Homework 3
Due: April 30th, 2020

1. Let A and B be sets with some universe U. Prove that $A \setminus B = A \cap \overline{B}$.

2. Let A, B, and C be sets. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3. Let A, B, and C be sets. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [1]

4. Let A and B be sets. Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$

5. Let A and B be sets. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ [2]

6. Let $m, n, p \in \mathbb{Z}$. Prove that, if $m + n$ and $n + p$ are even, then $m + p$ is even.

7. Using the notion of "Without Loss of Generality", prove $\min(x, y) = \dfrac{x + y - |x - y|}{2}$

8. Using the notion of "Without Loss of Generality", prove $\max(x, y) = \dfrac{x + y + |x - y|}{2}$

Definition 1. Let a and b be integers. a **divides** b if there exists an integer k such that $ak = b$. This is written as $a \mid b$

Consider definition **1** for the following questions:

9. Give the truth value of each the following statements. If true, write what value of k you used. If false, write a short explanation why.

   (a) "3 | 27"
   (b) "−2 | 8"
   (c) "4 | 1"
   (d) "−2 | 7"
   (e) "0 | − 17"
   (f) "3 | − 17"
   (g) "6 | 0"

10. Let $D(x, y)$ be the statement "x divides y". Give the truth values for the following statements:

    (a) $D(2, 9)$
    (b) $D(7, 49)$
    (c) $D(26, -52)$
    (d) $\forall x D(x, 0)$
    (e) $\forall y D(1, y)$
    (f) $\forall x \forall y D(x, y)$
    (g) $\exists x \forall y D(x, y)$

11. Let a, b, and c be integers. Use a direct proof to show that, if $a \mid b$ and $b \mid c$, then $a \mid c$.

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[1]Note: questions 3 and 4 are known as the **Distributive Laws for Sets**
[2]Note: questions 5 and 6 are known as **DeMorgan's Laws for Sets**