Midterm

Due: May 19th, 2020

- You may use your notes, previous lectures, and the textbook when working on this problem.
- You may not discuss your work with anybody else, and you are not allowed to look answers up online. This will result in a 0 on the midterm.
- You may post question to the discussion board, but any midterm-related question must be posted privately.
- Examples and scratch work may be considered for partial credit.
- I cannot assign partial credit if and only if you leave questions blank.
- In the work you turn in, make sure it is clear what problem you are working on.
- The list of theorems you can use without proof are given below. You must prove everything else for full credit, even if we proved it in class (unless you are given theorems in the question). You cannot use a given theorem if you are asked to prove it again.
- This test is difficult, and it is totally possible that you do not finish in time. Because of this, if necessary, I will curve the test up to reflect a grade distribution similar to that of my previous classes. I will also give plenty of partial credit, especially in higher-value questions :)
- Make sure to give each solution plenty of thought before you start writing your final proof. Scratch work will go a long way in this test. If you are uncertain at all about anything in your solution, ask me about it and get your questions resolved *before* starting your proof. Turning in only scratch work will give you a better grade than turning in only incomplete proofs or proofs that are half thought out.

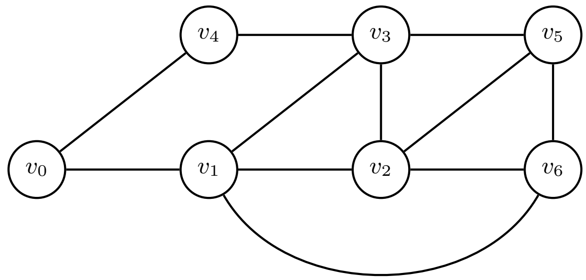
Good luck!!

Theorems you can use without proof unless asked to reprove:

- $p \to q \equiv \neg q \to \neg p$
- $p \equiv \neg p \to F$
- The Biconditional-Conjunction Equivalency
- The Conditional-Disjunction Equivalency
- Double Negation
- Generalized DeMorgan's Laws for propositions
- Generalized Distributive Law for propositions
- DeMorgan's Laws for Predicates (propositional functions)
- Let S be a set. $S \subseteq S$.
- Let S be a set. $\emptyset \subseteq S$.
- Let A and B be sets. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- Generalized Distributive laws for sets
- Generalized DeMorgan's laws for sets
- The three sums you were asked to memorize.
- A connected graph contains an Eulerian circuit if and only if it contains no vertices of odd degree
- A connected graph contains an Eulerian trail if and only if it contains exactly zero or two vertices of odd degree
- The Handshaking Lemma

1. (5 points) Is the statement "I cannot assign partial credit for a question if and only if you leave it blank" logically equivalent to "I can assign partial credit for a question if and only if you do not leave it blank"? Justify your reasoning with a truth table or logical equivalencies.

2. (5 points) Consider the following graph:



   Does it have an Eulerian trail? If yes, give one. If not, briefly justify why.

3. (10 points) Give a recursive definition for the set containing all positive integers that are not divisible by 3 and 5.

For questions 4 and 5, consider the following definition:

Definition 1. Let A and B be sets with universe U. The **symmetric difference** of A and B is the set

$$A \triangle B = \{x \in U \mid (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\} = (A \setminus B) \cup (B \setminus A)$$

4. (10 points) Prove that $A \triangle B = A \cup B$ if and only if $A \cap B = \emptyset$

5. (15 points) Prove that $A \times B \cap B \times A = \emptyset$ if and only if $A \triangle B = A \cup B$

For questions 6 through 8, consider the following definition:

Definition 2. Let $n \in \mathbb{Z}^+$. A **complete graph** on n vertices is a simple graph $K_n = (V, E)$ with $|V| = n$ where, for all vertices $u, v \in V$ such that $u \neq v$, $(u, v) \in E$.

6. (5 points) Let $n \in \mathbb{Z}^+$, $K_n = (V, E)$, and $v \in V$. What is $\deg_{K_n}(v)$?

7. (15 points) Let $n \in \mathbb{Z}^+$ and $K_n = (V, E)$. Prove that K_n is Eulerian if and only if n is odd.

8. (20 points) Let $n \in \mathbb{Z}_{\geq 3}$ and $K_n = (V, E)$. Prove that K_n has $\dfrac{(n-1)!}{2}$ unique cycles of length n.