Natural Computing

LMU Munich
summer term 2025

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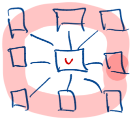
Definition 3 (Conway's game of life (standard)). Let $G = (V, E)$ be a graph with vertices V and (undirected) edges $E \subseteq V \times V$ with a fixed degree of 8 for all nodes. We define *surroundings* : $V \to \wp(V)$ via

$$\mathit{surroundings}(v) = \{w \mid (v, w) \in E\}$$

so that $|\mathit{surroundings}(v)| = 8$ and $v \notin \mathit{surroundings}(v)$ for all $v \in V$. A state $x \in \mathcal{X}$ is a mapping of vertices to the labels $\{\mathit{dead}, \mathit{alive}\}$, i.e., the state space $\mathcal{X}$ is given via $\mathcal{X} = (V \to \{\mathit{dead}, \mathit{alive}\})$. Let x_t be a state that exists at time step $t \in \mathbb{N}$. We define

$$|v|_{x_t} = |\{w \mid w \in \mathit{surroundings}(v) \wedge x_t(w) = \mathit{alive}\}|.$$



$|v|_{x_t}$

In the game of life, the evolution of a state x_t to its subsequent state x_{t+1} is given deterministically via

$$x_{t+1}(v) = \begin{cases} \mathit{dead} & \text{if } |v|_{x_t} \leq 1, \\ x_t(v) & \text{if } |v|_{x_t} = 2, \\ \mathit{alive} & \text{if } |v|_{x_t} = 3, \\ \mathit{dead} & \text{if } |v|_{x_t} \geq 4, \end{cases}$$

for all $v \in V$. A tuple (G, x_0) is called an instance of the game of life for initial state $x_0 \in \mathcal{X}$.

Let's try

conwaylife.com

Still Lifes

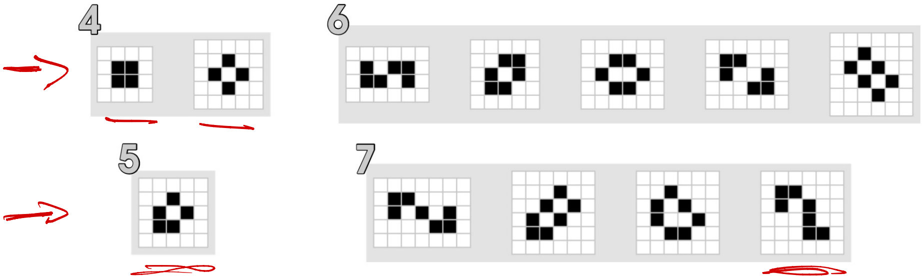


Figure 2.1: All still lifes with 7 or fewer live cells, arranged by their cell count. From left to right: (4 cells) block, tub, (5 cells) boat, (6 cells), snake, ship, beehive, aircraft carrier, barge, (7 cells) long snake, long boat, loaf, and eater 1 (which is sometimes called **fishhook**).

source: conwaylife.com/book/

Oscillators

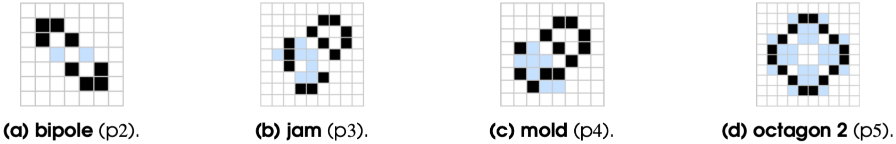


Figure 3.1: Some more small naturally occurring (but rare) oscillators that can be found via computer-assisted soup searches. These oscillators were found by (a) early Lifenthusiasts at M.I.T. in 1970, (b,c) Achim Flammenkamp in 1988,[2] and (d) Sol Goodman and Arthur Taber in 1971.

source: conwaylife.com/book/

Space Ships

glider

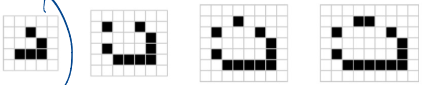
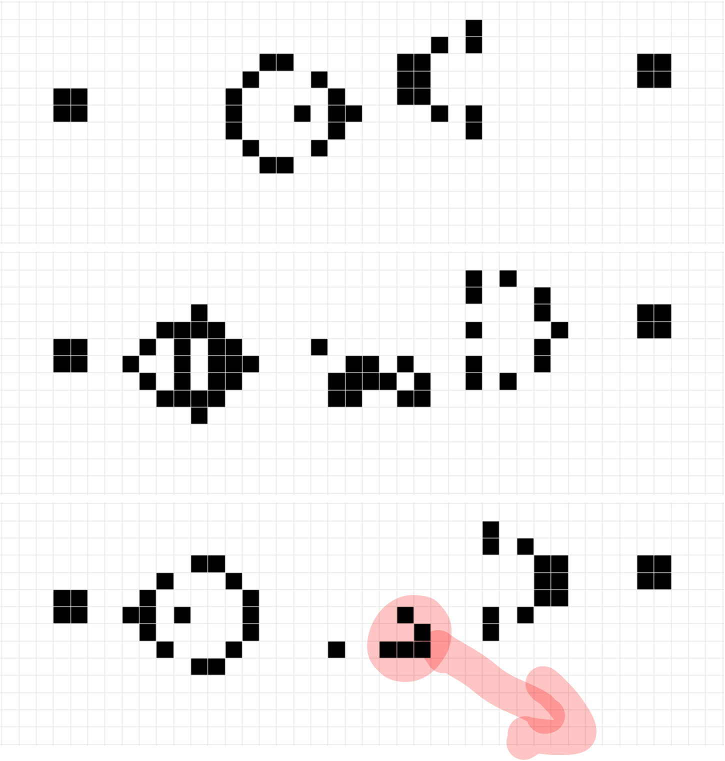


Figure 4.1: The four basic spaceships in Conway's Game of Life. From left to right, these are the glider (which moves diagonally at a speed of $c/4$) and the light/middle/heavyweight spaceships (which each move orthogonally[1] at a speed of $c/2$).

source: conwaylife.com/book/

Guns



t = 1

t = 14

t = 15

try online

source: conwaylife.com

Space Ship Maneuvering

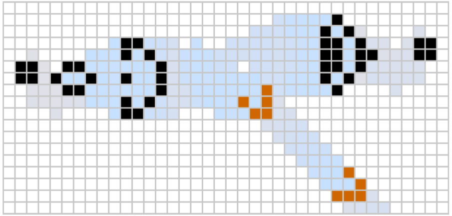


Figure 6.2: A Gosper glider gun producing gliders at a spacing of 30 generations.

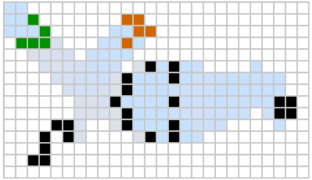


Figure 6.3: A buckaroo can reflect a glider by 90 degrees, from the position marked in green to the one in orange 30 generations later.

source: conwaylife.com/book/

More Space Ship Maneuvering

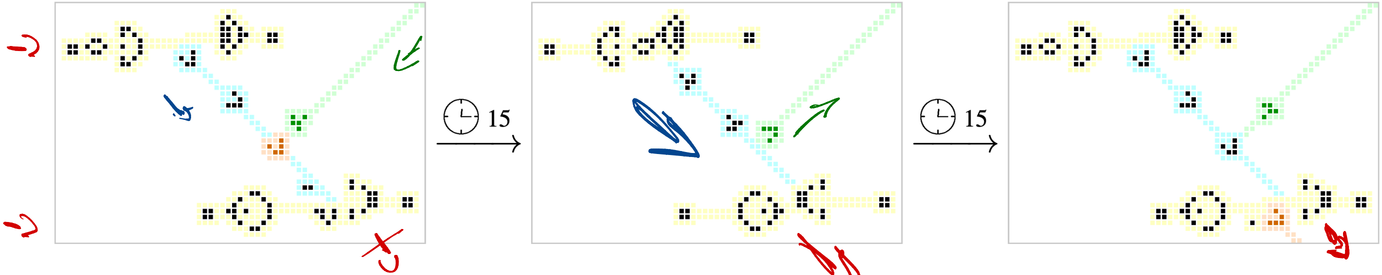


Figure 6.7: Two Gosper glider guns can be placed near each other so as to create a finite stream of gliders (highlighted here in aqua) between them. Here, we bounce a single glider (in green) off of one of those gliders (in orange) so that it is destroyed and thus released by the inline inverter to the southeast.

source: conwaylife.com/book/

conwaylife.com/book/periodic_circuitry

- No initial pattern with simply proven unlimited growth
- Initial patterns with apparently unlimited growth
- Simple initial patterns with behavior for a long period of time

Gardner, 1970.

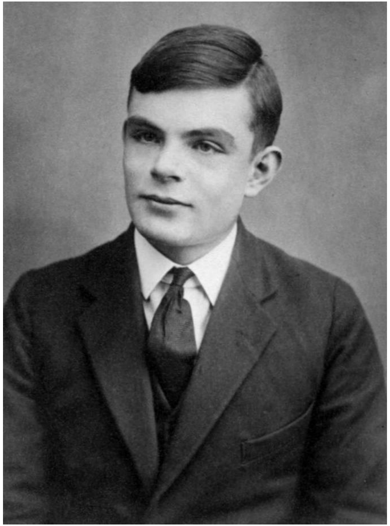
Why is this interesting?

Turing-Completeness

Alan Turing

1912 – 1954

- invented the field of (theoretical) computer science and artificial intelligence
- influential in breaking Enigma and thus shortening WWII



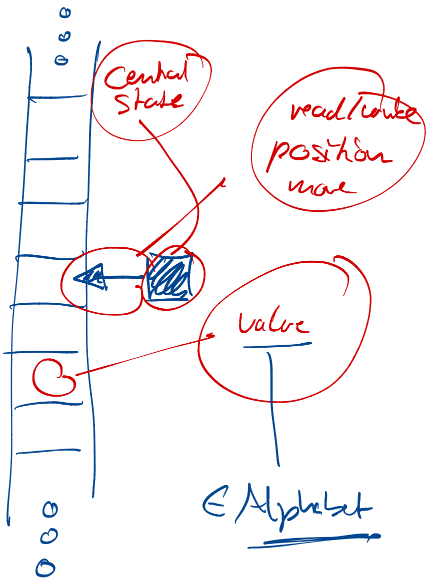
source: Wikipedia

The Turing Machine

initial state x_0

final states F

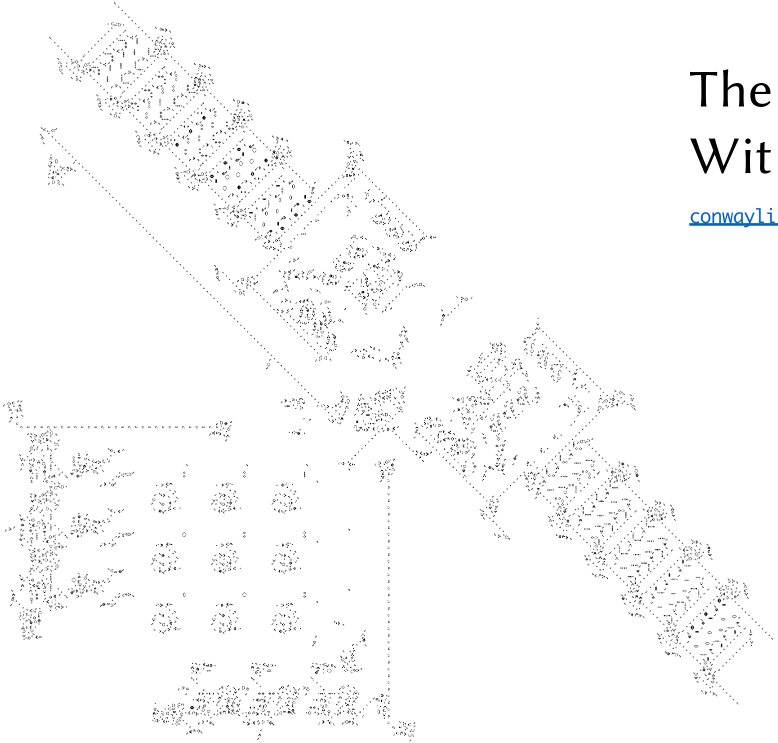
step function / transition function f



The Turing Machine

Definition 4 (Turing machine [1]). A Turing machine is a tuple $(Q, \Sigma, \Gamma, \llcorner\!\lrcorner, q_S, q_F, \delta)$ so that

- Q is a finite set of states,
- Σ is a finite input/output alphabet (e.g., $\Sigma = \{0, 1\}$),
- Γ is a finite tape alphabet with $\Sigma \subset \Gamma$,
- $\llcorner\!\lrcorner \in \Gamma$ is a blank symbol with $\llcorner\!\lrcorner \in \Sigma$,
- $q_S \in Q$ is a start state,
- $q_F \in Q$ is a stop state,
- and $\delta : Q \setminus \{q_F\} \times \Gamma \to Q \times \Gamma \times \{L, R, N\}$ is a partially defined transition function.



The Turing Machine Within the Game of Life

conwaylife.com/wiki/Turing_machine

But… why?

Theorem 1 (Church-Turing thesis)**.** Any computable function can be computed by a Turing machine.

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Theorem 2 (extended Church-Turing thesis). Any computable function can be computed by a Turing machine *with at most polynomial overhead*.

Can cellular automata explain the universe?