Natural Computing

LMU Munich
summer term 2025

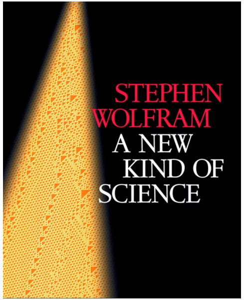
Thomas Gabor

Who knows Conway’s Game of Life?

Natural Computing, summer term 2025, LMU Munich 31

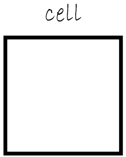
Cellular Automata

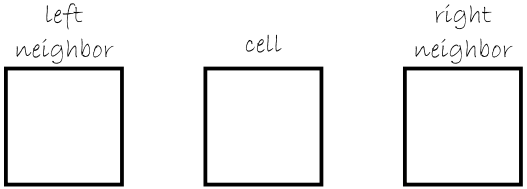
Wolfram.
A New Kind of Science.
Online, 2002.
www.wolframscience.com/nks/



Shiffman.
The Nature of Code.
Online, 2012.
natureofcode.com/book/

What was an automaton again?


cell


left neighbor
cell
right neighbor

Definition 1 (cellular automaton). Let $G = (V, E)$ be a graph with vertices V and edges $E \subseteq V \times V$. Let *neighborhood* : $V \to V^{d+1}$ for some neighborhood degree $d \in \mathbb{N}$ be a function that returns an ordered vector of neighbors of a given node v, always including v itself. A state $x \in \mathcal{X}$ is a mapping of vertices to the values $\{0, 1\}$, i.e., the state space $\mathcal{X}$ is given via $\mathcal{X} = (V \to \{0, 1\})$. Let x_t be a state that exists at time step $t \in \mathbb{N}$. The evolution of a state x_t to its subsequent state x_{t+1} is given deterministically via a function $f : \{0, 1\}^{d+1} \to \{0, 1\}$ so that

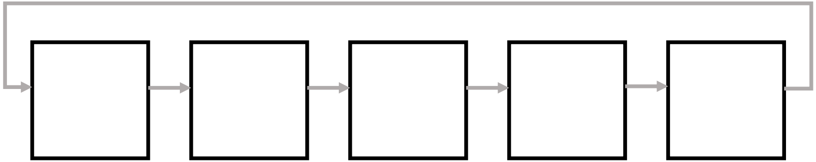
$$x_{t+1}(v) = f\big(x_t(u_1), \ldots, x_t(u_{d+1})\big)$$
$$\text{where } \vec{u} = \langle u_1, ..., u_{d+1} \rangle = \textit{neighborhood}(v).$$

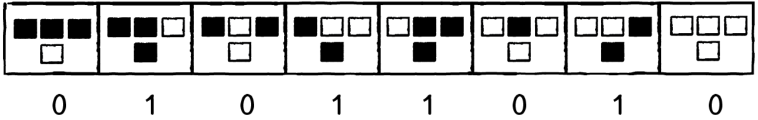
A tuple (G, f, x_0) is called a cellular automaton with initial state $x_0 \in \mathcal{X}$.

Definition 2 (1D cellular automaton)**.** A cellular automaton (G, f, x_0) is called a 1D cellular automaton iff all vertices have exactly one incoming edge ("left neighbor") and one outgoing edge ("right neighbor") and $\mathit{neighbors}(v) = \langle u, v, w \rangle$ where $(u, v) \in E$ and $(v, w) \in E$ and thus $d = 2$. Subsequently, we usually write

$$x_{t+1}(v) = f\big(x_t(u), x_t(v), x_t(w)\big)$$

for the evolution where u is the left neighbor of v and w is the right neighbor of v.





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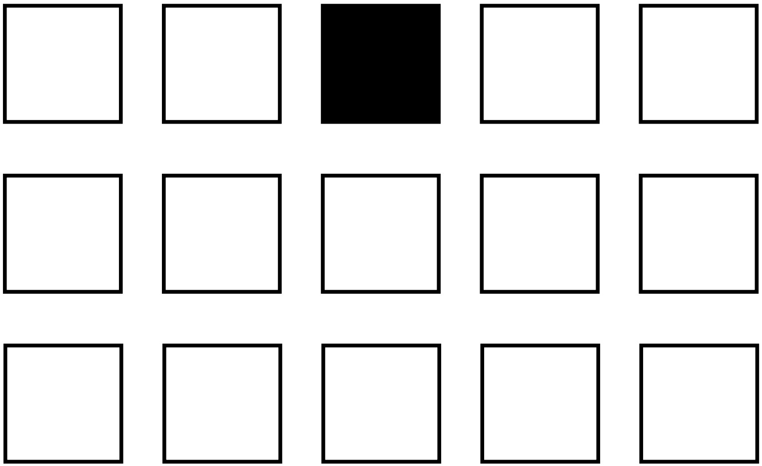
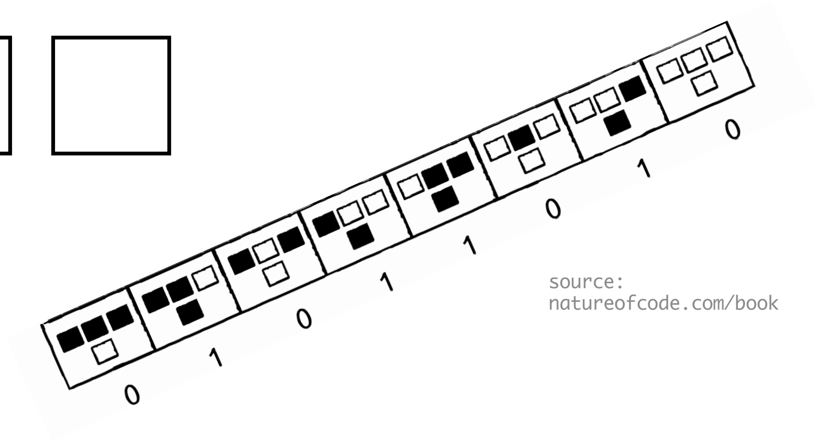
natureofcode.com/book





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Why?

Experiment Time!

Experiment 1

What is the average number

in the room?

Experiment 2

What is the most frequent number in the room?

But whispering only!

<u>Experiment 3</u>

What is the highest number
in the room?

But whispering only!

Experiment Results

	your result	my result
<u>Experiment 1:</u> average	______	______
<u>Experiment 2:</u> most frequent – whisper only	______	______
<u>Experiment 3:</u> highest – whisper only	______	______

Is local information sufficient
for meaningful computation?