

## Homework 6

**Due: May 24th, 2020**

For questions 1-5, let  $f : \mathbb{R} \rightarrow \mathbb{R}$ . Show your results in the form of a proof.

1. Determine whether  $f(n) = n^2 + 1$  is one-to-one.
2. Determine whether  $f(n) = n^5$  is one-to-one.
3. Determine whether  $f(n) = n - 1$  is onto.
4. Determine whether  $f(n) = n^2 + 1$  is onto.
5. Determine whether  $f(n) = n^5$  is onto.

For questions 6-8, give an example of a function from  $\mathbb{N}$  to  $\mathbb{N}$  that satisfies the specified criteria. Prove your result. Try to give a function you haven't seen in class or in the homework yet.

6. one-to-one but not onto
7. onto but not one-to-one
8. both one-to-one and onto.
9. Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x) = x^2 + 1$  and  $g(x) = e^x$ . Find:
  - (a)  $f \circ g$
  - (b)  $g \circ f$
10. Let  $f : B \rightarrow C$  and  $g : A \rightarrow B$ . Prove that, if  $f$  and  $g$  are onto, then  $f \circ g$  is onto.
11. let  $f : B \rightarrow C$  and  $g : A \rightarrow B$  and suppose  $f \circ g$  is onto.
  - (a) Prove or disprove:  $g$  is onto.
  - (b) Prove or disprove:  $f$  is onto.