E 10.5.1

4 nodes 4 edges

$\binom{4}{2} = \frac{4\times3}{2\times1} = 6$

Possible edges: $\frac{d}{dp}\left[p^4(1-p)^2\right] = 0$

$2p^3(1-p)(2-3p) = 0$

$\therefore$ $p = 0$ or 1 or $\frac{2}{3}$

$\therefore$ consider $p = \frac{2}{3}$, so

Probability $= (\frac{2}{3})^4 (1-\frac{2}{3})^2 = \frac{16}{729}$

E 10.5.2

$C = \{w, x\}$ $D = \{y, z\}$

$\therefore P_{wx} = P_C$

$P_{yz} = P_D$

$\therefore L(P_C, P_D) = P_{wx} \cdot P_{yz} \cdot (1-P_{wy}) \cdot (1-P_{wz}) \cdot (1-P_{xy}) \cdot (1-P_{xz})$

$= P_C \cdot P_D$

$= (P_C + P_D - P_C P_D)^4$

$\because$ To find maximum, and when $P_C = 1$ $L = 1$

$\therefore$ $P_D = 0.5$ to get a maximum

$\therefore$ $P_C = 1$ $P_D = 0.5$

E 10.5.3 $C = \{w, x, y\}$ $D = \{w, y, z\}$ $\therefore L(P_C, P_D) = P_C^2 \cdot P_D \cdot (1-(P_C + P_D - P_C P_D))^2$

$\because$ When $P_C = 1$ and $P_D = 0.5$ makes it maximum

$\therefore$ $z: D \to C:$ $C = \{x, w, y, z\}$ $D = \{w, y\}$ $\therefore L(P_C) = P_C^4$ $P_C = 1$ max $= 1$

$x: C \to D$ same, $P_D = 1$ max $= 1$

w: delete $\exists\, \epsilon$, $L < 1$ $\therefore$ Finally $C = \{w, x, y, z\}$ $D = \{w, y\}$

others: $\exists\, \epsilon$, $L < 1$

or $C = \{w, y\}$, $D = \{w, x, y, z\}$ $\Rightarrow L = 1$