I. INTRODUCTION:

A. Motivation: Why QFT?

- Particle physics:

* Laws of NATURE require the action is <u>LOCAL</u>.

Force felt by electron cannot be simultaneous when one moves the positron.

$\Longrightarrow$ Concept of "FIELD" (Maxwell, Einstein)

$\{\vec{E}, \vec{B}\}$ electromagnetic fields

$\rightsquigarrow$ Special Relativistics

* Laws of NATURE require "objects" are eventually

QUANTIZED.

Many observations, thought experiments, theoretical arguments (ultraviolet catastrophe, photonelectic effects ..)

$\Rightarrow$ Concept of "QUANTA"

$\rightsquigarrow$ Quantum Mechanics (Schrödinger, Heisenberg, plank, Bohr, Einstein ..)

S.R. Q.M.

"Marriage"

"THEORY OF QUANTIZED FIELDS."

Many Implications:

1. Particle number is NOT conserved.

Consider a particle of mass m in Box of size L.

Heisenberg's uncertainty Principle. $\rightsquigarrow \Delta p \geq \hbar/L$

Relativistics $\rightsquigarrow (E, p)$ are on an equal footing

$$\Rightarrow \quad \Delta E \geq \hbar c/L.$$

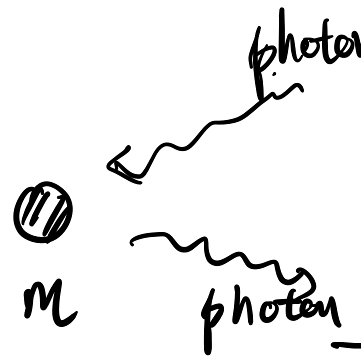
So starting from a Box of NOTHING.

if $\Delta E \geq \hbar c/L \geq 2mc^2$

$\rightsquigarrow$ a pair of particle & anti-particle.

$$|0\rangle \longrightarrow |p, \bar{p}\rangle$$

On the other hand,



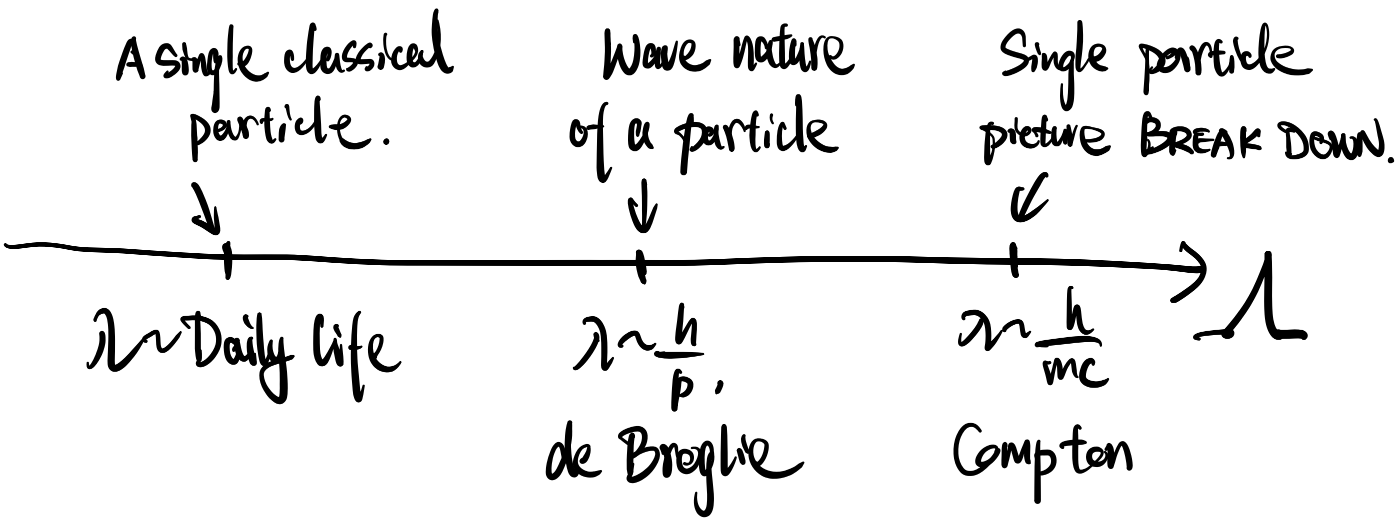
$$E = h\nu \geq mc^2, \quad h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda \leq \frac{h}{mc}$$

For $\lambda \lesssim \frac{h}{mc}$ (Compton length), highly possible to produce a new particle of mass m.

$\Rightarrow$ A single particle picture is broken down.

Energy scale. $\Lambda \sim \lambda^{-1}$



Remark: any attempt to write down a relativistic single particle Schrödinger Equation will encounter serious problems. (Negative possibility, break down of Causality, etc.)

2. All particles of the same type are

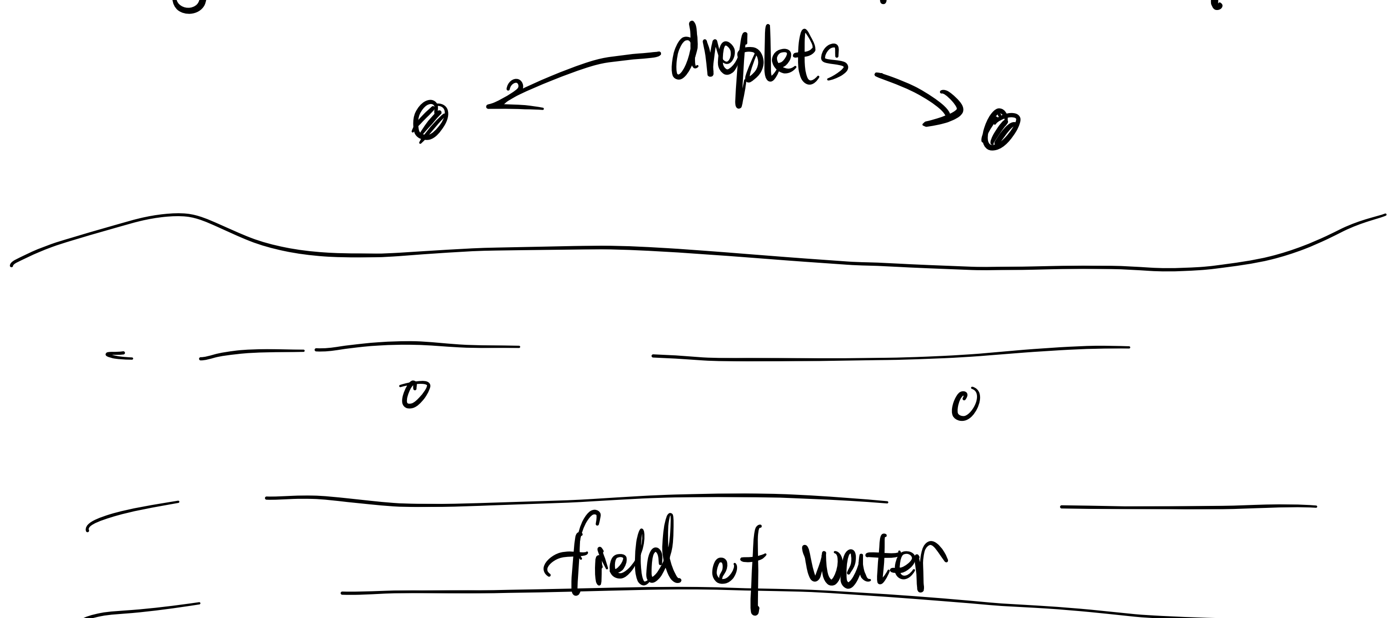
THE SAME.

For example, all electron are the same.

No matter They are at earth or Somewhere.

in the universe. Why ?

A anologue situation is droplets out of sea.



⤳ Imagine the universe is filled by

the "sea of electrons" (field of electrons)

Observe electrons at different positions is to excavate (excite) two electrons (quantum states) out of the Dirac sea (vaccum).

In addition, quantum mechanically, the "same" means interchanging the two particle is indistinguishable, up to a "±" sign (corresponding to Boson/Fermi statistics.)

In QM, the statistics are imposed by hand, while in QFT, we'll show this point from "FIRST PRINCIPLE".

In sum, people found $e^+, e^-, \gamma, \ldots$

S.R. + G.M.

QFT ← Defined in the UV regime (High energy)

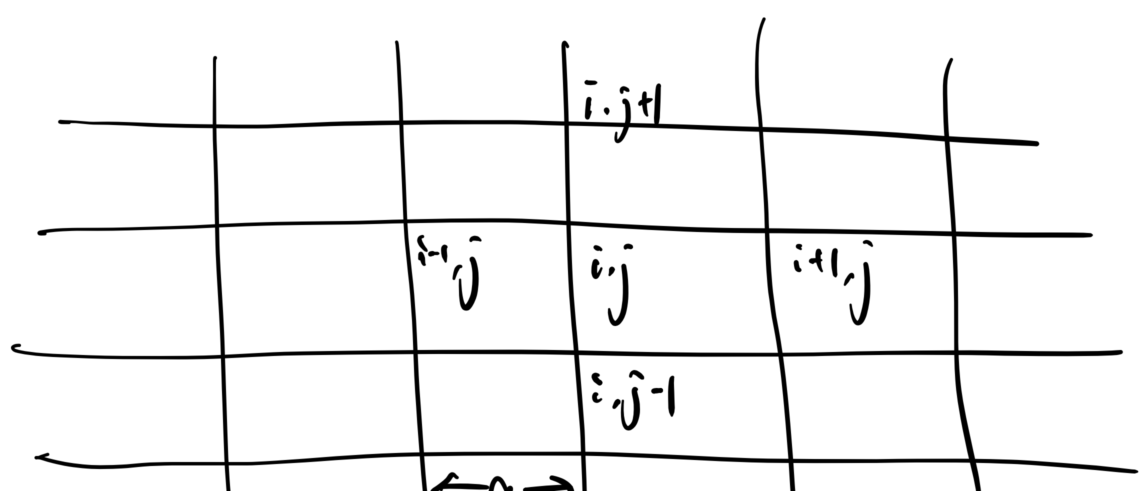
Many problems, e.g. renormalizations

Renormalizable QFT (Very restrictive)

Standard Model.

— Condense Matter / Statistical Mechanics.

e.g. Ising Md. is defined on a lattice



$s_{i,j} \in \{-1, 1\}$ c-number.

Partition Function :

$$Z(J) = \sum_{\{s_{\{i\}}\}} e^{-J \sum_{\langle\{i\},\{j\}\rangle} S_{\{i\}} S_{\{j\}}}.$$

adjacent neigher.

all possible configuration.

d=2 is solved Onsager.

d>2 NOT KNown yet. very hard. problem.

* For $J \gg 1$, 2 ground state $S_{\{i\}} = \pm 1$ for all i

* J is generic, $\langle S_{\vec{n}} S_{\vec{n}+\vec{x}} \rangle \sim e^{-|\vec{x}|/\xi}$.

ξ - correlation length. $|\vec{x}| \gg \xi$

$\langle \cdots \rangle \to 0$

$\xi \sim$ one lattice site, i.e. $\xi \sim a$.

* $\exists J_{critical}$. $J \to J_c$ $\xi \sim (J - J_c)^{-\nu} \to +\infty$

Spin S_i, S_j far away will see each other.

Deep idea (by Wilson. Nobel prize)

near J_c. lattice site "ci" doesn't matter with the physics.

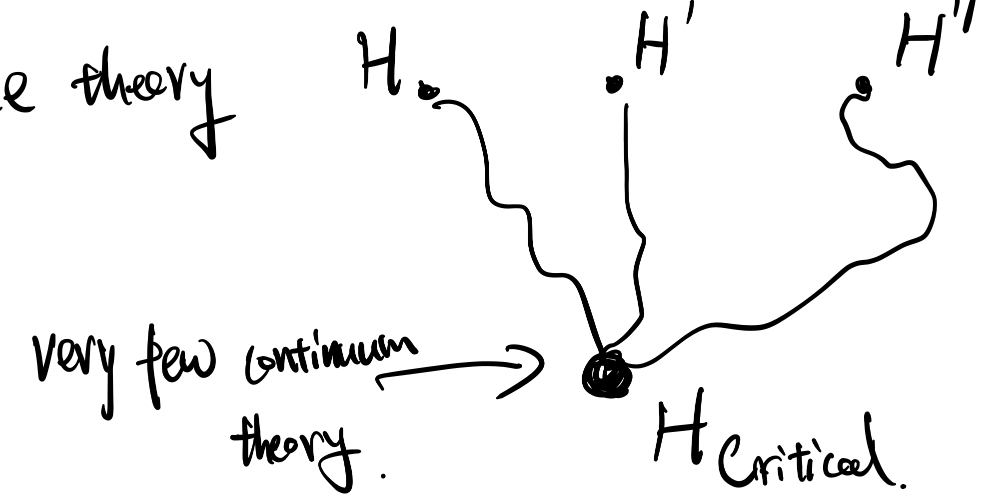
$\rightsquigarrow$ Forget about lattice, go to the Continuum.

$\rightsquigarrow$ $\boxed{\text{UNIVERSALITY.}}$: you can add much more interactions on the lattice.

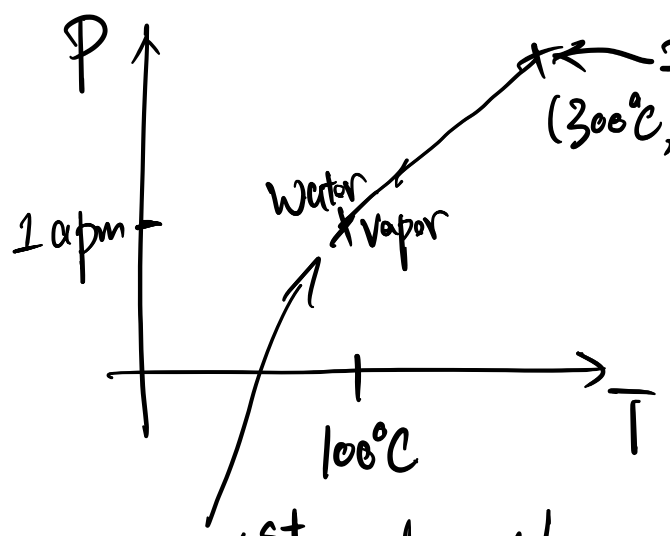
e.g. $H = J\sum_{\langle ij\rangle} S_i S_j + J'\sum_{\langle i,j,k,l\rangle} S_i S_j S_k S_l$

$+ \cdots$

When $J \to J_c$ they go to the same theory



e.g. 2. Vapor – Water phase transition



2^{nd} order phase transition, $\xi \to \infty$

(300°C, 200 atm) despcrition under QFT, $Ising_3$.

(No quanta. No $\hbar$

QFT is a formalism

to describe critical phys.)

1^{st} order phase transition. $\xi \ll 1$.

— PHILOSOPHY of QFT.

1. Starting from a UV regime

S.R. + Q.M. + Locality/Causality + Renormalization

+ Anomaly Free

+ etc. . .

$\Downarrow$

Restricted choices to define a QFT_{UV}

2. In material Science, starting from a Lattice model / Condense matter model, tuning parameters so that $\xi \to \infty$

RG Flow

Universal continuous theory defined at IR.

QFT_{IR}
(CFT)

B. Natural units. & Scales.

Our units for measurement are very much

<u>Artificial</u>

	CHINA	U.S.
Length	meter	feet / yard / mile
Area	meter^2	$---^2$.
Volume	meter^3/Liter	$---^2$/Gallon
Mass	gram	pound.
Temperature	$\#^\circ C$	$\#^\circ F$
Time	h/m/s.	same.

For convenience, we set a "natural unit"

$$c = \hbar = 1$$

$$\Rightarrow \quad [c] = LT^{-1} \qquad L \sim T.$$

$$[\hbar] = L^2 M T^{-1} \qquad M \sim T^{-1} \sim L^{-1}$$

$$[G] = L^3 M^{-1} T^{-2} \sim M^{-2}$$

$$\rightsquigarrow [X] = d \iff M^d, \quad d=0 \text{ dimensionless}$$

In high energy physics, common unit is eV

$$E_e = 0.511\ \text{MeV} = m_e c^2$$

$$m_e = \frac{0.511\,\text{MeV}}{c^2} = 0.511 \times 1.782 \times 10^{-30}\ \text{kg}$$

$$= 0.91 \times 10^{-30}\ \text{kg}$$

Compton wavelength

$$\lambda_e = \frac{\hbar}{c} \cdot \frac{1}{m_e} = \frac{\hbar}{m_e c} \sim \frac{10^{-34}}{10^{-30} \times 10^{8}}\ \text{m}$$

$$L \cdot M \quad M^{-1}$$

$$\sim 10^{-12}\ \text{m}.$$

$$[G] : L^3 M^{-1} T^{-2} = [\hbar c \cdot M^{-2}]$$

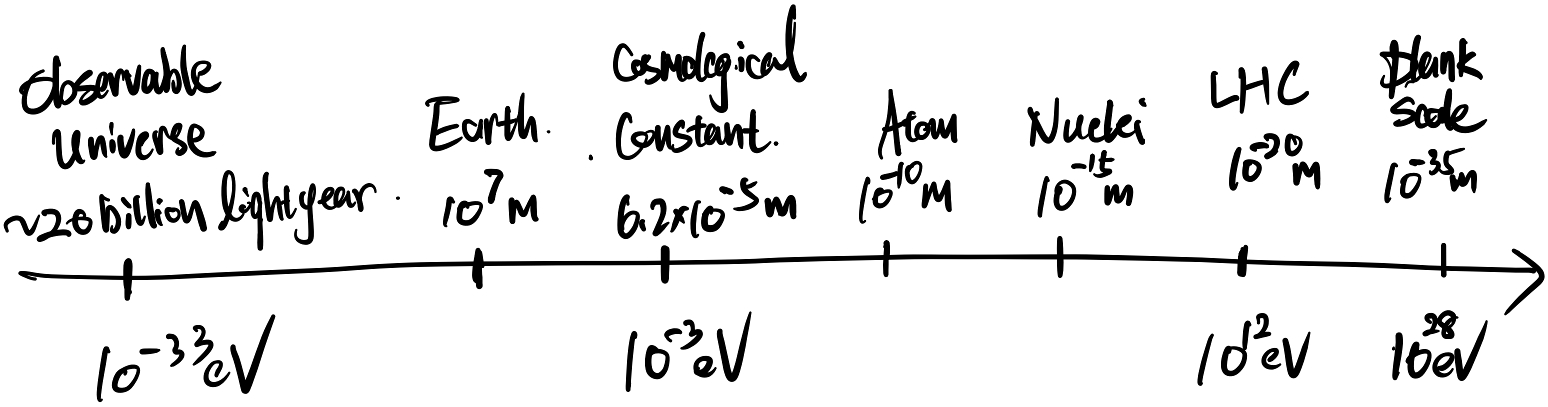
$[G] = L\,M\,T = [\hbar c \cdot M_p]$

$$\Rightarrow M_p = \left(\frac{\hbar c}{G}\right)^{1/2} \sim 10^{19}\,\text{GeV} \sim 10^{-35}\,\text{m}.$$

M_p: Plank Scale, the smallest scale physics make sense.

Beyond this scale, Quantum Gravity becomes important,

and not clear if the concept of spacetime make sense!



Scratch

$$J/s = 1A \cdot 1V = \frac{1e}{s} \cdot 1V =$$

$1\,J = 6.2 \times 10^{18}\ eV.$

$$\left[\frac{eV}{6.2\times10^{18}} \times \frac{1}{c^2}\right] = M$$

$[\hbar] = E \cdot T$

$[c] = L \cdot T^{-1}$

$[\hbar c] = E \cdot L$

$$\left[\frac{eV}{\ldots^{18}} \cdot \frac{1}{\ldots}\right] = L^{-1}$$

$[6.2 \times 10^{18} \ \hbar c] = L$

Something interesting to know:

neutrino	$\sim 10^{-2}$ eV
electron	0.5 MeV
Muon	100 MeV
Pion	140 MeV
p, n	1 GeV
Tau	2 GeV
W, Z	80-90 GeV
Higgs Boson	125 GeV.

C. Langrangian & Hamiltonian Formalism.

— A single particle:

$$\ddot{q} = -\frac{\partial V}{\partial q} \quad \xrightarrow{\text{Free theory}} \quad \ddot{q} = 0$$

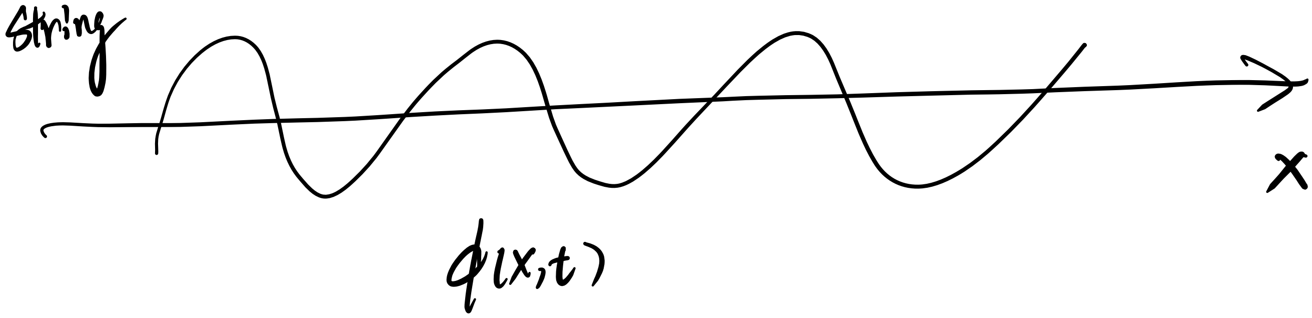
Langrange Mech.: $L = \frac{1}{2}\dot{q}^2 - V(q) = T - V$

$$S = \int dt\, L \Rightarrow \frac{\delta S}{\delta q} = 0$$

Hamilton Mech. $p = \frac{\partial L}{\partial \dot{q}}$ $\quad H = p\dot{q} - L$ (Legendre transf.)

$$\frac{\partial H}{\partial q} = -\dot{p}, \quad \frac{\partial H}{\partial p} = \dot{q}$$

— Continuum : A String



a Free string satisfies:

$$\frac{\partial^2 \phi}{\partial t^2} - v^2 \frac{\partial^2 \phi}{\partial x^2} = 0$$

A Lagrangian for this E.O.M.

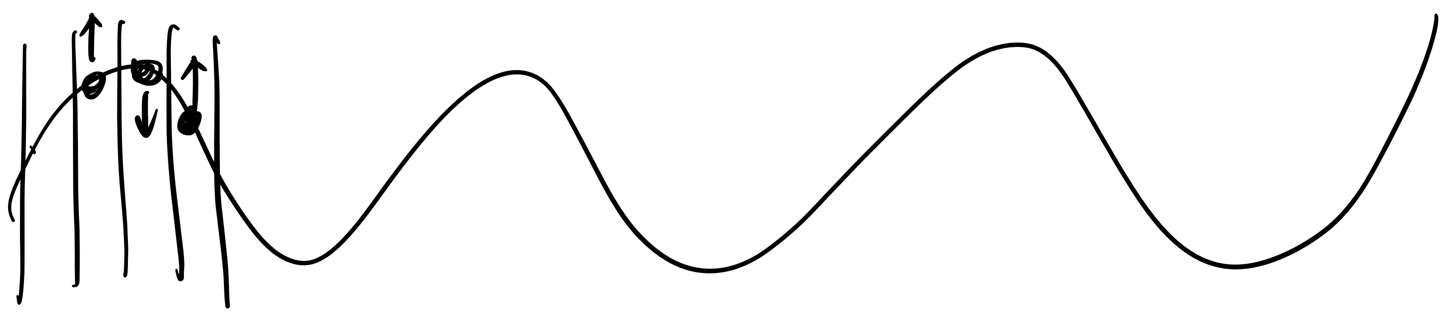
$$\mathcal{L}(x,t) = \frac{1}{2}(\partial_t \phi)^2 - \frac{v^2}{2}(\partial_x \phi)^2$$

$$S = \int \underbrace{d^2x}_{dtdx} \mathcal{L}(x,t) \qquad \frac{\delta S}{\delta \phi} = 0$$

$$\delta \mathcal{L} = \partial_t \delta\phi \, \partial_t \phi - v^2 \partial_x \delta\phi \, \partial_x \phi$$

$$= -\delta\phi \left(\partial_t^2 \phi - v^2 \partial_x^2 \phi \right)$$

A string can be regarded as a system with infinitely many D.O.F.



kinetic term. $K = \sum_a \frac{1}{2} \dot{q}_a^2 \qquad V = \frac{k}{2} \sum_{a,b} (q_a - q_b)^2$

$$\phi(x,t) \rightsquigarrow \int_a^{a+1} dx \, \phi(x,t) = q_a(t)$$

$$\sum_a \frac{1}{2} \dot{q}_a^2 \longrightarrow \frac{1}{2} \int dx \, (\partial_t \phi)^2 \qquad \frac{1}{2} \sum_{a,b} (q_a - q_b)^2 \longrightarrow \frac{v^2}{2} \int dx \, (\partial_x \phi)^2$$

$$L(t) = \int dx \, \mathcal{L}(x,t) = \frac{1}{2} \int dx \, (\partial_t \phi)^2 - v^2 (\partial_x \phi)^2$$

— Electromagnetic Field, Scalar Field, . . .

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2\right) \vec{E}, \vec{B} = 0$$

$\downarrow$ natural unit.

$$\partial_t^2 - \sum_i \partial_i^2 = \eta^{\mu\nu} \partial_\mu \partial_\nu \equiv \partial_\mu \partial^\mu \equiv \partial^2$$

$\partial_\mu = (\partial_t, \vec{\partial_i})$ $\eta^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$ <u>West Coast</u>

$\eta^{\mu\nu}_{East} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ Weinberg.

Rmk: $\partial^2 \equiv \partial_\mu \partial^\mu$ is a "<u>Lorentz invariant</u>" Operator.

Very Roughly. one may expect

Will come back later.

$$\mathcal{L}_{\vec{E},\vec{B}} = \frac{1}{2} \partial_\mu \vec{E} \cdot \partial^\mu \vec{E} + \frac{1}{2} \partial_\mu \vec{B} \cdot \partial^\mu \vec{B}.$$

NOT REALLY. !

Because of Gauge symmetry (Redundancy).

But For a scalar Field (3+1 D)

1+1 D String

2+1 D Drum.

3+1 D Continuum Filling all space-time

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi}_{\text{Free.}} - \underbrace{V(\phi)}_{\text{Potential/interaction.}}$$

Rmk: The String, Drum, ..., are toy model to give you a taste on what QFT looks like.

In a modern perspective, Due to Landau-Ginzburg. we start with fields (ϕ). Satisfying necessary "symmetries", and cook up all possible interactions in the Langrangians, and study the RG flow of them.

e.g. ϕ. $\partial_\mu\phi\,\partial^\mu\phi$ Lorentz invariants.

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4 + \cdots$$

Boiling water (at 200 atm, 300°C), 2nd order phase transition (CFT)

$\Uparrow$ RG-flow

$$S=\int d^3x \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Ising_3. 2+1 D QFT.

— Generics:

* Starting with a Langrangian $\mathcal{L}(\phi, \partial_\mu\phi)$

or $S = \int d^Dx\, \mathcal{L}(\phi, \partial_\mu\phi)$

$$\delta S = \int d^Dx\ \frac{\delta\mathcal{L}}{\delta\phi}\delta\phi + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta(\partial_\mu\phi)$$

$$= \int d^Dx\ \delta\phi\left(\frac{\delta\mathcal{L}}{\delta\phi} - \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\right)\right) + \underbrace{\partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi\right)}_{\text{full derivative}}$$

E.O.M. : $\frac{\delta\mathcal{L}}{\delta\phi} - \partial_\mu\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} = 0$

$$\int_B d^Dx\ \partial_\mu(\cdots) = \oint_{\partial B} d^{D-1}\vec{x}\cdot(\cdots) = 0 \text{ if } \partial B = 0$$

* From $\mathcal{L}$, we can go to Hamiltonian $\mathcal{H}$.

$$\Pi_\phi^a = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_a} \quad \text{momentum conjugate to } \phi$$

$$\mathcal{H} = \pi^a \dot{\phi}_a - \mathcal{L}$$

$$H = \int d^3x \ \mathcal{H}.$$

e.g. $\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$

$$\Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

$$\mathcal{H} = \Pi_\phi \dot{\phi} - \mathcal{L} = \frac{1}{2}\Pi_\phi^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

Similarly $\dot{\phi}(x,t) = \dfrac{\partial H}{\partial \Pi(x,t)}$, $\dot{\Pi}(x,t) = -\dfrac{\partial H}{\partial \phi(x,t)}$

Rmk: defining "Π" as well as "$\mathcal{H}$" will distinguish the time direction from other spatial directions,

$\rightsquigarrow$ $\mathcal{H}$ is NOT Lorentz invariant

$\Rightarrow$ $\mathcal{H}$ is NOT Lorentz invariant

$\mathcal{L}$-formalism is Lorentz invariant. (L.I.)

$\mathcal{H}$-formalism emphasize the dynamical aspect of the the theory, and thus NOT L.I. manifestly.

D. NOETHER Theorem:

* Recall in particle case

$$S = \int dt \, L(q, \dot{q})$$

Suppose there is a symmetry. $\delta q = \epsilon X(q, \dot{q}, t)$

$$\delta L = 0 = \frac{\delta L}{\delta q}\delta q + \frac{\delta L}{\delta \dot{q}}\delta \dot{q}$$

$$= \delta q \left(\frac{\delta L}{\delta q} - \frac{d}{dt}\frac{\delta L}{\delta \dot{q}} \right) + \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}} \delta q \right)$$

Apply E.O.M. $= 0 + \epsilon \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}} X \right)$

$Q \equiv \frac{\delta L}{\delta \dot{q}} X$ conserved charge., $\frac{dQ}{dt} = 0$

e.g. 2-body motion $L = \frac{1}{2}\dot{\theta}^2 + V(r)$

obviously $\theta \to \theta + \alpha$ is invariant.

$\delta\theta = \alpha \Rightarrow X = 1$

$$\Rightarrow \quad Q = \frac{\delta L}{\delta \dot{\theta}} X$$

$$= \dot{\theta}$$ angular momentum. conserved.

More generically, one can require the sym.

$\delta q = \epsilon X(q, \dot{q}, t)$ leads L to be

$\delta L = \epsilon \frac{d}{dt} F(q, \dot{q}, t)$ so that $S = \int dt\, L$

is invariant.

$$\Rightarrow \quad Q = \frac{\delta L}{\delta \dot{q}} X - F.$$

e.g. suppose $t \to t + \epsilon$ is a sym. of L

$q(t) \to q(t+\epsilon) \rightsquigarrow \delta q = q(t+\epsilon) - q(t)$

$= \epsilon\, \dot{q}(t)$

i.e. $X = \dot{q}(t)$

On the other hand, $L(t)$ under $t \to t+\epsilon$

On the other hand, $L(t)$ under $t \to t+\epsilon$

$$\delta L = \epsilon \dot{L}(t) = \epsilon \frac{d}{dt} L \Rightarrow F = L$$

$$\Rightarrow Q = \frac{\delta L}{\delta \dot{q}} X - F = \frac{\delta L}{\delta \dot{q}} \dot{q} - L = p\dot{q} - L = H$$

i.e. Hamilton H is conserved, $\frac{dH}{dt} = 0$

e.g. 2 (non-trivial)

Laplace – Runge – Lenz vector in 2-body problem

Consider $\quad L = \frac{\vec{I}^2}{2m} + V(r)$

angular momentum. $\vec{I} = (I_x, I_y, I_z)$ conserved

$$\uparrow \epsilon$$

$\rightsquigarrow SO(3)$ sym.

A hidden sym. $\delta_j x_i = \frac{\epsilon}{2}\left(2p_i x_j - x_i p_j - \delta_{ij} (\vec{x} \cdot \vec{p})\right)$

$$\Rightarrow \vec{A} = \vec{p} \times (\vec{r} \times \vec{p}) - \frac{\vec{x}}{|\vec{r}|}$$

$SO(3) \to SO(4)$. $\vec{A}$ conserved.

all $\{\vec{I}, \vec{A}\}$ are 6 generators of $SO(4)$

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* For continuum case.

$$A \text{ Sym. } \delta\phi = \epsilon X(\phi, \partial_\mu\phi, t)$$

leads Langrangian to $\delta\mathcal{L} = \epsilon\partial_\mu F^\mu$

On the other hand, $\delta\mathcal{L} = \frac{\delta\mathcal{L}}{\delta\phi}\delta\phi + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\partial_\mu\delta\phi$

$$= \left(\frac{\delta\mathcal{L}}{\delta\phi} - \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\right)\right)\delta\phi + \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi\right)$$

$$\Rightarrow \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} X - F^\mu\right) = 0$$

Conserved current $j^\mu = \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} - F^\mu$

From conserved current j^μ, one can define

Conserved charge $Q(t) = \int d^{D-1}x \; j^0(\vec{x}, t)$

$$\frac{dQ}{dt} = \int d^{D-1}x \, \frac{dj^0}{dt} = -\int d^{D-1}x \, \vec{\nabla} \cdot \vec{j} = 0$$

(Stokes theorem)

Q is conserved.

e.g. $\mathcal{L} = \partial_\mu \bar{\phi} \, \partial^\mu \phi - V(|\phi|)$

ϕ is a complex field.

$\phi \to e^{i\alpha}\phi$, $\mathcal{L} \to \mathcal{L}$

$$\Downarrow$$

$\delta\phi = i\alpha\phi = \alpha X(\phi) \Rightarrow X(\phi) = i\phi, \quad X(\bar{\phi}) = -i\bar{\phi}$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} X(\phi) + \frac{\partial \mathcal{L}}{\partial \partial_\mu \bar{\phi}} X(\bar{\phi})$$

$$= i \partial^\mu \bar{\phi} \, \phi - i \bar{\phi} \, \partial^\mu \phi .$$

one can check $\partial_\mu j^\mu = 0$

A cute trick (NOT only trick)

Boost $\alpha \to \alpha(x)$, global sym. $\Rightarrow$ local. sym (gauge redundancy)

$$\delta\mathcal{L} = \partial_\mu\alpha \cdot h^\mu(x) \quad (\text{Assuming } \delta\mathcal{L} = 0 \text{ for } \alpha = \text{Const})$$

$$= \partial_\mu(\alpha h^\mu) - \alpha\,\partial_\mu h^\mu$$

$$\Rightarrow \delta S = \delta\int d^Dx\,\mathcal{L} = -\int d^Dx\,\alpha\,\partial_\mu h^\mu$$

$$\Rightarrow \partial_\mu h^\mu = 0. \quad \text{i.e. } j^\mu = h^\mu.$$

e.g. $\mathcal{L} = \partial_\mu\bar\phi\,\partial^\mu\phi - V(\phi)$

$$\delta\mathcal{L} = -\partial_\mu(i\alpha\bar\phi)\,\partial^\mu\phi + \partial_\mu\bar\phi\,\partial^\mu(i\alpha\phi)$$

$$= -i\,\partial_\mu\alpha\,\bar\phi\,\partial^\mu\phi + i\,\partial^\mu\alpha\,\partial_\mu\bar\phi\,\phi$$

$$= \partial_\mu\alpha\underbrace{(-i\bar\phi\,\partial^\mu\phi + i\,\partial^\mu\bar\phi\,\phi)}_{j^\mu}$$

E. Canonical Quantization.

* Particle case :

Start from $L(q^a, \dot q^a) \rightsquigarrow H(p_b, q^a)$

Define Poisson bracket.

$$\{f, g\}_{P.B.} := \sum_a \frac{\partial f}{\partial q^a}\frac{\partial g}{\partial p_a} - \frac{\partial f}{\partial p_a}\frac{\partial g}{\partial q^a}\Big|_t$$

$$\Rightarrow \{q^a, p_b\} = \delta^a{}_b, \quad \{q^a, q^b\} = \{p_a, p_b\} = 0$$

Hamilton Eq. $\dot q = -\frac{\partial H}{\partial p}$ $\quad \dot p = \frac{\partial H}{\partial q}$ in terms of P.B.

$$\frac{dq^a}{dt} = \{H, q^a\} \qquad \frac{dp_a}{dt} = \{H, p_a\}$$

Canonical Quantization (by Dirac)

change P.B. to Quantum Bracke

$$\{\cdot, \cdot\}_{P.B} \longrightarrow -\frac{i}{\hbar}[\hat\cdot, \hat\cdot]$$

$$\Rightarrow -\frac{i}{\hbar}[\hat q^a, \hat p_b] = \delta^a{}_b \quad \text{or} \quad [\hat q^a, \hat p_b] = i\hbar\delta^a{}_b$$

$$\frac{d\hat q^a}{dt} = \frac{i}{\hbar}[\hat H, \hat q^a], \quad \frac{d\hat p_a}{dt} = \frac{i}{\hbar}[\hat H, \hat p_a]$$

Which is in the Heisenberg Picture, i.e. Operators which is in the Heisenberg picture, the operators are time dependent. $(\hat{q}(t), \hat{p}(t))$, and state

$|\psi\rangle_H$ is time independent $\frac{d}{dt}|\psi\rangle_H = 0$

Switch back to Schrödinger picture.

$$|\psi\rangle_S = e^{-i\hat{H}t}|\psi\rangle_H \Rightarrow i\frac{d|\psi\rangle_S}{dt} = \hat{H}|\psi\rangle_S$$

$$\hat{O}_H = e^{i\hat{H}t}\hat{O}_S e^{-i\hat{H}t}.$$

In Schrödinger Picture. $\hat{O}_S = e^{-i\hat{H}t}\hat{O}_H e^{i\hat{H}t}$

is time independent.

* Continuum case:

From single particle to continuum field.

$$q(t) \longrightarrow \phi(t, \vec{x})$$

$$p(t) \longrightarrow \pi(t, \vec{x})$$

Therefor the commutation relation

$$[q, p]\Big|_{\text{equal time}} = i$$

will be promoted to

$$[\phi(t,\vec{x}), \Pi(t,\vec{y})]\Big|_{\text{equal time}} = i\,\delta^{(3)}(\vec{x}-\vec{y})$$

Rmk 1: In Schrödinger Pic. $\phi_S(\vec{x}), \Pi_S(\vec{y})$ are time independent, the commutator above still hold.

Rmk 2: If you feel uncomfortable about the δ-function, Recall the dimensionality of ϕ and Π, $[\phi]=1$, $[\Pi]=2$

$$\therefore [\phi(\vec{x}), \Pi(\vec{y})]\Big|_{e.t.} = i \underbrace{?}_{3}$$

the only candidate with dimensionality 3 is $\delta^{(3)}(\vec{x}-\vec{y})$.