

## Homework 2

**Due: April 23rd, 2020**

1. Explain why  $f : \mathbb{R} \rightarrow \mathbb{R}$  is not a function when:
  - (a)  $f(x) = \sqrt{x}$
  - (b)  $f(x) = \pm\sqrt{x}$
  - (c)  $f(x) = \pm\sqrt{|x|}$
  - (d)  $f(x) = |\pm\sqrt{x}|$
  - (e)  $f(x) = \pm\sqrt{x^2}$
2. Let  $F(x, y)$  be the statement, “ $x$  has fooled  $y$ ”. Express the following statements in predicate logic. Quantifiers and DeMorgan’s Laws may be needed.
  - (a) Josephine has fooled Tom
  - (b) Aiden has been fooled by someone
  - (c) Nobody has fooled Bethany
  - (d) Alejandro has not fooled everyone
  - (e) Everybody has fooled somebody
  - (f) Nobody has fooled everybody
3. Let  $n, m \in \mathbb{Z}$ . Determine the truth value of the following quantified statements and briefly explain your answer:
  - (a)  $\forall n \exists m (n^2 < m)$
  - (b)  $\exists m \forall n (n^2 < m)$
  - (c)  $\exists n \exists m (nm = m)$
  - (d)  $\exists n \exists m (n^2 + m^2 = 3)$
4. Let  $x, y \in \mathbb{Z}$ . Find a counterexample to the following quantified statements and briefly explain your answer:
  - (a)  $\forall x \forall y ((x^2 = y^2) \rightarrow (x = y))$
  - (b)  $\forall x \exists y (y^2 = x)$
  - (c)  $\forall x \forall y (xy \geq x)$
5. Let  $A = \{a, b, c, d, e\}$  and  $B = \{a, b, c, d, e, f, g\}$ . Find:
  - (a)  $A \cup B$
  - (b)  $A \cap B$
  - (c)  $A \setminus B$
  - (d)  $B \setminus A$
6. Determine the truth value for the following statements:
  - (a)  $0 \in \emptyset$
  - (b)  $\{0\} \in \{0\}$
  - (c)  $\{\emptyset\} \subseteq \{\emptyset\}$
  - (d)  $\{0\} \subset \{0\}$

For questions 7 through 11, consider the following definition:

**Definition 1.** Let  $A$  and  $B$  be sets. The **Cartesian product** of  $A$  and  $B$  (written  $A \times B$ ) is the set  $\{(a, b) \mid a \in A \text{ and } b \in B\}$

For example, Let  $A = \{1, 2, 3\}$  and  $B = \{a, b\}$ .  $A \times B = \{(1, a), (2, a), (3, a), (1, b), (2, b), (3, b)\}$

Note that the elements of a Cartesian product are ordered 2-tuples.

7. Let  $A = \{1, 3, 5, 7\}$ ,  $B = \{1, 2, 6, 7\}$ , and  $C = \{a, e, f\}$ . Give the following sets:
  - (a)  $(A \cup B) \times C$
  - (b)  $(B \setminus A) \times C$
  - (c)  $A \times (C \setminus B)$
  - (d)  $A \times (B \times C)$
8. Let  $A$  and  $C$  be sets as defined in question 7. Is the statement  $A \times C = C \times A$  true? Briefly explain why.
9. Let  $A$ ,  $B$ , and  $C$  be defined as in question 7. What is the set  $(C \times A) \cap ((B \setminus A) \times C)$ ?
10. Let  $A$  and  $C$  be defined as in question 7 and  $D = \{1, 3\}$ . Is the statment  $D \times C \subseteq A \times C$  true? Briefly explain why.
11. Let  $A$  be the set as defined in question 7. What is  $\emptyset \times A$ ?