ELEN 4810 Homework 1

Due **Wednesday, November 4**. Please submit your responses via CourseWorks. Please combine your responses to the analytical and computational problems in a single pdf file.

For the analytical section of the assignment, submit a single PDF of name `yourunihere hw01.pdf` - e.g., "`abc1234 hw01.pdf`". For the coding problems. please put your code in a single script file `yourunihere hw01.m` - e.g., "`abc1234 hw01.m`". On execution, the two aforementioned figures and each the with six plots (or stem plots) should pop out. Also, use the Publish option on MATLAB to create a PDF that will publish your script as well as its figure outputs. Please submit this as `yourunihere hw01_m.pdf` - e.g., "`abc1234 hw01_m.pdf`"

Thanks!

Analytical Questions

Please complete problems 2.23, 2.43, 2.49 in Oppenheim and Schafer (3rd Edition). Justify your answers!

Computational Questions

1 Get Familiar with MATLAB

Problem Statement. This problem will ask you to practice a few basic operations in MATLAB. Generate 3 signals and use the `stem()` function of MATLAB to demonstrate your result. Although the `plot` function often generates smoother and better looking figures, the `stem()` function is in fact the right representation for the discrete-time signals (*why?*).

Generate the three sequences

$$x_1[n] = \sin\left(\frac{6n\pi}{N} + \frac{4\pi}{3}\right), \tag{1}$$

$$x_2[n] = \exp\left(\frac{j6n\pi}{N}\right), \tag{2}$$

$$x_3[n] = \frac{\sqrt{3}}{2}\mathrm{Re}\left(x_2\left[n\right]\right) + \frac{1}{2}\mathrm{Im}\left(x_2\left[n\right]\right), \tag{3}$$

over the set of integers $n \in \{-30, \ldots, 30\}$ with $N = 60$. As a reminder, the MATLAB variable for π is simply `pi`. A few things to take note of:

- x_1 is real valued, whereas x_2 is complex valued.

- MATLAB indexes arrays with $1, 2, \ldots$, i.e. `array(1)`, `array(2)`, etc. So to keep track of the "time" $\{-30, \ldots, 30\}$ a seperate array needs to be created.

Although complex signals are a mathematical device, they provide a concise representation for sinusoids and in some situations have very direct applications – e.g., the *quadrature receiver* in digital communication systems.

Submission Requirements. Please write a script that draws the following six plots with the `stem()` function in Matlab:

$$x_1, \quad \mathrm{Re}(x_2), \quad \mathrm{Im}(x_2), \quad |x_2|, \quad \angle x_2, \quad x_3. \tag{4}$$

Put all six plots in *one single figure* with function `subplot()`, and append an appropriate title, x-axis label and y-axis label *for each subplot* with functions `title`, `xlabel` and `ylabel`. For instance, the plot of x_1 should be appended with x-axis label `n`, y-axis label `x_1[n]`, and title `x_1`.

Please use the `'.'` option for the `stem()` function. We emphasize that the x-axis on the stem plots should keep track of sample indexing, i.e. the "time" $\{-30, \ldots, 30\}$.

Observations. *No submission required for these questions.*

- Check $x_1[-30]$ and $x_1[30]$, what do you find and why?
- Check the generated sequence x_1 and x_3, what do you find and why?

2 Practicing Discrete Convolution

Problem Statement. Please define the LTI systems (or filters) with impulse responses

$$h_1[m] = \begin{cases} 1 & 2 \le |m| \le 6 \\ 0 & \text{otherwise} \end{cases}, \tag{5}$$

$$h_2[m] = \begin{cases} -1 & m = 0 \\ 1 & m = 1 \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

$$h_3[m] = \begin{cases} \exp\{-\frac{m^2}{2\sigma^2}\} & m = -10, \ldots, 10 \\ 0 & \text{otherwise} \end{cases}, \tag{7}$$

over the set of integers $m \in \{-10, \ldots, 10\}$ with $\sigma = 2$ for the last system h_3. Now utilize the signal x_1 generated in the last question to convolve x_1 with the three systems h_1, h_2, h_3 to obtain output signals y_{11}, y_{12}, and y_{13} respectively.

Please use the MATLAB function `conv()` to output the *full convolution*, i.e. the default or `'full'` setting.

Submission Requirements. In the *same script*, produce *a single, seperate, figure* with six *stem plots* using the `stem()` function (again using the `'.'` option) from your output. Please generate these six stem plots of following signals

$$h_1, \quad y_{11}, \quad h_2, \quad y_{12}, \quad h_3, \quad y_{13}. \tag{8}$$

and append an appropriate x-label, y-label and title as before.

Please be extremely careful with correct indexing of output sequences, *it is not trivial.* As a toy example, consider a system characterized by $h[m] = \delta_{10}[m]$ was convolved with x_1 to delay it by 10 steps. At what "time" would the output end? Similarly, if a system characterized by $\delta_{-10}[m]$ is used to bring x_1 forward by 10 steps, when would the output begin? In general, for what index range whould one plot over for the output? Alternatively, a lazier way to find the correct indexing is to note that $\delta_0\{x\} * \delta_0\{h\} = \delta_0\{y\}$.

Observations. *No submission required for these questions.*

- What is the length of output sequence? How is it related to the length of the input sequence and length of filter?
- During the class we mentioned that sinusoids are *eigenfunctions* to LTI systems, what do you see in the output compared to the input sequence?
- What happens to y_{13} as $\sigma \to 0$? Why?

Code Submission

Please put your code in a single script file `yourunihere_hw01.m` - e.g., ``` ``abc1234_hw01.m'' ```. On execution, the two aforementioned figures and each the with six plots (or stem plots) should pop out. Upload your code to Courseworks before the start of class on **Monday, September 23**.