ASSIGMENT

a) Consider the problem of fitting a polynomial of degree d to data points $(x_i,\ y_i),\ i = 1,\ \dots\ ,\ m$ in the plane. Let the polynomial be written $p(x) = \sum_{j=0}^{d} a_j x^j$.
Choose $d = 14,\ a_j = 1,\ j = 0,\ \dots\ ,\ 14$ and 21 equally spaced points on the interval $[0,\ 1]$.
Compute $y_i = p(x_i),\ i = 1,\ \dots\ ,\ 21$.
Now we want to recover the coefficients a_j by solving the overdetermined system $A\hat{a} = y$, where the columns of A are powers of the vector x. Compare the error $\|a - \hat{a}\|_2$ in the following methods for solving the system of equations:
1. The normal equations
2. QR factorization
3. A regularization method defined like this: QR factorization of the enlarged system $\begin{bmatrix} A \\ \alpha I \end{bmatrix} = QR$ and then $\hat{a} = R \backslash Q^T \begin{bmatrix} y \\ 0 \end{bmatrix}$ with the properly chosen value $\alpha = 10^{-7}$. Discuss the effect of a smaller or a larger value of α.
4. Truncated least-squares with the properly chosen number of 12 terms in the SVD of A. Discuss the effect of more or less terms in the SVD.

Draw conclusions on the results and try to explain the differences between the methods.

b) Use QR factorization to solve the following problem as a least-squares problem with linear constraints: Fit a quartic (a fourth degree) polynomial to the points (1,1) (2,2) (4,3) (5,4) (6,5) (7,4) (9,5), and (10,6) with the restriction that the curve has to pass exactly through the points (4,3), (6,5) and (10,6). Plot points and the fitting curve. Also, compare with the least-squares approximation without constraints. Which of the fits seams the more reasonable?

Note about grading: To obtain the highest grade on this exercise you should write as efficient codes as you masters, for instance without explicit inverses.