Simulation

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Assignment 6

① Algorithm for Gibbs

- Start with a random (x_0, x_1, x_2) point
- repeat until you get N samples:
  - Calculate $p(x_0 | x_1, x_2)$
  - Calculate $p(x_1 | x_0', x_2)$
  - Calculate $p(x_2 | x_0', x_1')$
  - If point (x_0', x_1', x_2') is to be discarded because of burn-in or thinning then toss it and repeat
  - Else store it and repeat

The only tricky part for the above problem was to calculate correctly Σ_{11}, Σ_{12}, Σ_{22}^{-1}, Σ_{21} from the covariance matrix. Each time we calculate a probability (i.e. $p(x_0 | x_1, x_2)$) we get the correct values to build these matrices, we generate μ and sigma and we get a sample from the normal distribution which we scale by these values.

② Algorithm for Metropolis

- Start with a random point and initiate the chain
- Feed the point to q which is the Multivariate normal distribution with mean = point and covariance equal to the identity 3x3 matrix
- Calculate the ratio $\frac{\pi(Y)}{\pi(X)}$ where π is the target distribution. We don't need $\frac{q(X|Y)}{q(Y|X)}$ since the multivariate normal distribution is symmetric (essentially this is random walk)
- Get the min probability of $\{1, \frac{\pi(Y)}{\pi(X)}\}$

- Set $x+1 = x$ with probability a
- Otherwise with probability $1-a$ set $x+1 = x$
- Like before stack the sample only if burn-in and thinning allow it
- Repeat the whole process until you get 1000 samples

(3) Algorithm for Metropolis within Gibbs

The algorithm is identical with the one of (1) with a difference in the calculation of x_1 in (x_0', x_1, x_2). When we reach that point we call Metropolis.
The $q(x)$ of Metropolis is the Normal distribution with mean = x_1 and covariance ≡ std ≡ 1.
The $\pi(x)$ of Metropolis changes in every iteration since x_0', x_2 keep changing. It is the Normal distribution scaled by mu and sigma which are calculated like in (1)
After this point, we have successfully completed a step in our Markov chain and we resume the Gibbs algorithm by returning (x_0', x_1', x_2) where x_1' is our step.