Consider a Lagrangian consisted of N real scalar fields ϕ_i,

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{N} \partial_\mu \phi_i \partial^\mu \phi_i - V(\phi^2)\,,$$

where $\phi^2 \equiv \sum_{i=1}^{N} \phi_i \phi_i$. Answering the following questions:

- Show that the Lagrangian enjoys a $SO(N)$ global symmetry.
  (Hint: It means that you need to find the infinitesimal transformations of the $SO(N)$ symmery on the fields ϕ_i, verify the Lagrangian is indeed invariant respect to these transformations, and compute the commutation relation of these generators. It would be helpful to first understand how many independent transformations you could have, and thus find proper basis to express them. Recall what I have shown to you the generators of $SO(1,3)$ in the class.);

- Write down the currents as well as the charges respect to the $SO(N)$ symmetry;

- Use canonical quantization to quantize the system, spell out the Hamiltonian, and verify that the charge operators found in the previous subquestion indeed commute with the Hamilton;

- Further verify that the charge operators themselves satisfy the same commutation relation of the $SO(N)$ symmetry as they should be.