CSC 348 – Homework #2

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1. (a) The entirety of $\mathbb{R}$ is not in the domain (for example, $f(-1) = \sqrt{-1}$ which is invalid).
   (b) For some inputs, there are multiple outputs (for example, $f(1) = 1, -1$)
   (c) $f(1) = \pm\sqrt{|1|} = \pm\sqrt{1} = \pm 1$ (multiple outputs, not a function)
   (d) $f(-1) = |\pm\sqrt{-1}|$ which has $\sqrt{-1}$, making it invalid
   (e) $f(1) = \pm\sqrt{1^2} = \pm 1$ has multiple outputs
2. (a) $F(\text{Josephine}, \text{Tom})$
   (b) $\exists x \text{``}F(x, \text{Aiden})\text{''}$
   (c) $\forall x \text{``}\neg F(x, \text{Bethany})''$
   (d) $\neg\forall x \text{``}F(\text{Alejandro}, x)''$
   (e) $\forall x \exists y \text{``}F(x, y)''$
   (f) $\neg\exists x \forall y \text{``}F(x, y)''$
3. (a) True, $m = n^2 + 1$ will always work
   (b) This is false for all $m < 0$ because $\forall n(n^2 > 0)$.
   Let $n = \lceil\sqrt{m} + 1\rceil$ for some $m \geq 0$.
   By way of contradiction, assume $n^2 < m$.
   Rearrange terms:
   $$n^2 \geq m + 2\sqrt{m} + 1$$
   $$n^2 - m \geq 2\sqrt{m} + 1$$
   $2\sqrt{m} + 1 > 0$ therefore $n^2 - m > 0$
   Therefore $n^2 > m$ contradicts the original proposition, so this statement is false.
   (c) True. If $n = 1$ it holds true for all m.
   (d) False. All squares are positive, and there are no two squares that add up to 3.

4. (a) Let $x = -1$ and $y = 1$.
   The first half of the conditional is true:
   $$(-1)^2 = 1^2 = 1$$
   However, the second half of the conditional is false:
   $$-1 = 1$$
   Therefore, the conditional is false.

   (b) Let $x = 3$.
   $$y^2 = x = 3$$
   $$y = \sqrt{3} \notin \mathbb{Z}$$
   However, this contradicts $y \in \mathbb{Z}$. Therefore, the statement is false.

   (c) Let $x = 1$ and $y = -1$.
   $$(1)(-1) = 1$$
   $$-1 = 1$$
   However, this is false, therefore there is a contradiction.

5. (a) $A \cup B = \{a, b, c, d, e, f, g\}$
   (b) $A \cap B = \{a, b, c, d, e\}$
   (c) $A \setminus B = \emptyset$
   (d) $B \setminus A = \{f, g\}$

6. (a) False
   (b) False
   (c) True
   (d) False

7. (a)
   $$A \cup B = \{1, 2, 3, 5, 6, 7\}$$
   $$\begin{aligned}(A \cup B) \times C = \{&(1, a), (1, e), (1, f),\\ &(2, a), (2, e), (2, f),\\ &(3, a), (3, e), (3, f),\\ &(5, a), (5, e), (5, f),\\ &(6, a), (6, e), (6, f),\\ &(7, a), (7, e), (7, f)\}\end{aligned}$$

   (b)
   $$B \setminus A = \{6, 7\}$$
   $$\begin{aligned}(B \setminus A) \times C = \{&(6, a), (6, e), (6, f),\\ &(7, a), (7, e), (7, f)\}\end{aligned}$$

(c)

$$\begin{aligned}A \times (C \setminus B) = \{&(1,a),(1,e),(1,f),\\&(2,a),(2,e),(2,f),\\&(3,a),(3,e),(3,f),\\&(5,a),(5,e),(5,f),\\&(6,a),(6,e),(6,f),\\&(7,a),(7,e),(7,f)\}\end{aligned}$$

(d)

$$\begin{aligned}A \times (B \times C) = \{&(1,(1,a)),(1,(1,e)),(1,(1,f)),\\&(1,(2,a)),(1,(2,e)),(1,(2,f)),\\&(1,(6,a)),(1,(6,e)),(1,(6,f)),\\&(1,(7,a)),(1,(7,e)),(1,(7,f)),\\\\&(3,(1,a)),(3,(1,e)),(3,(1,f)),\\&(3,(2,a)),(3,(2,e)),(3,(2,f)),\\&(3,(6,a)),(3,(6,e)),(3,(6,f)),\\&(3,(7,a)),(3,(7,e)),(3,(7,f)),\\\\&(5,(1,a)),(5,(1,e)),(5,(1,f)),\\&(5,(2,a)),(5,(2,e)),(5,(2,f)),\\&(5,(6,a)),(5,(6,e)),(5,(6,f)),\\&(5,(7,a)),(5,(7,e)),(5,(7,f)),\\\\&(7,(1,a)),(7,(1,e)),(7,(1,f)),\\&(7,(2,a)),(7,(2,e)),(7,(2,f)),\\&(7,(6,a)),(7,(6,e)),(7,(6,f)),\\&(7,(7,a)),(7,(7,e)),(7,(7,f))\}\end{aligned}$$

this meme was made by the python gang

8. No, this statement is false. $A \times C = \{(1,a),(3,f),...\}$ while $C \times A = \{(e,1),(a,5),...\}$. In the former, the numbers precede the letters, but in the latter, the letters precede the numbers.

9. $C \times A$ is a set of (letters, numbers) while $(B \setminus A) \times C$ is a set of (numbers, letters).

10. By their values, $D \subseteq A$.

$(D \times C) \subseteq (A \times C)$	
$\equiv \forall (d,c) \in (D \times C)[(d,c) \in (A \times C)]$	Definiton of subset
$\equiv \forall (d,c) \in (D \times C)[d \in A \wedge c \in C]$	Definition of cartesian product
$\equiv \forall (d,c) \in (D \times C)[d \in A \wedge T]$	Definition of c
$\equiv \forall (d,c) \in (D \times C)[d \in A]$	Absorption
$\equiv \forall (d,c) \in (D \times C)[T]$	Implied by $D \subseteq A$
$\equiv T$	Every sub-proposition is true

Therefore, $(D \times C) \subseteq (A \times C)$ is true.

11. $\emptyset$