * z - 2 * p * \ z^2 + 2 * p * z^3 + z^2 - \ z^3) \quad \leftrightarrow$$

$$q = \frac{p^2*z^2*W-2*p^2*z*W+ p^2*W}{-p^2*z^2+p^2*z-2*p* z^2+2*p*z^3+z^2- z^3} \quad \leftrightarrow$$

$$q = \frac{p^2 * W * (z^2 - 2 * z + 1)}{z * (-p^2 * z + p^2 - 2 * p * \ z + 2 * p * z^2 + z - \ z^2)} \quad \leftrightarrow$$

$$q = \frac{p^2 * W * (z - 1)^2}{z * (-p^2 * z + p^2 - 2 * p * \ z + 2 * p * z^2 + z - \ z^2)} \quad \leftrightarrow$$

$$q = \frac{p^2 * W * (1 - z)}{z * (p^2 - 2 * p * z + z)} = q *$$

Now that we have found the critical point, we calculate the second derivative:

- $f''(q) = -(\frac{p*(1-z)^2}{4*(q-z*q)^{\frac{3}{2}}} + \frac{(1-p)*z^2}{4*(W-z*q)^{\frac{3}{2}}})$

It is trivial to observe that f''< 0 for every q e (0,W] since every quantity inside the parenthesis is greater than zero. This means that f is concave and therefore the critical point that we calculated earlier is a total maximum when q e (0,W] and this means that we have found the value that we were looking for and it is q*.

Finally, we will add some extra proof that will ensure that the q* that we found is valid and within the given bounds.

It can be proved that q* belongs in (0,W] in the following way:

- q* > 0 $\rightarrow$ $\frac{p^2*W*(1-z)}{z*(p^2-2*p*z+z)} > 0$

We have $p^2,\ W,\ z,\ 1 - z\ >\ 0$ and we need to prove that $p^2 - 2 * p * z + z > 0$ which is trivial since $0\ <\ p^2 - 2 * p * z + z\ <\ p^2 - 2 * p * z + z^2$ when we have 0 < z << 1.

Also, the exercise wants us to prove that q* is less than W/z and this can be proved quite easily without even calculating q*. This is because the insurance company demands that q <= W and therefore, since z is a value between 0 and 1 it must stand that every q must be less than W/z. Nevertheless, we will prove with a few simple steps that q* is indeed less than $\frac{w}{z}$. Let's assume that this statement is not true:

- $q* \geq \frac{W}{z} \quad \rightarrow \quad \frac{p^2 * W * (1-z)}{z*(p^2 - 2*p*z + z)} \geq \frac{W}{z} \quad \rightarrow \quad p^2 - 2*p*z + z \leq p^2 * (1-z) \rightarrow$

$p^2 * z + 2*p*z - z \leq 0 \quad \rightarrow \quad p^2 + 2*p - 1 \leq 0 \quad \rightarrow \quad (p-1)^2 \leq 0$

Which can't be true since it's a positive value and in the case of equality the probability that a disaster happens has to be equal to 1. Therefore, q* must indeed be less than $\frac{w}{z}$.

ii. All we must do in this case is replace the real values and see if the equation that we set in the beginning stands. But before that, we will calculate q:

$$q = \frac{10^{-4} * 10^{6} * 0.97}{3 * 10^{-2} * (10^{-4} - 6 * 10^{-4} + 3 * 10^{-2})} = \frac{10^{6} * 0.97}{3 * (0.01 - 0.06 + 3)} = \frac{10^{6} * 0.97}{8.85} = 10^{6} * 0.11$$

So, we get that $q = 10^{6} * 0.11$

Replacing this value along with those for z, p and W in the initial relation:

$$p * \sqrt{q - z * q} + (1-p) * \sqrt{W - z * q} - (1-p) * \sqrt{W} \geq 0 \quad \leftrightarrow$$

$$10^{-2} * \sqrt{10^{6} * 11 * 10^{-2} - 3 * 10^{-2} * 10^{6} * 11 * 10^{-2}} + 99 * 10^{-2} * \sqrt{10^{6} - 3 * 10^{-2} * 10^{6} * 11 * 10^{-2}} - 99 * 10^{-2} * \sqrt{10^{6}} \geq 0 \quad \leftrightarrow$$

$$10^{-2} * \sqrt{11 * 10^{4} - 33 * 10^{-2}} + 99 * 10^{-2} * \sqrt{10^{6} - 33 * 10^{4}} - 99 * 10^{-2} * 10^{3} \geq 0$$

$$10^{-2} * \sqrt{10.67 * 10^{4}} + 99 * 10^{-2} * \sqrt{0.67 * 10^{6}} - 990 \geq 0 \quad \leftrightarrow$$

$$10^{-2} * 10^{2} * 3.27 + 99 * 10^{-2} * 0.82 * 10^{3} - 990 \geq 0 \quad \leftrightarrow$$

$$3.27 + 811.8 - 990 \geq 0 \quad \leftrightarrow$$

$$-174.93 \geq 0$$

So ultimately, we get that it's not in the client's interests to agree to this contract and therefore he will choose not to insure.

Exercise 5

A bank is thinking of investing an amount of 100,000 euros. The options are the following:

- Option α0: The bank won't invest the money anywhere.
- Option α1: The bank will loan the money to a client with a 25% interest rate for 1 year.
- Option α2: The bank will invest the money in stocks. The probability is 0.4 that the rate will be 10% and 0.6 that it will be -5%.

The Executive Officer of Loan Approvals is doing research into how trustworthy is the client that will be granted the loan and estimates that the probability that the bank gets back the money will be 75%. We assume that the rest of the probability will give no money whatsoever to the bank within that year. Also, assume that the trustworthiness of the client and the rate of the stock are independent.

The utility function for the bank:

$$u(x) = \begin{cases} -x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

i. If we assume that the bank will choose only one of α0, α1, α2 then what should the final decision of the bank be if the utility function is the above u(x) and they are solely interested in the total amount that they will get this year? If it exists, include the decision tree.

ii. If we assume that the bank will choose to invest an amount in α1, meaning that they might approve a lesser amount for the client's loan and the rest will be invested in α2 (this means that the whole amount will be invested), how should the bank invest the amount if the utility function is the above u(x) and they are solely interested in the total amount that they will get this year? If it exists, include the decision tree.

   In the following sub-exercises, we will use as a decision criterion the maximization of the expected income and not the expected value of u.

iii. If we assume that the bank will choose exactly one of α0, α1, α2 then design the decision tree and determine the bank's strategy if we assume that the bank will follow the criterion of the maximization of the expected income that they will get by the end of the year. Update the decision tree accordingly.

Assume that the bank has the ability to ask for a fully detailed trustworthiness report of the client that costs 5000 euros, since it demands for extra data. The outcome of this report can be either positive (Θ) meaning that the money will be paid back or negative (A) and the predictions of this report have 90% accuracy. This means that if the client is trustworthy (Φ) (which means that he will pay back) then the probability that the outcome of the report is (Θ) is 0.9, while on the other hand if the result is (A) then the probability will be 0.1. Similarly, if the client is not trustworthy (AΦ) (which means that he will not pay back) then the probability that the research was negative (A) will be 0.9 and the probability that it was positive (Θ) will be 0.1. We consider, as in the previous case, that if the client is not trustworthy it means that the bank will not get 0% of the amount back within that year. In the three following sub-exercises, we assume that the bank will follow the criterion of maximization of the expected income that the bank will get at the end of that year.

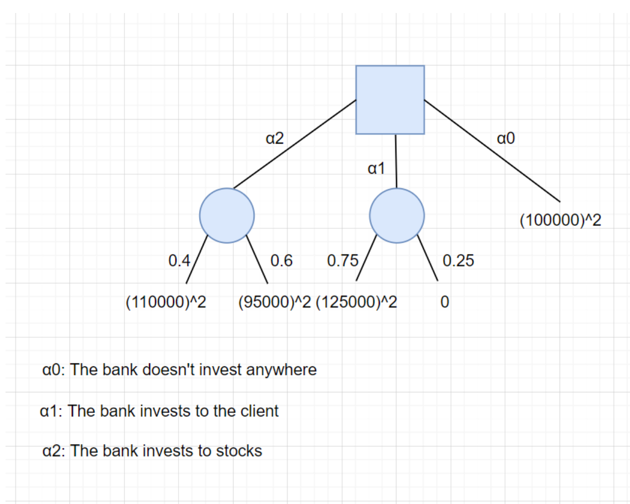
iv. Design the decision tree that includes the selection of further research and determine the bank's strategy. Fully update the decision tree if necessary.

v. Which is the maximum price that the bank is willing to pay for this research?

vi. What is the value of perfect intelligence? (Include all the random variables of the problem)

<u>Solution</u>

i. We start by designing the decision tree:



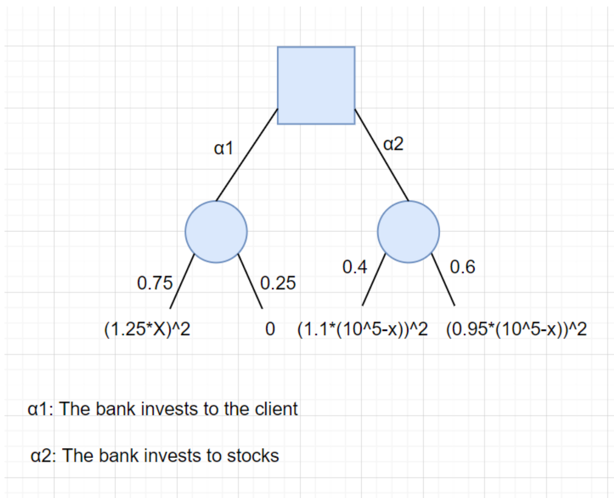
Then, we calculate the average expected income for each case:

- $\alpha 0 = {10^5}^2 = 10^{10}$
- $\alpha 1 = 0.75 * (10^5 + 25 * 10^3)^2 + 0.25 * 0 = 0.75 * 15625 * 10^6 = 1.711875 * 10^{10}$
- $\alpha 2 = 0.4 * (10^5 + 10^4)^2 + 0.6 * (10^5 - 5 * 10^3)^2 =$

  $= 0.4 * (12100 * 10^6) + 0.6 * (9025 * 10^6) = 1.0255 * 10^{10}$

So, it seems that the bank will choose to invest in the client asking for a loan in this case.

ii. Again, we start by designing the decision tree:



Then, we calculate once more the average expected income for each case:

- $\alpha 1 = 0.75 * (1.25 * x)^2 + 0.25 * 0 = 1.171875 * x^2$
- $\alpha 2 = 0.4 * \left(1.1 * (10^5 - x)\right)^2 + 0.6 * \left(0.95 * (10^5 - x)\right)^2 = 1.0255 * (x - 10^5)^2$

The sum of these α1, α2 represents the total expected income. We will define a function g and we will find the x that maximizes it:

- $g(x) = 1.171875 * x^2 + 1.0255 * (x - 10^5)^2, \; x \, e \, [0, 100.000]$

The first derivative of g:

- $g'(x) = 2.34375 * x + 2.051 * (x - 10^5)$

Next, we search for the critical points:

$g'(x) = 0 \quad \leftrightarrow \quad 2.34375 * x + 2.051 * (x - 10^5) = 0 \quad \leftrightarrow$

$1.171875 * x + 2.051 * x - 1.0255 * 10^5 = 0 \quad \leftrightarrow$

$2.197375 * x = 1.0255 * 10^5 \quad \leftrightarrow$

$x = 0.4667 * 10^5$

Moving on, we calculate the second derivative of g:

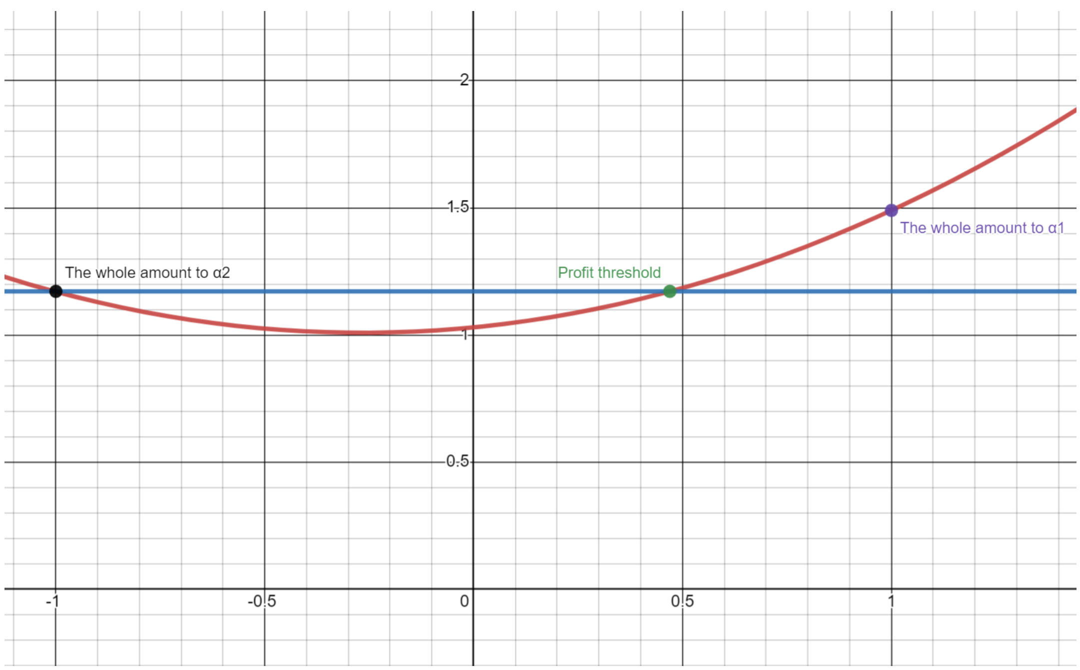
- $g''(x) = 4.39475$

Which is clearly positive for every x since its constant. Therefore, the critical point we found before is the total minimum. This means that the function is maximized for either $x = 0 \ or \ x = 10^5$. If we plug these values in g we get:

- $g(0) = 1.0255 * 10^{10}$
- $g(10^5) = 1.171875 * 10^{10}$

So finally, this means that the bank prefers to give the entire amount for a loan and give nothing to stocks. This is a point where the exercise is not exactly clear since it does not state if it is mandatory to invest in both options. This is why we will give an extra solution.

Assume that we had to invest at least 1 euro in both options and of course invest in total the entire amount. We will add a graph that will help with the explanation.

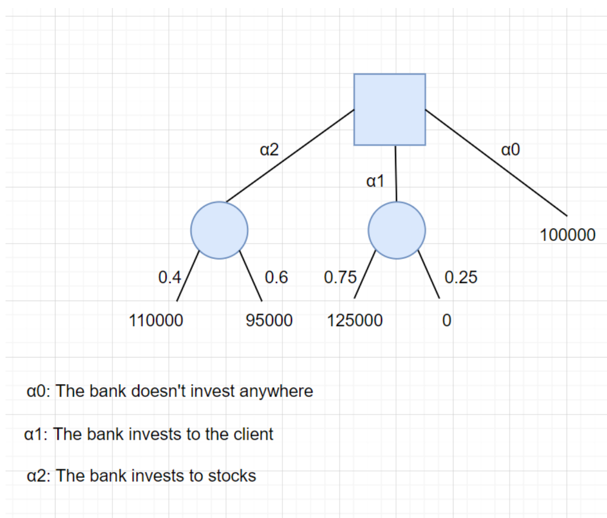


In the above graph we see in red our function g and we also see 3 points. The black point is the case where the entire percentage of the money goes to α2 and similarly the purple point is the case where the entire percentage of the money goes to α1. The graph supports our case, since obviously the maximum is the purple point (in the graph where x=1 it means that we have invested the entire amount and x=-1 means that we invested the entire amount to the opposite party, so we ignore the plot outside of [-1, 1]).

If the bank wanted to invest in both cases, then they would want to invest in the next best possible case. So, they would still invest the biggest proportion to α1 because as we proved it's the most profitable, but they would choose an amount that would go to α2. That amount will also be subtracted from the money that will be invested in α1.

What we just described is the interval between the green point and the purple point. The green point is where the average expected income becomes exactly equal to giving all the money to α2, which means that if we keep decreasing the proportion that goes to α1 then we will never achieve anything better. So, the proportion that the bank will choose to redirect from α1 to α2 will be the least possible - since the more they invest to α1 the more the expected income rises – and that amount will belong in the interval between the green point and the purple point.

iii. We have already designed the tree for this case but before it included the values that were influenced by the utility function. The exact decision tree that the exercise wants this time is:

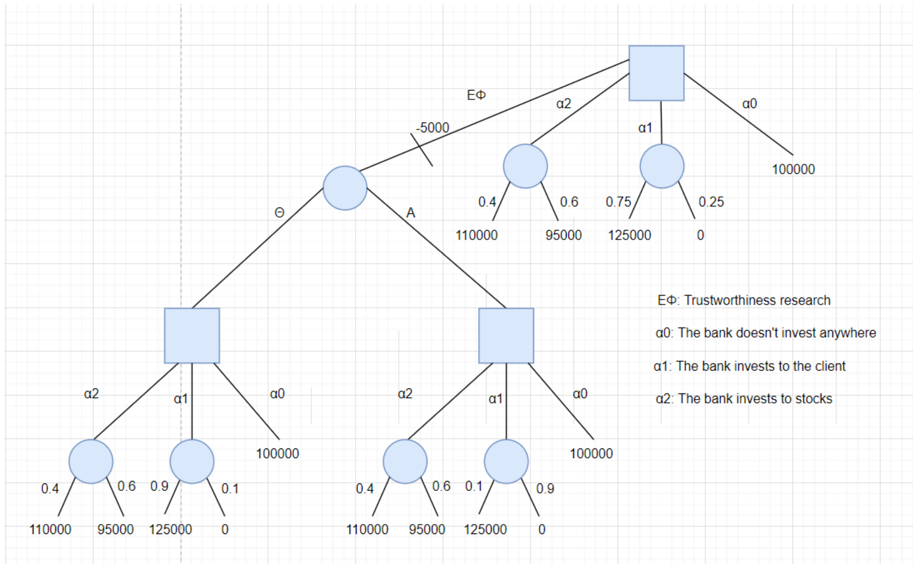


The expected average income for each branch:

- $\alpha 0 = 10^5 = 100000$
- $\alpha 1 = 0.75 * 125000 + 0.25 * 0 = 75 * 1250 = 93750$
- $\alpha 2 = 0.4 * 110000 + 0.6 * 95000 = 4 * 11000 + 6 * 9500 = 101000$

So, by using the exact values that achieve the maximization of the expected income the bank ends up investing in α2. We also observe that α1, which was the first option before, is now the least preferable.

iv. The tree that the exercise wants us to design is the following:



To calculate the best possible option for the bank we need to calculate the average expected income of new option EΦ.

- For the left subtree:
  - $a0 = 100{,}000$
  - $a1 = 0.9 * 125{,}000 + 0.1 * 0 = 112{,}500$
  - $a2 = 101{,}000$

So, we keep α1 from the left subtree.

- For the right subtree:
  - $a0 = 100{,}000$
  - $a1 = 0.1 * 125{,}000 + 0.9 * 0 = 12{,}500$
  - $a2 = 101{,}000$

So, we keep α2 from the right subtree.

Next, we calculate EΦ we finally need the probabilities P(Θ) and P(A).

- $P(\Theta) = P(\Theta|\Phi) * P(\Phi) + P(\Theta|A\Phi) * P(A\Phi) = 0.9 * 0.75 + 0.1 * 0.25 = 0.7$
- $P(A) = 1 - P(\Theta) = 0.3$

Using all the above we get:

- $E\Phi = 0.3 * 101{,}000 + 0.7 * 112{,}500 - 5000 = 109{,}050 - 5000 = 104{,}050$

Ultimately, the bank will choose to do trustworthiness research since the average expected income in this case is bigger than the other cases.

v. The bank is willing to fund the research with up to 8,500 euros. That is because if they don't use the research at all, the best expected average income is 101,000 like we calculated before and the total benefit of the research has an expected average income of 109,050. This means that if we subtract these values, we get 8,500, which is the maximum amount that the bank is willing to pay. We can see that 5000 is less than 8,500 and that is why they chose to make the research in the first place.

vi. We can calculate the value of the optimal intelligence in the following way:

- $EVPI = 0.75 * 125{,}000 + 0.25 * 101{,}000 - 101{,}000 = 93750 + 25{,}250 - 101{,}000 = 18000$

The explanation behind this calculation is that if we had the perfect intelligence, then we would know exactly when the client is trustworthy and when they are not. The client is trustworthy with probability 0.75 so with this probability we would loan the money. But since we know the truth now, we won't have to suffer the loss of not getting our money back with the rest 0.25. Instead, with that probability, we choose to invest somewhere else, and we choose the next best global option, which is α2, and it has an expected average income equal to 101,000. Ultimately, we sum these two values, and we subtract 101,000 which is the best option without any kind of information. It should be noted that we don't subtract 104,050 which is the best global option in general when we also use research, because in this question we are evaluating the best information. To do that fairly we need to compare it with a baseline that doesn't contain any kind of information, and research is a type of intelligence, just not the perfect one.