Natural Computing

LMU Munich
summer term 2025

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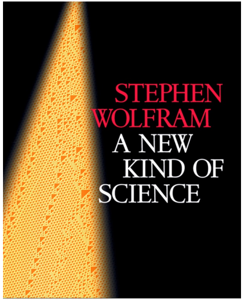
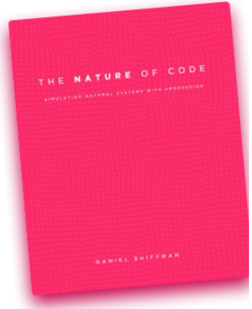
Who knows Conway's Game of Life?

Cellular Automata

Wolfram.
A New Kind of Science.
Online, 2002.
www.wolframscience.com/nks/



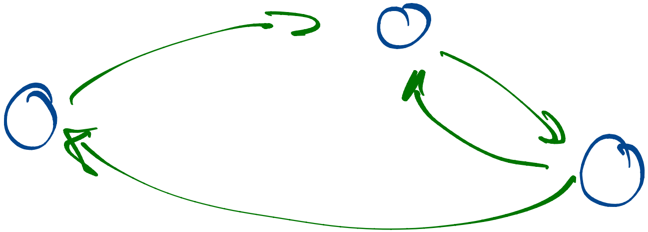
Shiffman.
The Nature of Code.
Online, 2012.
natureofcode.com/book/

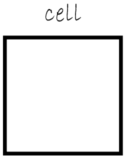


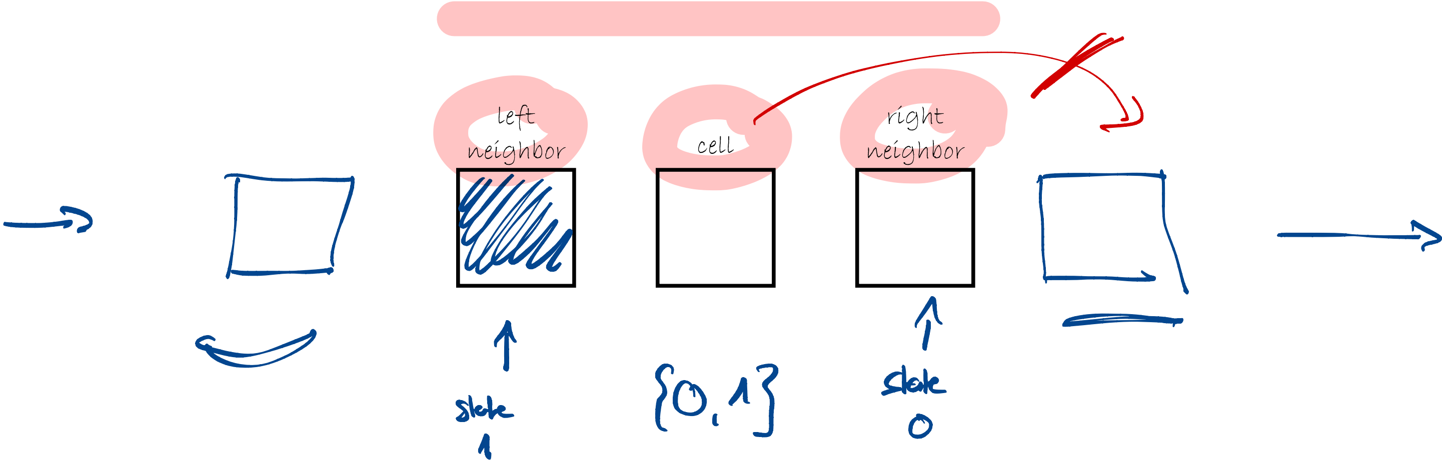
What was an automaton again?

states

transitions




cell



left neighbor
cell
right neighbor
state 1
{0, 1}
state 0

Definition 1 (cellular automaton)**.** Let $G = (V, E)$ be a graph with vertices V and edges $E \subseteq V \times V$. Let $neighborhood : V \rightarrow V^{d+1}$ for some neighborhood degree $d \in \mathbb{N}$ be a function that returns an ordered vector of neighbors of a given node v, always including v itself. A state $x \in \mathcal{X}$ is a mapping of vertices to the values $\{0, 1\}$, i.e., the state space $\mathcal{X}$ is given via $\mathcal{X} = (V \rightarrow \{0, 1\})$. Let x_t be a state that exists at time step $t \in \mathbb{N}$. The evolution of a state x_t to its subsequent state x_{t+1} is given deterministically via a function $f : \{0, 1\}^{d+1} \rightarrow \{0, 1\}$ so that

$$x_{t+1}(v) = f\big(x_t(u_1), \ldots, x_t(u_{d+1})\big)$$
$$\text{where } \vec{u} = \langle u_1, ..., u_{d+1} \rangle = neighborhood(v).$$

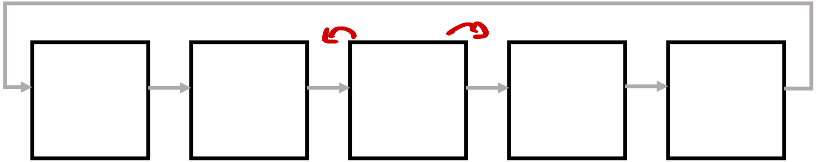
A tuple (G, f, x_0) is called a cellular automaton with initial state $x_0 \in \mathcal{X}$.

Definition 2 (1D cellular automaton)**.** A cellular automaton (G, f, x_0) is called a 1D cellular automaton iff all vertices have exactly one incoming edge ("left neighbor") and one outgoing edge ("right neighbor") and $neighbors(v) = \langle u, v, w \rangle$ where $(u, v) \in E$ and $(v, w) \in E$ and thus $d = 2$. Subsequently, we usually write

$$x_{t+1}(v) = f\big(x_t(u), x_t(v), x_t(w)\big)$$

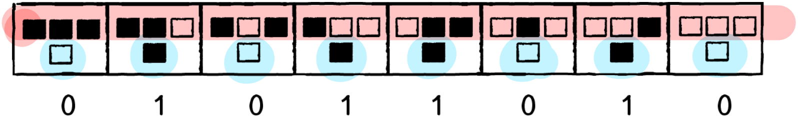
for the evolution where u is the left neighbor of v and w is the right neighbor of v.

$G =$



+ neighborhood

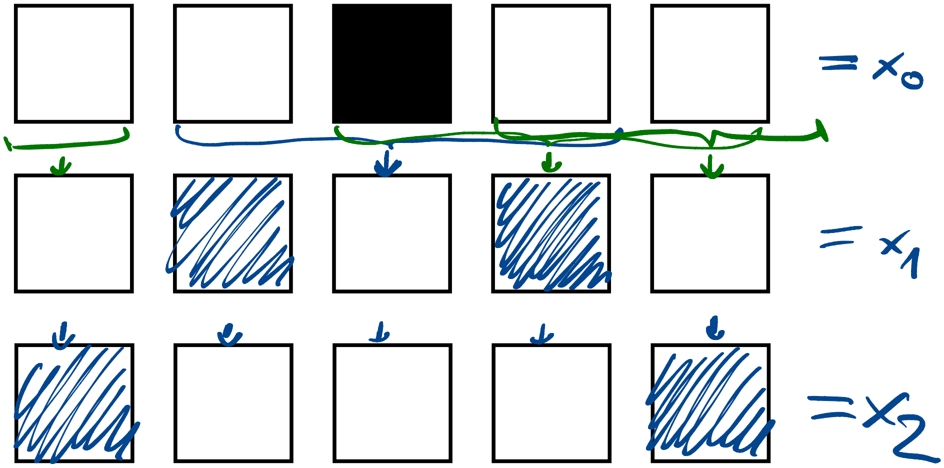
$f =$

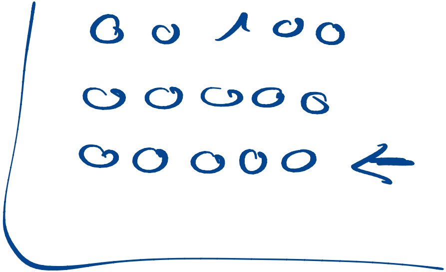


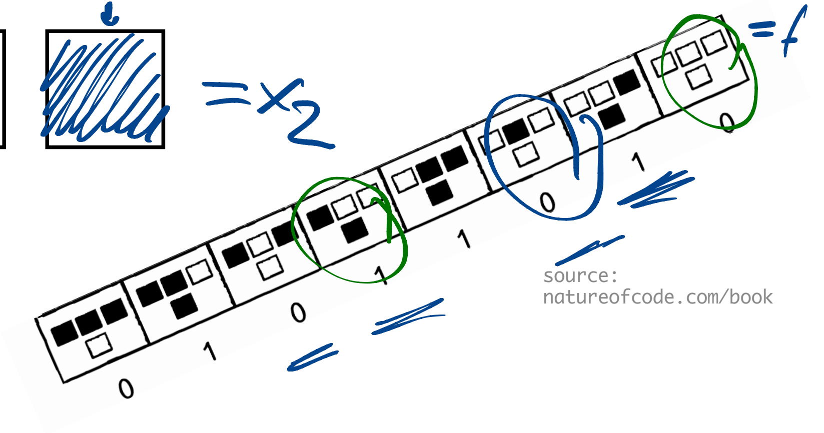
source:
natureofcode.com/book

$x_0 =$

0 0 1 0 0







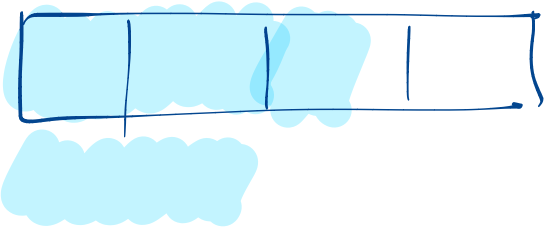
source:
natureofcode.com/book

Why?

Experiment Time!

≤ 16

≤ 10



Experiment 1

What is the average number in the room?

13

12.8

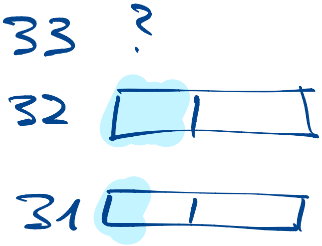
15 ✓

28

Experiment 2

What is the most frequent number in the room?

But whispering only!



Experiment 3

What is the highest number
in the room?

But whispering only!

Experiment Results

	your result	my result
Experiment 1: average	13	12.8
Experiment 2: most frequent – whisper only	28	28
Experiment 3: highest – whisper only	32	32

local information is a constraint on what we can do efficiently

Is local information sufficient for meaningful computation?

When I can use only local information, I might be more ...
- physical - robust