Computational Intelligence WS24/25

Exercise Sheet 7 — January 23rd, 2025

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1 Finding Nash Equilibria

In the lecture we have covered Definition 13 (normal-form game), Definition 18 (best response), and Definition 19 (Nash equilibrium). Now consider the following quite literal Definition[1]:

[***Pure*** **Nash equilibrium**] Let $(G, \mathcal{A}, \mathcal{T}, \chi)$ be a normal-form game. A joint strategy π is a *Nash equilibrium* iff for all agents $G^{[i]}$ it holds that $\pi^{[i]}$ is the best response to $\pi^{[-i]}$. A Nash equilibrium $(\pi^{[i]}, \pi^{[-i]})$ is called *pure* iff both $\pi^{[i]}$ and $\pi^{[-i]}$ are pure strategies.

(i) In each of the following situations write down the corresponding two-player normal-form game.

- *Pure coordination.* Two firms (Smith and Brown) decide whether to design the computers they sell to use large or small floppy disks. Both players will sell more computers if their disk drives are compatible. If they both choose large disks, the payoffs will be 2 for each of them. If they both choose small disks, the payoffs will be 1 for each. If they choose different sizes the payoffs will be -1 for each.

[1] Exercises taken from *Hans Peter: Game Theory – Multi-Leveled Approach (Springer 2015, Second Edition)*

- *Marketing game.* Two firms sell a similar product. Each percent of market share yields a net payoff of 1. Without advertising, both firms have 50% of the market. The cost of advertising is equal to 10 payoff pts., but leads to an increase in market share of 20% at the expense of the other firm. The firms make their advertising decisions simultaneously and independently. The total market for the product is of fixed size.

(ii) To *find* the pure Nash equilibria in a two-player normal-form game, one can

1. determine the pure best response of player 2 (the column player) to every pure strategy of player 1,
2. determine the pure best response of player 1 (the row player) to every pure strategy of player 2,
3. identify all pairs of pure strategies that are mutual best responses.

Test this simple algorithm by finding the/all pure Nash equilibrium/-a in the following two-player normal-form game with $\mathcal{A}^{[1]} = \{\mathrm{T}, \mathrm{M}, \mathrm{B}\}, \mathcal{A}^{[2]} = \{\mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$. Underline your choice for each step. Are there *Pareto-optimal* strategies in this game?

$$
\begin{array}{c c}
 & \begin{array}{cccc} \mathrm{W} & \mathrm{X} & \mathrm{Y} & \mathrm{Z} \end{array} \\
\begin{array}{c} \mathrm{T} \\ \mathrm{M} \\ \mathrm{B} \end{array} & \begin{pmatrix} 2,2 & 4,0 & 1,1 & 3,2 \\ 0,3 & 1,5 & 4,4 & 3,4 \\ 2,0 & 2,1 & 5,1 & 1,0 \end{pmatrix}
\end{array}
$$

2 The Power-Node Game

Consider a power grid made up of different power nodes necessary to keep the grid working. However, these nodes are susceptible to failures and have a generally difficult life. But when they need fixing, they can call upon a *RepairBot* to help them.

All nodes can choose between two different strategies on how to react to observed technical problems: Strategy E signals an `alarming` state as early as possible while strategy L signals an `alarming` state as late as possible. When all power nodes signal early, the *RepairBot*, who wants help all the power nodes, gets overwhelmed and, when all nodes signal late, the chance of the grid breaking down increases. Thus, when two power nodes need to decide on a strategy of how to attract the *RepairBot*, formally, they play the following two-player normal-form game we call *alarm signaling*:

$$\begin{array}{cc} & \begin{array}{cc} \text{E} & \text{L} \end{array} \\ \begin{array}{c} \text{E} \\ \text{L} \end{array} & \begin{pmatrix} 2,2 & 3,1 \\ 1,3 & 1,1 \end{pmatrix} \end{array}$$

(i) Show that (E, E) is the *only* pure Nash equilibrium for *alarm signaling* by denoting all best responses.

(ii) From the perspective of the grid designer, is it desirable that the power nodes play this game in regard to getting the attention of the *RepairBot*? Why (not)? Is there a different game (given by a different two-player normal-form payoff matrix) that we would rather have them playing?

(iii) Bonus: Show that the strategy E is an *evolutionary stable strategy* in a population of power nodes.

3 Believe in the Power (Node)!

Now, imagine a pair of particularly damaged power nodes that – as a measure of preserving energy – can only focus on themselves for most of the times. These agents are now oblivious to the payoffs obtained or obtainable by other agents. We do, however, assume that each one of these nodes still knows their own payoff matrix in the stage game (i.e., the payoff it would get in each action profile, whether or not encountered in the past)....

Fictitious play is an instance of model-based learning, in which the learner explicitly maintains beliefs about the opponent's strategy. The structure of such techniques is straightforward, where the agent

1. starts with initial *beliefs* about the opponent's strategy,
2. always plays the best response to the assessed strategy of the opponent, and then
3. observes the opponent's actual play and updates beliefs accordingly.

More formally, for all $a \in \mathcal{A}$ we let $N(a)$ be the number of times that the opponent has played action a and suggest the agent to assess the probability $P(a)$ of the opponent's mixed strategy as

$$P(a) = \frac{N(a)}{\sum_{a' \in \mathcal{A}} N(a')}.$$

(i) In such a scenario, power node `CARst-3N` observes the history of his neighboring opponent node *BEP-0* to be: E, E, L, E, L, L, E. Starting from a belief that *BEP-0* has only played each action once before this, what is the best response of `CARst-3N`'s final move in round 8? Fill in the table below.

Round	BEP-0's action	CARst-3N's belief
0		
1	E	
2	E	
3	L	
4	E	
5	L	
6	L	
7	E	
8		Action =

(ii) If we assume that both parties are utilizing fictitious play, briefly explain why the *initial belief* parameter plays an important role in the convergence of the game? What is the only *initial belief* parameter that <u>cannot</u> be chosen (and why)?