

## Homework 8

**Due: June 8th, 2020**

1. Give pseudocode for a recursive algorithm that takes, as input, a positive integer  $n$  and outputs the sum of the first  $n$  integers. For example, for  $n = 4$ , your algorithm should output  $1 + 2 + 3 + 4 = 10$ .
2. State the lemma that you need to show the correctness of your algorithm in question 1
3. Prove your algorithm in question 1 is correct.
4. Give pseudocode for an algorithm that takes, as input, a finite sequence of  $n$  integers and returns the number of negative elements of the sequence. For example, given the sequence:

$$(a_1, a_2, a_3, a_4, a_5) = (-1, 0, -8, 5, 7)$$

your algorithm should return 2.

5. Prove your algorithm in question 4. is correct.
6. Give pseudocode that takes, as input, a finite sequence of  $n$  integers and returns the largest difference between two consecutive elements of the sequence.
7. Prove your algorithm in question 4 is correct.

For problems 8 and 9, consider the sum of all powers of 2 up through the  $n$ th. For example, if  $n = 4$ , then this sum is  $2^0 + 2^1 + 2^2 + 2^3 + 2^4$ .

8.
  - (a) Write a recursive algorithm to solve this problem.
  - (b) Prove your algorithm in part (a) correct.
9. Use one of the previously-proved sums you were asked to memorize to write an algorithm that solves this problem using only one line.
10. Find the time complexity of the following problems:
  - (a) problem 1
  - (b) problem 4
  - (c) problem 6
  - (d) problem 9
11. For the following recurrence relations, use the tree method to solve them. When possible, verify with the Master Theorem
  - (a)  $T(n) = T(n/2) + O(n)$
  - (b)  $T(n) = T(n/2) + O(n^2)$
  - (c)  $T(n) = T(n-2) + O(1)$