Homework 7

Due: June 1st, 2020

For all of the following questions, you may cite $\mathbb{N}$ is countably infinite.

For questions 1-7, you may also cite that $\mathbb{Z}^+_{odd}$ and $\mathbb{Z}$ are countably infinite without proof (all proven in class).

1. Determine if $\mathbb{Z}_{\geq 10}$ is finite or countably infinite.
2. Determine if $\mathbb{Z}^-_{odd}$ is finite or countably infinite.
3. Determine if $10\,\mathbb{Z}^+ = \{10n \mid n \in \mathbb{Z}^+\}$ is finite or countably infinite.
4. Determine if $\{x \in \mathbb{Z} \mid |x| < 1,000,000\}$ is finite or countably infinite.
5. Determine if $\{1,2\} \times \mathbb{N}$ is finite or countably infinite.
6. Give an example of two countably infinite sets A and B such that $A \cap B$ is:
   (a) finite
   (b) countably infinite
7. Give an example of two countably infinite sets A and B such that $A \setminus B$ is:
   (a) finite
   (b) countably infinite
8. Recall the following theorems from class:
   - Let A and B be sets. If A and B are countable, then $A \cup B$ is countable.
   - Let A and B be sets. If A and B are infinite, then $A \cup B$ is infinite.
   - Let A and B be sets. If A and B are countably infinite, then $A \cup B$ is countably infinite.

   Find nontrivial (sets that are not empty and do not equal $\mathbb{Z}$) sets A and B such that $A \cup B = \mathbb{Z}$, then use these theorems to show $\mathbb{Z}$ is countably infinite.
9. Let A, B, and C be sets. Prove that, if $|A| = |B|$ and $|B| = |C|$, then $|A| = |C|$.
10. Let A and B be sets. Prove that $|A \cup B| = |A| + |B \setminus A|$
11. Let A and B be sets. Prove that $|A \times B| = |A| \cdot |B|$