CSC 348 Homework #1

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1. (a) It is a simple graph because there are no self-loops, and all edges only occur once.
   (b) $V = \{v_0, v_1, v_2, v_3\}$
   $E = \{(v_0, v_1), (v_1, v_2), (v_2, v_3), (v_3, v_0), (v_0, v_2), (v_1, v_3)\}$
2. (a) No, it is not a simple graph. v_2 has a self-loop.
   (b) $V = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$
   $E = \{(v_0, v_1), (v_1, v_2), (v_1, v_4), (v_2, v_2), (v_2, v_4), (v_4, v_3), (v_3, v_0), (v_5, v_7), (v_5, v_8), (v_7, v_8), (v_7, v_6)\}$
3. (a) $\{0, 5\}$
   (b) $\{\emptyset\}$
   (c) $\{\emptyset, 3, 2, 9, \text{“}m''\}$
   (d) $\emptyset$
4. (a) T by definition
   (b) F because natural numbers cannot be negative by definition
   (c) $-\frac{4}{2} = -2$, and $-2 \in \mathbb{Z}$ because $-(-2) \in \mathbb{N}$. Therefore, $-\frac{4}{2} \in \mathbb{N} \equiv T$.
   (d) $-\frac{2}{4} = \frac{p}{q}$ for $p = -1, q = 2$. p and q are both in $\mathbb{Z}$ and they share no common factors except 1. Therefore, $\frac{p}{q} = -\frac{2}{4} \in \mathbb{Q}$ and the statement is equivalent to T.
   (e) $\pi \in \mathbb{R} \equiv T$ by definition.
   (f) $-4i$ has an imaginary component, so $-4i \in \mathbb{R} \equiv F$ by definition.
5. (a) Yes
   (b) Yes
   (c) No, because we don't know what x is.
   (d) No, because it is a question.
   (e) Yes
   (f) Yes
   (g) No, because it is a command.

6. (a) Logic is not important or proofs are not important.
   (b) There are not 30 problems in this homework.
   (c) There are quizzes in this class.
   (d) 6 is a prime number.
7. (a) $p \wedge q$
   (b) $p \wedge \neg q$
   (c) $p \to q$
   (d) $\neg p \vee q$
8. (a)

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

   (b)

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

9. (a)

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

   (b)

p	q	r	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

10. (a) The two columns differ at $(p, q, r) \equiv (F, T, F)$ and $(p, q, r) \equiv (F, F, F)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

(b) The proof is as follows:

i. $(p \wedge q) \to r \equiv (p \to r) \wedge (q \to r)$

ii. $\neg(p \wedge q) \vee r \equiv (\neg p \vee r) \wedge (\neg q \vee r)$ (CDE)

iii. $\neg(p \wedge q) \vee r \equiv (\neg p \wedge \neg q) \vee r$ (Distributive)

iv. $\neg(p \wedge q) \vee r \vee \neg r) \equiv (\neg p \wedge \neg q) \vee r \vee \neg r$ (OR both sides with $\neg r$)

v. $\neg(p \wedge q) \equiv \neg p \wedge \neg q$ (Idempotence, $a \vee \neg a \equiv T$)

vi. $\neg(p \wedge q) \equiv \neg(p \vee q)$ (DeMorgan's)

vii. $\neg\neg(p \wedge q) \equiv \neg\neg(p \vee q)$ (Negate both sides)

viii. $p \wedge q \equiv p \vee q$ (Double Negative)

ix. This statement is false by the following truth table. r does not matter.

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

11. (a) $p \leftrightarrow q$
$\equiv (p \to q) \wedge (q \to p)$ (BDE)
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ (CDE)
$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$ (Associativity)

(b) $\neg q \to \neg p$
$\equiv \neg\neg q \vee \neg p$ (CDE)
$\equiv q \vee \neg p$ (Double Negative)
$\equiv \neg(\neg q \wedge \neg\neg p)$ (DeMorgan's)
$\equiv \neg(\neg q \wedge p)$ (Double Negative)

(c) $p \to (q \wedge r)$
$\equiv \neg p \vee (q \wedge r)$ (CDE)
$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ (Distributive)
$\equiv (p \to q) \wedge (p \to r)$ (CDE)