Copying

Flash Cards for Math 2250

"Differential Equations and Linear Algebra"

ODEs and Linear Algebra

Definition

order

ODEs and Linear Algebra

Definition

ODE and PDE

ODEs and Linear Algebra

Definition

directly integrable

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Definition

general solution

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Definition

initial value problem
(IVP)

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Definition

particular solution

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Algorithm

slope field plot

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Theorem

existence

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Theorem

uniqueness

ODEs and Linear Algebra

The **order** of a differential equation (DE) is the order of the highest derivative which occurs in the equation.

The DE, $y' = F(x,y)$, is **directly integrable** if

$$F(x,y) = f(x).$$

Such an equation may be solved by computing:

$$y(x) = \int f(x)\,dx.$$

An **ODE** (ordinary differential equation) is a DE that only contains total derivatives.

A **PDE** (partial differential equation) is a DE which contains at least one partial derivative.

An **initial value problem** or **IVP** is a differential equation coupled with at least one initial condition, for example,

$$y' = F(x,y) \qquad y(0) = y_0.$$

A second order IVP will require two initial conditions, $y(0) = y_0$ and $y'(0) = v_0$. In general, an nth order IVP will require n initial conditions.

A **general solution** to a DE is any continuous function which satisfies the DE and contains at least one constant of integration.

A general solution is actually a collection or *family* of functions parametrized by the integration constant(s).

Given a first–order DE, $y' = F(x,y)$, one can graphically approximate solutions to the DE by generating a **slope field plot**:

1. Divide a region of the xy–plane into a grid.
2. For each cell in the grid compute $m = F(\bar{x},\bar{y})$, where $(\bar{x},\bar{y})$ is the midpoint of the cell, and plot a small bar with slope m centered on the point $(\bar{x},\bar{y})$.

A **particular solution** is a solution to an IVP. In other words it is a single function which satisfies both the DE and any initial condition(s).

$$y' = F(x,y) \qquad y(a) = b \tag{1}$$

If $F(x,y)$ and $F_y(x,y)$ are both continuous on some rectangle R containing the point (a,b) in its interior, **then** there exists an open interval I which contains a such that (1) has a *unique* solution on I. Note that the width of I may be shorter than the width of R.

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Definition

separable

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Definition

population model

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Definition

diffusion model
(Newton cooling and heating)

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Definition

linear DE

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Definition

integrating factor

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Algorithm

integrating factor method

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Definition

autonomous

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The **population model** describes exponential growth and decay. It is an IVP:

$$P' = kP \qquad P(0) = P_0.$$

This model has particular solution:

$$P(t) = P_0 e^{kt}.$$

Using the particular solution to extrapolate values may not be reasonable because exponential growth is often unphysical (impossible) over a long time scale.

The DE $y' = F(x, y)$ is called **separable** if the right hand side function can be expressed as a product of two single variable functions, for example,

$$F(x, y) = f(x)g(y).$$

Such equations may be solved by "separating the variables", that is, computing:

$$\int \frac{1}{g(y)}\, dy = \int f(x)\, dx.$$

An ODE is **linear** if it can be written in the form:

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = f(x).$$

Example of first and second order, *linear* ODEs:

$$\begin{aligned} a_1(x)y' + a_0(x)y &= f(x) \\ a_2(x)y'' + a_1(x)y' + a_0(x)y &= f(x) \end{aligned}$$

The **diffusion model** describes how something diffuses or spreads into its *ambient* environment.

$$y' = k(A - y) \qquad y(0) = y_0$$

This model has solution,

$$y(t) = A - Ce^{-kt} \qquad C = A - y_0.$$

Do not try to memorize the solution! Solve the model via separation of variables.

1. Put in standard form: $y' + p(x)y = q(x)$.
2. Multiply both sides by $I(x) = e^{\int p(x)dx}$, and integrate:
   $yI(x) = \int q(x)I(x)\, dx$
3. $y = e^{-\int p(x)dx}\left[\int \left(q(x)e^{\int p(x)dx}\right) dx\right]$

   Don't forget the $+C$ when integrating!

Given a linear, first order ODE:

$$a_1(x)y' + a_0(x)y = f(x),$$

first divide by $a_1(x)$ to yield:

$$y' + p(x)y = q(x).$$

The **integrating factor** is: $I(x) = e^{\int p(x)dx}$.

The DE $y' = F(x, y)$ is called **autonomous** if $F(x, y)$ is just a function of the dependent variable y, that is if:

$$F(x, y) = g(y).$$

Such equations can be analyzed by finding the roots of the equation $g(y) = 0$ and then creating a phase diagram.