Computational Intelligence

LMU Munich
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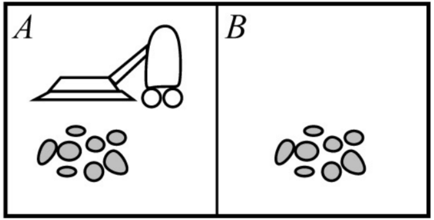
First: What are states?

Second: What are state values?

(encoding policies)

running example #1

The Vacuum World



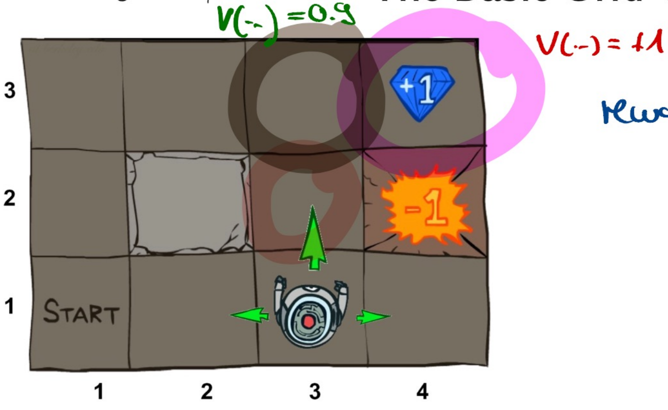
reward function: +1 if dirty room becomes clean

$\Rightarrow$ states after we cleaned often are good

Russel, Norvig. Artificial Intelligence – A Modern Approach. Third Edition. 2016.

running example #2

The Basic Grid World



reward function : get reward according to grid field

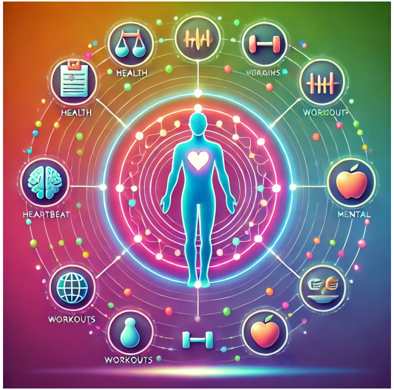
https://inst.eecs.berkeley.edu/~cs188/fa22/

running example #3

Resource/Stock Trading

chatgpt.com

Personal Life Assistant



chatgpt.com

Third: Where do we get
a state value function?

(finding policies)

Theorem 2 (Bellman equation)**.** Let $(\mathcal{S}, \mathcal{A}, \mathcal{T}, P, R)$ be a Markov decision process. Let $R : \mathcal{S} \times \mathcal{A} \to \mathcal{T}$ be the expected reward of executing an action in a given state, i.e., $R(s, a) = \mathbb{E}[R(s, a, s')]$ where $s' \sim P(s'|s, a)$. Let $\gamma \in [0; 1) \subseteq \mathbb{R}$ be a temporal discount factor.
The expected reward of a policy π being executed starting from state s is given via π's value function

$$V^\pi(s) = R(s, \pi(s)) + \gamma \cdot \sum_{s' \in \mathcal{S}} P(s'|s, \pi(s)) \cdot V^\pi(s').$$

The value function of the optimal policy π^* is given via

$$V^{\pi^*}(s) = \max_{a \in \mathcal{A}} \left(R(s, a) + \gamma \cdot \sum_{s' \in \mathcal{S}} P(s'|s, a) \cdot V^{\pi^*}(s') \right).$$

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Algorithm 7 (optimal policy)**.** Let $V^* : \mathcal{S} \to \mathcal{T}$ be the *true value function* of a Markov decision process $(\mathcal{S}, \mathcal{A}, \mathcal{T}, P, R)$. The optimal policy $\pi^* : \mathcal{S} \to \mathcal{A}$ is given via

$$\pi^*(s_t) = \arg\max_{a \in \mathcal{A}} V^*(s')$$

where $s' \sim P(s'|s_t, a)$ is the follow-up state when executing action a in state s_t.

Let's try that!

running example #1

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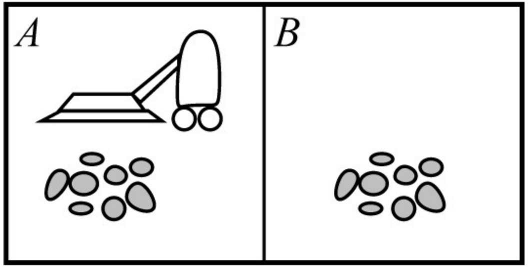

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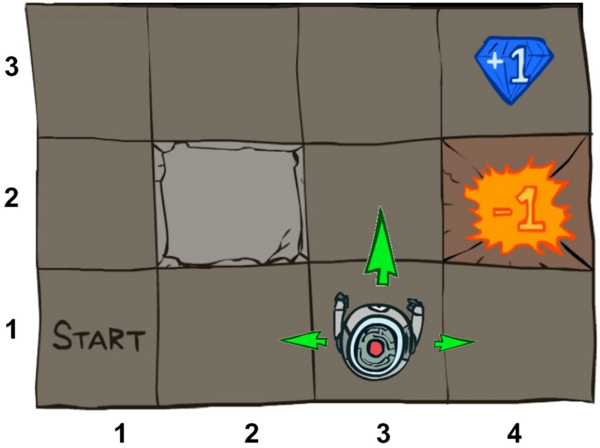
running example #2

The Basic Grid World



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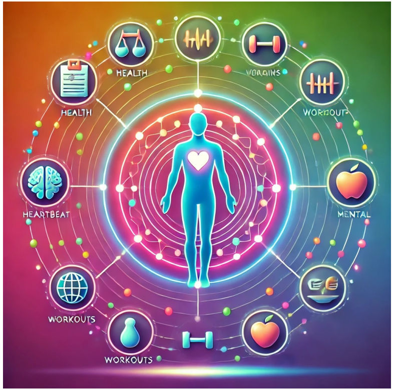
running example #3

Resource/Stock Trading

chatgpt.com

running example #4

Personal Life Assistant



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Putting it together...

Definition 11 (training of a neural network)**.** Let $\mathcal{N} : \mathbb{R}^p \to \mathbb{R}^q$ be a neural network with n weights $\overline{\mathcal{N}} = \mathbf{w} +\!\!+\, \mathbf{b} \in \mathbb{R}^n$ as in Definition 8. Note that thus $|\overline{\mathcal{N}}| = n$. Let $\tau : \mathbb{R}^n \to \mathbb{R}$ be a target function as in Definition 2. Note that thus $\mathcal{T} = \mathbb{R}$. The process of optimizing the network weights $\overline{\mathcal{N}}$ so that $\tau(\overline{\mathcal{N}})$ becomes minimal is called training.

- Let $\mathbb{T} = \{(\mathbf{x}_i, \mathbf{y}_i) \, : \, i = 1, ..., N\}$ be a set of N points of training data, where $\mathbf{x}_i \in \mathbb{R}^p, \mathbf{y}_i \in \mathbb{R}^q$ for all i.
  If τ is of the form
  $$\tau(\overline{\mathcal{N}}) = \sum_{i=1}^{N} (\mathcal{N}(\mathbf{x}_i) - \mathbf{y}_i)^2$$
  or a similar form, the process of training $\mathcal{N}$ is called supervised learning.

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- Let $(\mathcal{O}, \mathcal{A}, \mathcal{T}, e, R)$ be a decision process (cf. Definition 9) for which policy $\pi_{\overline{\mathcal{N}}} : \mathcal{O} \to \mathcal{A}$ yields (possibly randomized or non-deterministic) rewards $\langle r_t \rangle_{t \in \mathcal{Z}}$. Note that $\pi_{\overline{\mathcal{N}}}$ in some way calls $\mathcal{N}$ to produce its output, for example
  $$\pi_{\overline{\mathcal{N}}}(o) = \mathcal{N}(o)$$
  for $\mathcal{O} \subseteq \mathbb{R}^p, \mathcal{A} \subseteq \mathbb{R}^q$ or if suitable translations exist.
  If τ is of the form
  $$\tau(\overline{\mathcal{N}}) = -\mathbb{E}\left[\sum_{t \in \mathcal{Z}} \gamma^t \cdot r_t\right]$$
  or a similar form, the process of training $\mathcal{N}$ is called policy-based reinforcement learning.

- Let $(\mathcal{S}, \mathcal{A}, \mathcal{T}, P, R)$ be a Markov decision process (cf. Definition 10) for which we run policy $\pi_{\overline{\mathcal{N}}} : \mathcal{S} \to \mathcal{A}$. Note that $\pi_{\overline{\mathcal{N}}}$ in some way calls $\mathcal{N}$ to produce its output, for example

$$\pi_{\overline{\mathcal{N}}}(s) = \arg\max_{a \in \mathcal{A}} \mathbb{E}_{s' \sim P(s'|s,a)} \left[\mathcal{N}(s') \right]$$

for $\mathcal{S} \times \mathcal{A} \subseteq \mathbb{R}^p$ with $q = 1$ or if suitable translations exist.
Let $R : \mathcal{S} \times \mathcal{A} \to \mathcal{T}$ be the expected reward of executing an action in a given state, i.e., $R(s,a) = \mathbb{E}[R(s,a,s')]$ where $s' \sim P(s'|s,a)$. Let $T : \mathcal{S} \times \mathcal{A} \to \mathcal{S}$ be a (possibly randomized or non-deterministic) transition function, i.e., $T(s,a) = s'$ where $s' \sim P(s'|s,a)$. Let $\gamma \in [0;1]$ be a discount factor. Let $V_{\pi_{\overline{\mathcal{N}}}} : \mathcal{S} \to \mathbb{R}$ be the total discounted reward that policy $\pi_{\overline{\mathcal{N}}}$ generates when starting in state s, i.e.,

$$V_{\pi_{\overline{\mathcal{N}}}}(s) = R\big(s, \pi_{\overline{\mathcal{N}}}(s)\big) + \gamma \cdot V_{\pi_{\overline{\mathcal{N}}}}\big(T(s, \pi_{\overline{\mathcal{N}}}(s))\big).$$

Note that for $\gamma < 1$ we can abort this recursive computation once the effect of the further recursive part is sufficiently small. Note that we may also have a fixed recursion depth or that $T(s^\dagger, \_)$ might not be defined for all $s^\dagger \in \mathcal{S}$, which are then called terminal states and also cause the recursion to end.
Let $\mathbb{S} = \{\mathbf{s}_i \,:\, i = 1, ..., N\} \subseteq S$ be a set of training states. If τ is of the form

$$\tau(\overline{\mathcal{N}}) = -\frac{1}{N} \cdot \sum_{i=1}^{N} V_{\pi_{\overline{\mathcal{N}}}}(\mathbf{s}_i)$$

or a similar form, the process of training $\mathcal{N}$ is called value-based reinforcement learning.

Reinforcement Learning

Variations of Value Functions

name	network	policy
policy-based	$\mathcal{N} : \mathcal{S} \to \mathcal{A}$	$\pi_{\overline{\mathcal{N}}}(s) = \mathcal{N}(s)$
value-based (V)	$\mathcal{N} : \mathcal{S} \to \mathbb{R}$	$\pi_{\overline{\mathcal{N}}}(s) = \arg\max_{a \in \mathcal{A}} \mathbb{E}_{s' \sim P(s' \mid s,a)} \left[\mathcal{N}(s') \right]$
value-based (Q)	$\mathcal{N} : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$	$\pi_{\overline{\mathcal{N}}}(s) = \arg\max_{a \in \mathcal{A}} \mathcal{N}(s, a)$

...

On-Policy vs. Off-Policy Learning

The Goal Class Hierarchy

Goal Class 5: State Values

Goal Class 4: Rewards and Costs

Goal Class 3: Goal Direction

Goal Class 2: Goal Valuation | Goal Class 2.5: Multiple Goal Valuations

Goal Class 1: Goal Predicate | Goal Class 1.5: Multiple Goal Predicates

Goal Class 0: No Goals