An excellent example of a physical system that inherently solves differential equations is an electronic analog computer, specifically a simple RLC circuit consisting of a resistor (R), an inductor (L), and a capacitor (C).

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How an RLC Circuit Solves a Differential Equation

The behavior of an RLC circuit is governed by Kirchhoff's voltage law, which results in a second-order linear ordinary differential equation. The equation describing the current, I(t), in a series RLC circuit with a driving voltage, V(t), is:

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = \frac{dV(t)}{dt}$$

This equation is mathematically identical in form to the equation for many other physical systems, such as a damped harmonic oscillator (e.g., a mass on a spring with a damper):

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

Here's how the analogy works:

RLC Circuit Component	Mathematical Term	Mechanical System Analog
Inductance (L)	L	Mass (m)
Resistance (R)	R	Damping Coefficient (c)

1 / Capacitance (1/C)	1/C	Spring Constant (k)
Current (I)	I(t)	Position (x)
Rate of change of Voltage (dV(t)/dt)	dV(t)/dt	Driving Force (F(t))

By selecting the values for the resistor, inductor, and capacitor, an engineer can build a circuit that directly models a specific mechanical system. When the voltage is applied, the resulting current I(t) in the circuit over time is the physical solution to the differential equation. You can measure this current with an oscilloscope to find the "answer," which corresponds to the position x(t) of the mass in the mechanical system.

The circuit isn't *calculating* a solution step-by-step; its physical evolution over time is the solution itself.