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Contest (1)

template.cpp

```cpp
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define trav(a, x) for(auto& a : x)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
  cin.sync_with_stdio(0); cin.tie(0);
  cin.exceptions(cin.failbit);
}
```

.bashrc

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++14 \
  -fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps = <>
```

.vimrc

```
set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul
sy on   |   im jk <esc>   |   im kj <esc>   |   no ; :
" Select region and then type :Hash to hash your selection.
" Useful for verifying that there aren't mistypes.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
 \| md5sum \| cut -c-6
```

hash.sh

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

troubleshoot.txt

```
Pre-submit:
Write a few simple test cases, if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all datastructures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a team mate.
Ask the team mate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a team mate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your team mates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all datastructures between test cases?
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where A'_i is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b + c}\right)^2\right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

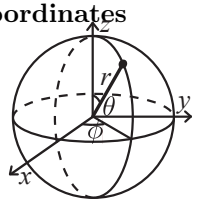
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned} area &= 4\pi R^3/3 \\ x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \operatorname{atan2}(y,x) \end{aligned}$$

2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1+\tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

1 ↓ Natural numbers, $n = C(n, 1)$
1 1 ↓ Triangular numbers, $T_n = C(n+1,2)$
1 2 1 ↓ Tetrahedral numbers, $Te_n = C(n+2,3)$
1 3 3 1 ↓ Pentatope numbers $= C(n+3,4)$
1 4 6 4 1 ↓ 5-simplex ({3,3,3,3}) numbers
1 5 10 10 5 1 ↓ 6-simplex ({3,3,3,3,3}) numbers
1 6 15 20 15 6 1 ↓ 7-simplex ({3,3,3,3,3,3}) numbers
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\operatorname{Bin}(n,p)$, $n = 1, 2, \ldots, 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\operatorname{Bin}(n,p)$ is approximately $\operatorname{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\operatorname{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\operatorname{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\operatorname{U}(a,b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\operatorname{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki} t_k$.

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

HashMap.h

Description: Hash map with the same API as unordered_map, but ~3x faster. Initial capacity must be a power of 2 (if provided).

1443bc, 7 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash {
  const uint64_t C = ll(2e18 * M_PI) + 71; // large odd number
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16});
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming.
Time: $\mathcal{O}(\log N)$

95e223, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
Time: $\mathcal{O}\left(\log^2 N\right)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" b28c27, 22 lines

```
struct FT2 {
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
  }
  void init() {
    trav(v, ys) sort(all(v)), ft.emplace_back(sz(v));
  }
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()); }
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }
};
```

RMQ.h

Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
Usage: `RMQ rmq(values);`
`rmq.query(inclusive, exclusive);`
Time: $\mathcal{O}(|V| \log |V| + Q)$

1f8996, 17 lines

```
template<class T>
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) {
    int N = sz(V), on = 1, depth = 1;
    while (on < N) on *= 2, depth++;
    jmp.assign(depth, V);
    rep(i,0,depth-1) rep(j,0,N)
      jmp[i+1][j] = min(jmp[i][j],
      jmp[i][min(N - 1, j + (1 << i))]);
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};
```

MaxSparseTable.java

Description: Sparse table for max function.

c41092, 36 lines

```
class MaxSparseTable {
  static final int maxn = (int) 1e5;
  static int[] logs;
  int[][] st;

  MaxSparseTable(int[] a) {
    initLogs();
    int n = a.length;
    int maxLog = 1;
    while (1 << maxLog <= n) maxLog++;
    st = new int[n][maxLog];
```

```
    for (int i = n - 1; i >= 0; i--) {
      st[i][0] = a[i];
      for (int log = 1; i + (1 << log) <= n; log++) {
        st[i][log] = max(
          st[i][log - 1],
          st[i + (1 << (log - 1))][log - 1]);
      }
    }
  }

  void initLogs() {
    if (logs != null) return;
    logs = new int[maxn + 1];
    for (int i = 1, log = 0; i <= maxn; i++) {
      if ((1 << (log + 1)) == i) log++;
      logs[i] = log;
    }
  }

  int get(int l, int r) {
    int len = r - l + 1;
    int log = logs[len];
    return max(st[l][log], st[r - (1 << log) + 1][log]);
  }
}
```

RangeAddSingleGetFenwickTree.java

Description: Fenwick tree with add on segment.

9cee6c, 35 lines

```
class RangeAddSingleGetFenwickTree {
  int n;
  long[] sum, fun;

  RangeAddSingleGetFenwickTree(int n) {
    this.n = ++n;
    sum = new long[n];
    fun = new long[n];
  }

  void add(int at, long sumValue, long funValue) {
    for (at++; at < n; at += at & -at) {
      sum[at] += sumValue;
      fun[at] += funValue;
    }
  }

  void addConstSegment(int l, int r, long value) {
    add(l, value, 0);
    add(r + 1, -value, 0);
  }

  void addFunctionSegment(int l, int r, long value) {
    add(l, -(l - 1) * value, value);
    add(r + 1, (l - 1) * value, -value);
  }

  long get(int at) {
    int at0 = at;
    long result = 0;
    for (at++; at > 0; at -= at & -at)
      result += sum[at] + fun[at] * at0;
    return result;
  }
}
```

Treap.java

Description: Treap with tree reverse and k-th order statistics

61915b, 97 lines

```
class Treap {
  static Random rnd = new Random(239);
  long priority = rnd.nextLong();
  int size = 1;
  boolean needSwap;
  int value;
  Treap left;
  Treap right;
  Treap parent;

  Treap(int value) {
    this.value = value;
  }

  static Treap merge(Treap t1, Treap t2) {
    update(t1);
    update(t2);
    if (t1 == null) return t2;
    if (t2 == null) return t1;
    if (t1.priority > t2.priority) {
      t1.right = merge(t1.right, t2);
      update(t1);
      return t1;
    } else {
      t2.left = merge(t1, t2.left);
      update(t2);
      return t2;
    }
  }

  static Split split(Treap t, int size) {
    if (t == null) return Split.empty;
    update(t);
    int leftSize = 1 + size(t.left);
    if (leftSize <= size) {
      Split split = split(t.right, size - leftSize);
      t.right = split.left;
      if (split.right != null) split.right.parent = null;
      update(t);
      return new Split(t, split.right);
    } else {
      Split split = split(t.left, size);
      t.left = split.right;
      if (split.left != null) split.left.parent = null;
      update(t);
      return new Split(split.left, t);
    }
  }

  static void reverse(Treap t, int l, int r) {
    int size = r - l + 1;
    Split leftRight = split(t, l);
    Split middleRight = split(leftRight.right, size);
    middleRight.left.needSwap = true;
    update(middleRight.left);
    merge(leftRight.left,
          merge(middleRight.left, middleRight.right));
  }

  static void update(Treap t) {
    if (t == null) return;
    t.size = 1 + size(t.left) + size(t.right);
    if (t.left != null) t.left.parent = t;
    if (t.right != null) t.right.parent = t;
    if (t.needSwap) {
      Treap tmp = t.left;
      t.left = t.right;
      t.right = tmp;
      if (t.left != null) t.left.needSwap ^= true;
      if (t.right != null) t.right.needSwap ^= true;
      t.needSwap = false;
    }
  }

  static void updateTillRoot(Treap t) {
    if (t == null) return;
    updateTillRoot(t.parent);
    update(t);
  }

  int index() {
    updateTillRoot(this);
    int index = size(left);
    for (Treap t = this; t.parent != null; t = t.parent)
      if (t.parent.right == t)
        index += size(t.parent.left) + 1;
    return index;
  }

  static int size(Treap t) { return t == null ? 0 : t.size; }

  static class Split {
    Treap left;
    Treap right;
    static final Split empty = new Split(null, null);
  }
}
```

LiChaoTree.java

Description: Li Chao (segment) Tree.

8912bf, 46 lines

```
class LiChaoTree {
  Line[] tree;
  int n;

  LiChaoTree(int size) {
    n = Math.max(1, Integer.highestOneBit(size - 1) << 1);
    tree = new Line[2 * n];
    Arrays.fill(tree, new Line(-1, 0, (long) 1e18));
  }

  void add(Line line) { add(1, 0, n - 1, line); }

  void add(int v, int tl, int tr, Line line) {
    if (tl == tr) {
      if (line.value(tl) < tree[v].value(tl)) tree[v] = line;
      return;
    }
    int tm = tl + tr >> 1;
    int signLeft = Long.compare(tree[v].value(tl), line.value(
        tl));
    int signMid  = Long.compare(tree[v].value(tm), line.value(
        tm));
    if (signLeft == signMid || signLeft == 0 || signMid == 0) {
      int x = signLeft == 0 ? tl + 1 : tl;
      Line lower = line.value(x) <= tree[v].value(x) ? line :
          tree[v];
      Line other = lower == line ? tree[v] : line;
      tree[v] = lower;
      add(v * 2 + 1, tm + 1, tr, other);
    } else {
      Line lower = line.value(tr) <= tree[v].value(tr) ? line :
          tree[v];
      Line other = lower == line ? tree[v] : line;
      tree[v] = lower;
      add(v * 2, tl, tm, other);
    }
  }
```

```
  Line getLowest(int x) { return getLowest(1, 0, n - 1, x); }

  Line getLowest(int v, int tl, int tr, int x) {
    if (tl == tr) return tree[v];
    int tm = tl + tr >> 1;
    Line res = x <= tm
      ? getLowest(v * 2, tl, tm, x)
      : getLowest(v * 2 + 1, tm + 1, tr, x);
    if (tree[v].value(x) < res.value(x)) res = tree[v];
    return res;
  }
}
```

Numerical (4)

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}(\log((b-a)/\epsilon))$

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
}
```

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for(int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" 2cf190, 23 lines

```
vector<double> poly_roots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = poly_roots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `BerlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$

"../number-theory/ModPow.h" 40387d, 20 lines

```
vector<ll> BerlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  trav(x, C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots \geq n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions.

f40e55, 16 lines

```
typedef array<double, 2> P;

double func(P p);

pair<double, P> hillClimb(P start) {
  pair<double, P> cur(func(start), start);
  for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
    rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {
      P p = cur.second;
      p[0] += dx*jmp;
      p[1] += dy*jmp;
      cur = min(cur, make_pair(func(p), p));
    }
  }
  return cur;
}
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

4756fc, 7 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
  double h = (b - a) / 2 / n, v = f(a) + f(b);
  rep(i,1,n*2)
    v += f(a + i*h) * (i&1 ? 4 : 2);
```

```
  return v * h / 3;
}
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.
Usage: `double sphereVolume = quad(-1, 1, [](double x) {`
`return quad(-1, 1, [&](double y) {`
`return quad(-1, 1, [&](double z) {`
`return x*x + y*y + z*z < 1; });});});`

92dd79, 15 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
  d c = (a + b) / 2;
  d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
}
```

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage: `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`
Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

aa8530, 68 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2 m)$

44c9ab, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" 08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}(n^2 m)$

fa2d7a, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank $< n$). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

4.1 Fourier transforms

NumberTheoreticTransform.h

Description: Can be used for convolutions modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. Inputs must be in [0, mod).
Time: $\mathcal{O}(N \log N)$

"../number-theory/ModPow.h" d75aad, 32 lines

```
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.

typedef vector<ll> vl;
void ntt(vl& a, vl& rt, vl& rev, int n) {
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
      a[i + j + k] = (z > ai ? ai - z + mod : ai - z);
      ai += (ai + z >= mod ? z - mod : z);
    }
}

vl conv(const vl& a, const vl& b) {
  if (a.empty() || b.empty())
    return {};
  int s = sz(a)+sz(b)-1, B = 32 - __builtin_clz(s), n = 1 << B;
  vl L(a), R(b), out(n), rt(n, 1), rev(n);
  L.resize(n), R.resize(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << B) / 2;
  ll curL = mod / 2, inv = modpow(n, mod - 2);
  for (int k = 2; k < n; k *= 2) {
    ll z[] = {1, modpow(root, curL /= 2)};
    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
  }
  ntt(L, rt, rev, n); ntt(R, rt, rev, n);
  rep(i,0,n) out[-i & (n-1)] = L[i] * R[i] % mod * inv % mod;
  ntt(out, rt, rev, n);
  return {out.begin(), out.begin() + s};
}
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

3de473, 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) trav(x, a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

FFT.java

Description: FFT on real and imag double arrays.

6181c7, 86 lines

```
static class FFTArrays {
```

```
void fft(double[] real, double[] imag, boolean inv) {
  int n = real.length;
  int m = 0;
  while (1 << m < n) m++;

  int[] rev = new int[n];
  for (int i = 1, j = -1; i < n; i++) {
    if ((i & (i - 1)) == 0) j++;
    rev[i] = rev[i ^ (1 << j)] | (1 << (m - 1 - j));
  }
  for (int i = 0; i < n; i++) {
    if (i < rev[i]) {
      swap(real, i, rev[i]);
      swap(imag, i, rev[i]);
    }
  }

  for (int len = 1; len < n; len <<= 1) {
    double root = Math.PI / len;
    if (inv) root = -root;
    double rootReal = Math.cos(root);
    double rootImag = Math.sin(root);
    for (int i = 0; i < n; i += len * 2) {
      double wReal = 1;
      double wImag = 0;
      for (int j = 0; j < len; j++) {
        double real1 = real[i + j];
        double imag1 = imag[i + j];
        double real2 = real[i + j + len];
        double imag2 = imag[i + j + len];
        double real3 = real2 * wReal - imag2 * wImag;
        double imag3 = real2 * wImag + imag2 * wReal;
        real[i + j] = real1 + real3;
        imag[i + j] = imag1 + imag3;
        real[i + j + len] = real1 - real3;
        imag[i + j + len] = imag1 - imag3;
        double nextwr = wReal * rootReal - wImag * rootImag;
        double nextwi = wReal * rootImag + wImag * rootReal;
        wReal = nextwr;
        wImag = nextwi;
      }
    }
  }

  if (inv) {
    for (int i = 0; i < n; i++) {
      real[i] /= n;
      imag[i] /= n;
    }
  }
}

void swap(double[] a, int i, int j) {
    double t = a[i];
    a[i] = a[j];
    a[j] = t;
}

double[] copy(int[] a, int n) {
    double[] b = new double[n];
    for (int i = 0; i < a.length; i++) b[i] = a[i];
    return b;
}

long[] multiply(int[] a, int[] b) {
  int n = Math.max(1,
    highestOneBit(max(a.length, b.length) - 1) << 2);
  double[] aReal = copy(a, n);
  double[] aImag = new double[n];
    double[] bReal = copy(b, n);
    double[] bImag = new double[n];
    fft(aReal, aImag, false);
    fft(bReal, bImag, false);
    for (int i = 0; i < n; i++) {
      double real = aReal[i] * bReal[i] - aImag[i] * bImag[i];
      double imag = aReal[i] * bImag[i] + aImag[i] * bReal[i];
      aReal[i] = real;
      aImag[i] = imag;
    }
    fft(aReal, aImag, true);
    long[] res = new long[n];
    for (int i = 0; i < n; i++) res[i] = round(aReal[i]);
    return res;
  }
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set `mod` to some number first and then you can use the structure.

"euclid.h" 35bfea, 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime. 6f684f, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

b83e45, 8 lines

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

ModLog.h

Description: Returns the smallest $x \geq 0$ s.t. $a^x = b \pmod m$. a and m must be coprime.

Time: $\mathcal{O}\left(\sqrt{m}\right)$ 49d606, 10 lines

```
ll modLog(ll a, ll b, ll m) {
  assert(__gcd(a, m) == 1);
  ll n = (ll) sqrt(m) + 1, e = 1, x = 1, res = LLONG_MAX;
  unordered_map<ll, ll> f;
  rep(i,0,n) e = e * a % m;
  rep(i,0,n) x = x * e % m, f.emplace(x, i + 1);
  rep(i,0,n) if (f.count(b = b * a % m))
    res = min(res, f[b] * n - i - 1);
  return res;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions. `modsum(to, c, k, m)` $= \sum_{i=0}^{to-1} (ki + c)\%m$. `divsum` is similar but for floored division.

Time: $\log(m)$, with a large constant. 5c5bc5, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b < c < 2^{63}$.

Time: $\mathcal{O}(1)$ for `mod_mul`, $\mathcal{O}(\log b)$ for `mod_pow` 88c37a, 12 lines

```
typedef unsigned long long ull;
typedef long double ld;
ull mod_mul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(ld(a) * ld(b) / ld(M));
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull mod_pow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = mod_mul(b, b, mod), e /= 2)
    if (e & 1) ans = mod_mul(ans, b, mod);
  return ans;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).

Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 19a793, 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
```

```
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

5.2 Primality

eratosthenes.h

Description: Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.
Time: lim=100'000'000 $\approx$ 0.8 s. Runs 30% faster if only odd indices are stored.

29cd0a, 11 lines

```
const int MAX_PR = 5'000'000;
bitset<MAX_PR> isprime;
vi eratosthenes_sieve(int lim) {
  isprime.set(); isprime[0] = isprime[1] = 0;
  for (int i = 4; i < lim; i += 2) isprime[i] = 0;
  for (int i = 3; i*i < lim; i += 2) if (isprime[i])
    for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
  vi pr;
  rep(i,2,lim) if (isprime[i]) pr.push_back(i);
  return pr;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to 2^{64}; for larger numbers, extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 6ab8e1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return n - 2 < 2;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  trav(a, A) {   // ^ count trailing zeroes
    ull p = mod_pow(a, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = mod_mul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$ gcd calls, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" f5adaa, 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return (mod_mul(x, x, n) + 1) % n; };
  if (!(n & 1)) return 2;
  for (ull i = 2;; i++) {
    ull x = i, y = f(x), p;
    while ((p = __gcd(n + y - x, n)) == 1)
      x = f(x), y = f(f(y));
    if (p != n) return p;
  }
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

FastC.java

Description: Linear c(n,k).

7fabd1, 6 lines

```
int fast_c(int n, int k) {
  double res = 1;
  for (int i = 1; i <= k; i++)
    res = res * (n - k + i) / i;
  return (int) (res + 0.01);
}
```

Rational.java

Description: Rational operations.

a04785, 96 lines

```
class Rational implements Comparable<Rational> {
  static final Rational MAX_VALUE = new Rational(MAX_VALUE, 1);
  static final Rational MIN_VALUE = new Rational(MIN_VALUE, 1);
  static final Rational ONE = new Rational(1, 1);
  static final Rational ZERO = new Rational(0, 1);

  long numerator;
  long denominator;

  Rational(long numerator, long denominator) {
    if (denominator == 0)
      throw new IllegalArgumentException();
    long gcd = gcd(abs(numerator), abs(denominator));
    if (denominator > 0) {
      this.numerator = numerator / gcd;
      this.denominator = denominator / gcd;
    } else {
      this.numerator = -numerator / gcd;
      this.denominator = -denominator / gcd;
    }
  }

  int compareTo(Rational other) {
    return compare(numerator * other.denominator,
                   denominator * other.numerator);
  }

  Rational add(Rational other) {
    return new Rational(
      numerator * other.denominator +
      denominator * other.numerator,
      denominator * other.denominator
    );
  }

  Rational reverse() {
    if (numerator == 0)
      throw new ArithmeticException();
    return new Rational(denominator, numerator);
  }

  Rational multiply(long number) {
    return new Rational(numerator * number, denominator);
  }

  Rational subtract(Rational other) {
    return new Rational(
      numerator * other.denominator -
       denominator * other.numerator,
      denominator * other.denominator
    );
  }

  @Override
  public boolean equals(Object o) {
    Rational rational = (Rational) o;
    if (denominator != rational.denominator) return false;
    if (numerator != rational.numerator)     return false;
    return true;
  }

  Rational divide(long number) {
    return new Rational(numerator, denominator * number);
  }

  long floor() {
    return numerator >= 0
      ? numerator / denominator
      : (numerator - denominator + 1) / denominator;
  }

  long ceil() {
    return numerator >= 0
      ? (numerator + denominator - 1) / denominator
      : numerator / denominator;
  }

  Rational divide(Rational other) {
    return new Rational(numerator * other.denominator,
                        other.numerator * denominator);
  }

  Rational multiply(Rational other) {
    return new Rational(numerator * other.numerator,
                        other.denominator * denominator);
  }

  double value() {
    return (double) numerator / denominator;
  }

  Rational abs() {
    if (numerator >= 0) return this;
    return new Rational(-numerator, denominator);
  }
}
```

SegmentedEratosthenesSieve.java

Description: Prime sieve for generating all primes up to a certain limit.
Time: $\mathcal{O}(n \log n)$

844248, 37 lines

```
class SegmentedEratosthenesSieve {
  int BLOCK_SIZE = 10000;  //1e4..1e5 is the best

  int sieve(int n) {
    int nsqrt = (int) sqrt(n + .1);
    boolean[] isPrime = new boolean[nsqrt + 1];
    fill(isPrime, true);
    int[] primes = new int[nsqrt + 1];
    int cnt = 0;
    for (int i = 2; i <= nsqrt; i++) {
      if (isPrime[i]) {
        primes[cnt++] = i;
        if (i * (long) i <= nsqrt)
          for (int j = i * i; j <= nsqrt; j += i)
```

```
            isPrime[j] = false;
        }
    }
    int result = 0;
    boolean[] block = new boolean[BLOCK_SIZE];
    for (int k = 0; k <= n / BLOCK_SIZE; k++) {
      fill(block, true);
      int start = k * BLOCK_SIZE;
      for (int i = 0; i < cnt; i++) {
        int startIdx = (start + primes[i] - 1) / primes[i];
        int j = max(startIdx, 2) * primes[i] - start;
        for (; j < BLOCK_SIZE; j += primes[i])
          block[j] = false;
      }
      if (k == 0)
        block[0] = block[1] = false;
      for (int i = 0; i < BLOCK_SIZE && start + i <= n; i++)
        if (block[i])
          result++;
    }
    return result;
  }
}
```

5.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

ee6239, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (b) { ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d; }
  return x = 1, y = 0, a;
}
```

Euclid.java

Description: Finds {x, y, d} s.t. ax + by = d = gcd(a, b).

e49211, 26 lines

```
static BigInteger[] euclid(BigInteger a, BigInteger b) {
  BigInteger x = BigInteger.ONE, yy = x;
  BigInteger y = BigInteger.ZERO, xx = y;
  while (b.signum() != 0) {
    BigInteger q = a.divide(b), t = b;
    b = a.mod(b); a = t;
    t = xx; xx = x.subtract(q.multiply(xx)); x = t;
    t = yy; yy = y.subtract(q.multiply(yy)); y = t;
  }
  return new BigInteger[]{x, y, a};
}
```

```
long _x;
long _y;
long extGcd(long a, long b) {
  if (a == 0) {
    _x = 0;
    _y = 1;
    return b;
  }
  long d = extGcd(b % a, a);
  long nx = _y - (b / a) * _x;
  _y = _x;
  _x = nx;
  return d;
}
```

CRT.h

Description: Chinese Remainder Theorem.

crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) :=$ # of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1}p_2^{k_2}...p_r^{k_r}$ then $\phi(n) = (p_1-1)p_1^{k_1-1}...(p_r-1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.

Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p \ \forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for(int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for(int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \ge 0$, finds the closest rational approximation p/q with $p, q \le N$. It will obey $|p/q - x| \le 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.

Time: $\mathcal{O}(\log N)$

dd6c5e, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives us a
      // better approximation; if b = a/2, we *may* have one.
      // Return {P, Q} here for a more canonical approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \le N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}

Time: $\mathcal{O}(\log(N))$

27ab3e, 25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ \ b = k \cdot (2mn), \ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.7 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.)
Time: $\mathcal{O}(n)$ e1b8ea, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  trav(x, v) r = r * ++i + __builtin_popcount(use & -(1 << x)),
    use |= 1 << x;                    // (note: minus, not ~!)
  return r;
}
```

6.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Binomials

binomialModPrime.h

Description: Lucas' thm: Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$. fact and invfact must hold pre-computed factorials / inverse factorials, e.g. from ModInverse.h.
Time: $\mathcal{O}(\log_p n)$ 81845f, 10 lines

```
ll chooseModP(ll n, ll m, int p, vi& fact, vi& invfact) {
  ll c = 1;
  while (n || m) {
    ll a = n % p, b = m % p;
    if (a < b) return 0;
    c = c * fact[a] % p * invfact[b] % p * invfact[a - b] % p;
    n /= p; m /= p;
  }
  return c;
}
```

multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!\ldots k_n!}$. a0a312, 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able).
$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$
$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$
$$E(n,0) = E(n,n-1) = 1$$
$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$
$$S(n,1) = S(n,n) = 1$$
$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

6.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Euler walk

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, also put it->second in s (and then ret).

Time: $\mathcal{O}(E)$ where E is the number of edges.

f8bd47, 27 lines

```
struct V {
  vector<pii> outs; // (dest, edge index)
  int nins = 0;
};

vi euler_walk(vector<V>& nodes, int nedges, int src=0) {
  int c = 0;
  trav(n, nodes) c += abs(n.nins - sz(n.outs));
  if (c > 2) return {};
  vector<vector<pii>::iterator> its;
  trav(n, nodes)
    its.push_back(n.outs.begin());
  vector<bool> eu(nedges);
  vi ret, s = {src};
  while(!s.empty()) {
    int x = s.back();
    auto& it = its[x], end = nodes[x].outs.end();
    while(it != end && eu[it->second]) ++it;
    if(it == end) { ret.push_back(x); s.pop_back(); }
    else { s.push_back(it->first); eu[it->second] = true; }
  }
  if(sz(ret) != nedges+1)
    ret.clear(); // No Eulerian cycles/paths.
  // else, non-cycle if ret.front() != ret.back()
  reverse(all(ret));
  return ret;
}
```

7.2 Network flow

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$

6c4045, 49 lines

```
typedef ll Flow;
struct Edge {
  int dest, back;
  Flow f, c;
};

struct PushRelabel {
  vector<vector<Edge>> g;
  vector<Flow> ec;
  vector<Edge*> cur;
  vector<vi> hs; vi H;
  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}

  void add_edge(int s, int t, Flow cap, Flow rcap=0) {
    if (s == t) return;
    g[s].push_back({t, sz(g[t]), 0, cap});
    g[t].push_back({s, sz(g[s])-1, 0, rcap});
  }

  void add_flow(Edge& e, Flow f) {
    Edge &back = g[e.dest][e.back];
    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
    e.f += f; e.c -= f; ec[e.dest] += f;
    back.f -= f; back.c += f; ec[back.dest] -= f;
  }
  Flow maxflow(int s, int t) {
    int v = sz(g); H[s] = v; ec[t] = 1;
    vi co(2*v); co[0] = v-1;
    rep(i,0,v) cur[i] = g[i].data();
    trav(e, g[s]) add_flow(e, e.c);

    for (int hi = 0;;) {
      while (hs[hi].empty()) if (!hi--) return -ec[s];
      int u = hs[hi].back(); hs[hi].pop_back();
      while (ec[u] > 0)  // discharge u
        if (cur[u] == g[u].data() + sz(g[u])) {
          H[u] = 1e9;
          trav(e, g[u]) if (e.c && H[u] > H[e.dest]+1)
            H[u] = H[e.dest]+1, cur[u] = &e;
          if (++co[H[u]], !--co[hi] && hi < v)
            rep(i,0,v) if (hi < H[i] && H[i] < v)
              --co[H[i]], H[i] = v + 1;
          hi = H[u];
        } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
          add_flow(*cur[u], min(ec[u], cur[u]->c));
        else ++cur[u];
    }
  }
};
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

Time: $\mathcal{O}\left(V^3\right)$

03261f, 31 lines

```
pair<int, vi> GetMinCut(vector<vi>& weights) {
  int N = sz(weights);
  vi used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    vi w = weights[0], added = used;
    int prev, k = 0;
    rep(i,0,phase){
      prev = k;
      k = -1;
      rep(j,1,N)
        if (!added[j] && (k == -1 || w[j] > w[k])) k = j;
      if (i == phase-1) {
        rep(j,0,N) weights[prev][j] += weights[k][j];
        rep(j,0,N) weights[j][prev] = weights[prev][j];
        used[k] = true;
        cut.push_back(k);
        if (best_weight == -1 || w[k] < best_weight) {
          best_cut = cut;
          best_weight = w[k];
        }
      } else {
        rep(j,0,N)
          w[j] += weights[k][j];
        added[k] = true;
      }
    }
  }
  return {best_weight, best_cut};
}
```

FlowWithRestrictionsAndCirculation.java

Description: Flow with restrictions and circulation.

a3fbf4, 53 lines

```
class FlowWithRestrictionsAndCirculation {
  /*
  * Add new S' and T'.
  * For each edge with Li > 0 do:
  * add S'->Bi(Li), add Ai->T'(Li), add Ai->Bi(Ri-Li).
  * Then add T->S(inf).
  *
  * Now run dinic and you're done.
  */
  List<Edge>[] buildGraphWithRestrictions(
      List<Edge0>[] g0, int s0, int t0) {
    int n0 = g0.length;
    int s = n0;
    int t = s + 1;
    int n = n0 + 2;
    List<Edge>[] g = new List[n];
    for (int i = 0; i < n; i++) g[i] = new ArrayList<>();
    for (List<Edge0> adj : g0) {
      for (Edge0 e : adj) {
        addEdge(g, s, e.to, e.min);
        addEdge(g, e.from, t, e.min);
        addEdge(g, e.from, e.to, e.max - e.min);
      }
    }
    //binary search on this to find min flow
    addEdge(g, t0, s0, MAX_VALUE);
    return g;
  }

  /*
  * Make a graph with restrictions and find max flow on it.
  * To find a real flow, add Li to all the edges.
  */
  List<Edge>[] buildCirculation(List<Edge>[] b, int s, int t) {
    return null;
  }

  void addEdge(List<Edge>[] g, int from, int to, int cap) {
    Edge e1 = new Edge(from, to, cap);
    Edge e2 = new Edge(to, from, 0);
    e1.rev = e2;
    e2.rev = e1;
    g[from].add(e1);
    g[to].add(e2);
  }
```

```
  class Edge {
    int from, to, capacity;
    Edge rev;   //(to, from, 0)
  }

  class Edge0 { int from, to, min, max; }
}
```

MinCostMaxFlow.java

Description: Min cost max flow (ADD FORD BELLMAN). 0689eb, 69 lines

```
class MinCostMaxFlow {
  int n, s, t;
  Edge[][] g;

  int[] used;
  int timer;
  int[] phi;
  int[] dist;
  Edge[] parent;
  boolean dijkstra() {
    timer++;
    fill(dist, MAX_VALUE);
    dist[s] = 0;
    parent = new Edge[n];
    while (true) {
      int curV = -1;
      for (int v = 0; v < n; v++)
        if (used[v] != timer &&
            (curV == -1 || dist[v] < dist[curV])) {
          curV = v;
        }
      if (curV == -1) break;
      used[curV] = timer;
      for (Edge e : g[curV]) {
        if (e == null) continue;
        int newDist = phi[e.from] +
                      e.cost -
                      phi[e.to] +
                      dist[curV];
        if (e.capacity - e.flow > 0 && dist[e.to] > newDist) {
          dist[e.to] = newDist;
          parent[e.to] = e;
        }
      }
    }
    return parent[t] != null;
  }

  Result minCostMaxFlow() {
    used = new int[n];
    dist = new int[n];
    parent = new Edge[n];
    phi = new int[n];
    int cost = 0;
    int flow = 0;
    while (dijkstra()) {
      for (int i = 0; i < n; i++)
        if (dist[i] != MAX_VALUE)
          phi[i] += dist[i]; //or just phi[i] = dist[i]
      int minEdge = MAX_VALUE;
      for (Edge e = parent[t]; e != null; e = parent[e.from])
        minEdge = min(minEdge, e.capacity - e.flow);
      for (Edge e = parent[t]; e != null; e = parent[e.from]) {
        cost += e.cost * minEdge;
        e.flow += minEdge;
        e.rev.flow -= minEdge;
      }
      flow += minEdge;
    }
    return new Result(cost, flow);
  }

  class Result { int cost, flow; }

  class Edge {
    int from, to, cost, capacity, flow;
    Edge rev;
  }
}
```

7.3 Matching

hopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.

Usage: `vi btoa(m, -1); hopcroftKarp(g, btoa);`

Time: $\mathcal{O}\left(\sqrt{V}E\right)$ 536939, 42 lines

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
  if (A[a] != L) return 0;
  A[a] = -1;
  trav(b, g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
      return btoa[b] = a, 1;
  }
  return 0;
}

int hopcroftKarp(vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);
    fill(all(B), 0);
    cur.clear();
    trav(a, btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a);
    for (int lay = 1;; lay++) {
      bool islast = 0;
      next.clear();
      trav(a, cur) trav(b, g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        }
        else if (btoa[b] != a && !B[b]) {
          B[b] = lay;
          next.push_back(btoa[b]);
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      trav(a, next) A[a] = lay;
      cur.swap(next);
    }
    rep(a,0,sz(g))
      res += dfs(a, 0, g, btoa, A, B);
  }
}
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" d0b3f2, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = dfsMatching(g, match);
  vector<bool> lfound(n, true), seen(m);
  trav(it, match) if (it != -1) lfound[it] = false;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    trav(e, g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

WeightedMatching.h

Description: Min cost bipartite matching. Negate costs for max cost.

Time: $\mathcal{O}\left(N^3\right)$ 055ca9, 75 lines

```
typedef vector<double> vd;
bool zero(double x) { return fabs(x) < 1e-10; }
double MinCostMatching(const vector<vd>& cost, vi& L, vi& R) {
  int n = sz(cost), mated = 0;
  vd dist(n), u(n), v(n);
  vi dad(n), seen(n);

  rep(i,0,n) {
    u[i] = cost[i][0];
    rep(j,1,n) u[i] = min(u[i], cost[i][j]);
  }
  rep(j,0,n) {
    v[j] = cost[0][j] - u[0];
    rep(i,1,n) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  L = R = vi(n, -1);
  rep(i,0,n) rep(j,0,n) {
    if (R[j] != -1) continue;
    if (zero(cost[i][j] - u[i] - v[j])) {
      L[i] = j;
      R[j] = i;
      mated++;
      break;
    }
  }

  for (; mated < n; mated++) { // until solution is feasible
    int s = 0;
    while (L[s] != -1) s++;
    fill(all(dad), -1);
    fill(all(seen), 0);
    rep(k,0,n)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    for (;;) {
      j = -1;
      rep(k,0,n){
        if (seen[k]) continue;
```

```
      if (j == -1 || dist[k] < dist[j]) j = k;
    }
    seen[j] = 1;
    int i = R[j];
    if (i == -1) break;
    rep(k,0,n) {
      if (seen[k]) continue;
      auto new_dist = dist[j] + cost[i][k] - u[i] - v[k];
      if (dist[k] > new_dist) {
        dist[k] = new_dist;
        dad[k] = j;
      }
    }
  }

  rep(k,0,n) {
    if (k == j || !seen[k]) continue;
    auto w = dist[k] - dist[j];
    v[k] += w, u[R[k]] -= w;
  }
  u[s] += dist[j];

  while (dad[j] >= 0) {
    int d = dad[j];
    R[j] = R[d];
    L[R[j]] = j;
    j = d;
  }
  R[j] = s;
  L[s] = j;
}
auto value = vd(1)[0];
rep(i,0,n) value += cost[i][L[i]];
return value;
}
```

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod.
Time: $\mathcal{O}\left(N^3\right)$

"../numerical/MatrixInverse-mod.h" bb8be4, 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  trav(pa, ed) {
    int a = pa.first, b = pa.second, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  }

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);

  if (M != N) do {
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
      mat[i].resize(M);
      rep(j,N,M) {
        int r = rand() % mod;
        mat[i][j] = r, mat[j][i] = (mod - r) % mod;
      }
    }
  } while (matInv(A = mat) != M);

  vi has(M, 1); vector<pii> ret;
  rep(it,0,M/2) {
    rep(i,0,M) if (has[i])
      rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;
    } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0;
    rep(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);
      rep(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
      }
      swap(fi,fj);
    }
  }
  return ret;
}
```

MaxMatchingInBipartiteGraph.java

Description: Fast max bipartite matching.

537b8f, 38 lines

```
class MaxMatchingInBipartiteGraph {
  int n;
  List<Integer>[] g;
  int[] parent;
  int[] pair;
  boolean[] used;

  boolean dfs(int v) {
    if (used[v]) return false;
    used[v] = true;
    for (int to : g[v]) {
      if (parent[to] == -1) {
        pair[v] = to;
        parent[to] = v;
        return true;
      }
    }
    for (int to : g[v]) {
      if (dfs(parent[to])) {
        pair[v] = to;
        parent[to] = v;
        return true;
      }
    }
    return false;
  }

  void run() {
    fill(pair, -1);
    fill(parent, -1);
    for (boolean run = true; run; ) {
      run = 0;
      fill(used, false);
      for (int i = 0; i < n; i++)
        if (pair[i] == -1 && dfs(i)) run = true;
    }
  }
}
```

HungarianAssignment.h

Description: Hungarian algorithm.

6bdc3d, 36 lines

```
vector<int> u (n+1), v (m+1), p (m+1), way (m+1);
for (int i=1; i<=n; ++i) {
  p[0] = i;
  int j0 = 0;
  vector<int> minv (m+1, INF);
  vector<char> used (m+1, false);
  do {
    used[j0] = true;
    int i0 = p[j0],  delta = INF,  j1;
    for (int j=1; j<=m; ++j)
      if (!used[j]) {
        int cur = a[i0][j]-u[i0]-v[j];
        if (cur < minv[j])
          minv[j] = cur,  way[j] = j0;
        if (minv[j] < delta)
          delta = minv[j],  j1 = j;
      }
    for (int j=0; j<=m; ++j)
      if (used[j])
        u[p[j]] += delta,  v[j] -= delta;
      else
        minv[j] -= delta;
    j0 = j1;
  } while (p[j0] != 0);
  do {
    int j1 = way[j0];
    p[j0] = p[j1];
    j0 = j1;
  } while (j0);
}

vector<int> ans (n+1);
for (int j=1; j<=m; ++j)
  ans[p[j]] = j;

int cost = -v[0];
```

7.4 DFS algorithms

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
Usage: `int eid = 0; ed.resize(N);`
`for each edge (a,b) {`
`ed[a].emplace_back(b, eid);`
`ed[b].emplace_back(a, eid++); }`
`bicomps([&](const vi& edgelist) {...});`
Time: $\mathcal{O}\left(E+V\right)$

cca7e6, 33 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
  int me = num[at] = ++Time, e, y, top = me;
  trav(pa, ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me)
        st.push_back(e);
    } else {
      int si = sz(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}
```

```
template<class F>
void bicomps(F f) {
  num.assign(sz(ed), 0);
  rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

Bridges.java

Description: Bridges finder.

b3085f, 31 lines

```
class Bridges {
  List<Integer>[] g;
  boolean[] used;
  int[] tin, fup;
  int timer;

  void dfs(int v, int p) {
    used[v] = true;
    tin[v] = fup[v] = timer++;
    for (int to : g[v]) {
      if (to == p)
        continue;
      if (used[to]) {
        fup[v] = min(fup[v], tin[to]);
      } else {
        dfs(to, v);
        fup[v] = min(fup[v], fup[to]);
        if (fup[to] > tin[v]) process(v, to);
      }
    }
  }

  void findBridges() {
    int n = g.length;
    used = new boolean[n];
    tin = new int[n];
    fup = new int[n];
    timer = 0;
    for (int i = 0; i < n; i++) if (!used[i]) dfs(i, -1);
  }
}
```

Cutpoints.java

Description: Cut points finder.

178f78, 33 lines

```
class Cutpoints {
  List<Integer>[] g;
  boolean[] used;
  int[] tin, fup;
  int timer;

  void dfs(int v, int p) {
    used[v] = true;
    tin[v] = fup[v] = timer++;
    int children = 0;
    for (int to : g[v]) {
      if (to == p) continue;
      if (used[to]) {
        fup[v] = Math.min(fup[v], tin[to]);
      } else {
        dfs(to, v);
        fup[v] = Math.min(fup[v], fup[to]);
        if (fup[to] >= tin[v] && p != -1) process(v);
        children++;
      }
    }
    if (p == -1 && children > 1) process(v);
  }

  void findCutpoints() {
    int n = g.length;
    used = new boolean[n];
    tin = new int[n];
    fup = new int[n];
    timer = 0;
    for (int i = 0; i < n; i++) if (!used[i]) dfs(i, -1);
  }
}
```

StronglyConnectedComponents.java

Description: Strongle connected components finder.

99e493, 39 lines

```
class StronglyConnectedComponents {
  int n;
  List<Integer>[] g, gr;

  boolean[] used;
  List<Integer> order;
  void dfs1(int v) {
    used[v] = true;
    for (int i = 0; i < g[v].size(); i++)
      if (!used[g[v].get(i)])
        dfs1(g[v].get(i));
    order.add(v);
  }

  List<Integer> component;
  void dfs2(int v) {
    used[v] = true;
    component.add(v);
    for (int i = 0; i < gr[v].size(); i++)
      if (!used[gr[v].get(i)])
        dfs2(gr[v].get(i));
  }

  void find() {
    used = new boolean[n];
    order = new ArrayList<>();
    for (int i = 0; i < n; ++i) if (!used[i]) dfs1(i);
    fill(used, false);
    component = new ArrayList<>();
    for (int i = 0; i < n; ++i) {
      int v = order.get(n - i - 1);
      if (!used[v]) {
        dfs2(v);
        process();
        component.clear();
      }
    }
  }
}
```

7.5 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

fbbef1, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    trav(v,r) v.d = 0;
    trav(v, r) trav(j, r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return;
      q.push_back(R.back().i);
      vv T;
      trav(v,R) if (e[R.back().i][v.i]) T.push_back({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
        C[1].clear(), C[2].clear();
        trav(v, T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) trav(i, C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vi maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  }
};
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

7.6 Trees

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" dabd75, 20 lines

```
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.dist));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.query(a, b));
  }
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.query(a, b)], b);
  }
  return ret;
}
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

693483, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void push_flip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (push_flip(); p; ) {
      if (p->p) p->p->push_flip();
      p->push_flip(); push_flip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    push_flip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    make_root(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    make_root(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void make_root(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
};
```

DirectedMST.h

Description: Edmonds' algorithm for finding the weight of the minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFind.h" a69883, 48 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

ll dmst(int n, int r, vector<Edge>& g) {
  UF uf(n);
  vector<Node*> heap(n);
  trav(e, g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n);
  seen[r] = r;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      path[qi++] = u, seen[u] = s;
      if (!heap[u]) return -1;
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u);
        heap[u] = cyc, seen[u] = -1;
      }
    }
  }
  return res;
}
```

MatrixTree.h

Description: To count the number of spanning trees in an undirected graph G: create an $N \times N$ matrix `mat`, and for each edge $(a, b) \in G$, do `mat[a][a]++, mat[b][b]++, mat[a][b]--, mat[b][a]--`. Remove the last row and column, and take the determinant.

CentroidDecomposition.java

Description: Centroid decomposition without saving.

f2d3b7, 43 lines

```
class CentroidDecomposition {
  List<Integer>[] g;
  boolean[] used;
  int[] time;
  int timer;
  int[] size;

  void findCentroid(int v, int size) {
    int p = -1;
    boolean any = true;
    while (any) {
      any = false;
      for (int to : g[v]) {
```

```
        if (to != p && !used[to] &&
            getSize(to, -1) > size / 2) {
          p = v;
          v = to;
          any = true;
          break;
        }
      }
      if (!any) break;
    }
    used[v] = true;
    process(v);
    for (int to : g[v]) {
      if (!used[to]) {
        timer++;
        findCentroid(to, getSize(to, -1));
      }
    }
  }

  int getSize(int v, int p) {
    if (time[v] == timer) return size[v];
    time[v] = timer;
    size[v] = 1;
    for (int to : g[v])
      if (to != p && !used[to])
        size[v] += getSize(to, v);
    return size[v];
  }
}
```

MultiTreeHeavyLightDecomposition.java

Description: Heavy Light Decomposition on multiple trees. 7eb498, 119 lines

```
class MultiTreeHeavyLightDecomposition {
  int n;
  Vertex[] g;

  class HLD {
    MaxSparseTable[] trees;
    List<List<Integer>> treeIndexes;
    int[] treeIndex;
    int[] indexInTree;

    HLD(List<Integer> roots) {
      treeIndex = new int[n];
      indexInTree = new int[n];
      treeIndexes = new ArrayList<>();
      for (int v : roots) {
        treeIndex[v] = treeIndexes.size();
        treeIndexes.add(new ArrayList<>());
        initIndexes(v);
      }
      initTables();
    }

    HLD() { this(findRoots()); }

    int getMax(int v, int u) {
      if (g[u].component != g[v].component) return -1;
      int max = -1;
      while (treeIndex[v] != treeIndex[u]) {
        if (g[head(v)].depth < g[head(u)].depth) {
          int t = v;
          v = u;
          u = t;
        }
        max = max(max,
                  trees[treeIndex[v]].get(0, indexInTree[v]));
        v = g[head(v)].parent;
      }
      if (v == u) return max;
      int l = min(indexInTree[v], indexInTree[u]) + 1;
      int r = max(indexInTree[v], indexInTree[u]);
      return max(max, trees[treeIndex[v]].get(l, r));
    }

    int head(int v) {
      return treeIndexes.get(treeIndex[v]).get(0);
    }

    void initTables() {
      int treeCount = treeIndexes.size();
      trees = new MaxSparseTable[treeCount];
      for (int i = 0; i < treeCount; i++) {
        int[] a = new int[treeIndexes.get(i).size()];
        for (int j = 0; j < a.length; j++)
          a[j] = g[treeIndexes.get(i).get(j)].inEdgeTime;
        trees[i] = new MaxSparseTable(a);
      }
    }

    void initIndexes(int v) {
      indexInTree[v] = treeIndexes.get(treeIndex[v]).size();
      treeIndexes.get(treeIndex[v]).add(v);
      for (Edge e : g[v].adj) {
        if (g[e.to].size > g[v].size / 2) {
          treeIndex[e.to] = treeIndex[v];
        } else {
          treeIndex[e.to] = treeIndexes.size();
          treeIndexes.add(new ArrayList<>());
        }
        initIndexes(e.to);
      }
    }
  }

  class Vertex {
    int index;
    List<Edge> adj = new ArrayList<>();
    int size;
    int inEdgeTime;
    int depth;
    int parent = -1;
    int component;

    Vertex(int index) {
      this.index = index;
    }

    void dfs() {
      size = 1;
      for (int i = 0; i < adj.size(); i++) {
        Edge e = adj.get(i);
        int to = e.to;
        if (to == parent) {
            adj.remove(i--);
            continue;
        }
        g[to].depth = depth + 1;
        g[to].parent = index;
        g[to].inEdgeTime = e.time;
        g[to].component = component;
        g[to].dfs();
        size += g[to].size;
      }
    }
  }

  List<Integer> findRoots() {
    List<Integer> roots = new ArrayList<>();
    for (Vertex v : g) {
      if (v.parent == -1) {
        v.component = roots.size();
        roots.add(v.index);
        g[v.index].dfs();
      }
    }
    return roots;
  }

  class Edge { int from, to, time; }
}
```

Geometry (8)

8.1 Geometric primitives

Angle.java

Description: Angle class. 8c990e, 14 lines

```
class Angle {
  Point o, a, b;

  // [0..2PI)
  double value() {
    double x1 = a.x - o.x, y1 = a.y - o.y;
    double x2 = b.x - o.x, y2 = b.y - o.y;
    double scal = x1 * x2 + y1 * y2;  //x(cos)
    double vect = x1 * y2 - x2 * y1;  //y(sin)
    double a = atan2(vect, scal);
    if (a < 0) a += 2 * PI;  //or abs to get a lower angle
    return a;
  }
}
```

CanonicalAngle.h

Description: Makes angle lie in [-PI..PI). d2b5d2, 5 lines

```
double canonicalAngle(double angle) {
  while (angle > +PI) angle -= 2 * PI;
  while (angle < -PI) angle += 2 * PI;
  return angle;
}
```

PositiveAngle.h

Description: Makes angle lie in [0..2PI). fb7ddf, 5 lines

```
double positiveAngle(double angle) {
  while (angle > 2 * PI - epsilon) angle -= 2 * PI;
  while (angle < -epsilon) angle += 2 * PI;
  return angle;
}
```

Point.java

Description: Point class. 57c5c5, 21 lines

```
class Point {
  double x, y;

  Line line(Point other) {
    if (equals(other)) return null;
    double a = other.y - y;
    double b = x - other.x;
    double c = -a * x - b * y;
    return new Line(a, b, c);
  }

  double angle() { return atan2(y, x); }

  Point rotate(double angle) {
    double sin = sin(angle);
    double cos = cos(angle);
    double nx = x * cos - y * sin;
    double ny = x * sin + y * cos;
    return new Point(nx, ny);
  }
}
```

Segment.java

Description: Segment class. 91e32b, 94 lines

```
class Segment {
  Point a;
  Point b;

  Segment(Point a, Point b) {
    this.a = a;
    this.b = b;
  }

  double length() { return a.distance(b); }

  double distance(Point point) {
    double length = length();
    double left = point.distance(a);
    if (length < epsilon) return left;
    double right = point.distance(b);
    if (left * left > right * right + length * length)
      return right;
    if (right * right > left * left + length * length)
      return left;
    return point.distance(line());
  }

  Point intersect(Segment other, boolean includeEnds) {
    Line line = line();
    Line otherLine = other.a.line(other.b);
    if (line.parallel(otherLine)) return null;
    Point intersection = line.intersect(otherLine);
    if (contains(intersection, includeEnds) &&
        other.contains(intersection, includeEnds)) {
      return intersection;
    }
    return null;
  }

  boolean contains(Point point, boolean includeEnds) {
    if (a.equals(point) || b.equals(point)) return includeEnds;
    if (a.equals(b)) return false;
    Line line = line();
    if (!line.contains(point)) return false;
    Line perpendicular = line.perpendicular(a);
    double aValue = perpendicular.value(a);
    double bValue = perpendicular.value(b);
    double pointValue = perpendicular.value(point);
    return (aValue < pointValue && pointValue < bValue) ||
           (bValue < pointValue && pointValue < aValue);
  }

  Line line() {
    return a.line(b);
  }

  Point middle() {
    return new Point((a.x + b.x) / 2,
                     (a.y + b.y) / 2);
  }

  Point[] intersect(Circle circle) {
    Point[] result = line().intersect(circle);
    if (result.length == 0) return result;
    if (result.length == 1)
      return contains(result[0], true) ? result : new Point[0];
    if (contains(result[0], true))
      return contains(result[1], true)
        ? result
        : new Point[]{result[0]};
    if (contains(result[1], true))
      return new Point[]{result[1]};
    return new Point[0];
  }

  Point intersect(Line line) {
    Line selfLine = line();
    Point intersection = selfLine.intersect(line);
    if (intersection == null) return null;
    if (contains(intersection, true)) return intersection;
    return null;
  }

  double distance(Segment other) {
    Line line = line();
    Line otherLine = other.line();
    Point p = line == null || otherLine == null
      ? null
      : line.intersect(otherLine);
    if (p != null &&
        contains(p, true) &&
        other.contains(p, true))
      return 0;
    return min(
      min(other.distance(a), other.distance(b)),
      min(distance(other.a), distance(other.b)));
  }
}
```

Polygon.java

Description: Polygon class (REWRITE CONVEX HULL). b778e7, 93 lines

```
class Polygon {
  Point[] vertices;

  double area() {
    double sum = 0;
    for (int i = 1; i < vertices.length; i++)
      sum += (vertices[i].x - vertices[i - 1].x) *
             (vertices[i].y + vertices[i - 1].y);
    sum += (vertices[0].x - vertices[vertices.length - 1].x) *
           (vertices[0].y + vertices[vertices.length - 1].y);
    return abs(sum) / 2;
  }

  boolean over(Point a, Point b, Point c) {
    return a.x * (b.y - c.y) +
           b.x * (c.y - a.y) +
           c.x * (a.y - b.y) < -epsilon;
  }

  boolean under(Point a, Point b, Point c) {
    return a.x * (b.y - c.y) +
           b.x * (c.y - a.y) +
           c.x * (a.y - b.y) > epsilon;
  }

  Polygon convexHull(Point[] points) {
    if (points.length == 1)
      return new Polygon(points);
    sort(points, new Comparator<Point>() {
      public int compare(Point o1, Point o2) {
        int value = compare(o1.x, o2.x);
        if (value != 0) return value;
        return Double.compare(o1.y, o2.y);
      }
    });
    Point left = points[0];
    Point right = points[points.length - 1];
    List<Point> up = new ArrayList<>();
    List<Point> down = new ArrayList<>();
    for (Point point : points) {
      if (point == left ||
```

```
        point == right ||
        !under(left, point, right)) {
      while (up.size() >= 2 && under(
          up.get(up.size() - 2),
          up.get(up.size() - 1),
          point))
        up.remove(up.size() - 1);
      up.add(point);
    }
    if (point == left ||
        point == right ||
        !over(left, point, right)) {
      while (down.size() >= 2 && over(
          down.get(down.size() - 2),
          down.get(down.size() - 1),
          point))
        down.remove(down.size() - 1);
      down.add(point);
    }
  }
  Point[] result = new Point[up.size() + down.size() - 2];
  int index = 0;
  for (Point point : up) result[index++] = point;
  for (int i = down.size() - 2; i > 0; i--)
    result[index++] = down.get(i);
  return new Polygon(result);
}

boolean contains(Point point) {
  return contains(point, false);
}

boolean contains(Point point, boolean strict) {
  for (Segment segment : sides())
    if (segment.contains(point, true))
      return !strict;
  double totalAngle = canonicalAngle(
    atan2(vertices[0].y - point.y,
          vertices[0].x - point.x) -
    atan2(vertices[vertices.length - 1].y - point.y,
          vertices[vertices.length - 1].x - point.x)
  );
  for (int i = 1; i < vertices.length; i++)
    totalAngle += canonicalAngle(
      atan2(vertices[i].y - point.y,
            vertices[i].x - point.x) -
      atan2(vertices[i - 1].y - point.y,
            vertices[i - 1].x - point.x)
    );
  return abs(totalAngle) > PI;
}
}
```

Circle.java

Description: Cicrle class.

91e81f, 82 lines

```
class Circle {
  Point center;
  double radius;

  boolean contains(Point point) {
    return center.distance(point) < radius + epsilon;
  }

  boolean strictContains(Point point) {
    return center.distance(point) < radius - epsilon;
  }

  Point[] findTouchingPoints(Point point) {
    double distance = center.distance(point);
    if (distance < radius - epsilon)
      return new Point[0];
    if (distance < radius + epsilon)
      return new Point[]{point};
    Circle power = new Circle(
      point,
      sqrt((distance - radius) * (distance + radius))
    );
    return intersect(power);
  }

  Point[] intersect(Circle other) {
    double x1 = center.x;
    double y1 = center.y;
    double r1 = radius;
    double x2 = other.center.x;
    double y2 = other.center.y;
    double r2 = other.radius;
    double a = 2 * (x2 - x1);
    double b = 2 * (y2 - y1);
    double c = (x1 * x1 - x2 * x2) +
               (y1 * y1 - y2 * y2) +
               (r2 * r2 - r1 * r1);
    Line line = new Line(a, b, c);
    return line.intersect(this);
  }

  double area() {
    return PI * radius * radius;
  }

  double sector(double angle) {
    return area() * angle / (2 * PI);
  }

  double triangle(double angle) {
    return radius * radius * sin(angle) / 2;
  }

  double segment(double angle) {
    return sector(angle) - triangle(angle);
  }

  double intersectionArea(Circle other) {
    if (radius > other.radius)
      return other.intersectionArea(this);
    if (center.distance(other.center) + radius <= other.radius)
      return area();
    if (center.distance(other.center) >= radius + other.radius)
      return 0;

    Point[] intersections = intersect(other);
    double angle1 = Point.angle(intersections[0],
                                intersections[1],
                                center);
    double angle2 = Point.angle(intersections[1],
                                intersections[0],
                                other.center);
    if (angle2 > PI) {
      angle1 = 2 * PI - angle1;
      angle2 = 2 * PI - angle2;
    }
    double area1 = segment(angle1);
    double area2 = other.segment(angle2);

    return area1 + area2;
  }
}
```

Line.java

Description: Line class.

1b3c06, 77 lines

```
class Line {
  double a, b, c;
  double b;
  double c;

  Line(Point p, double angle) {
    a = sin(angle);
    b = -cos(angle);
    c = -p.x * a - p.y * b;
  }

  Line(double a, double b, double c) {
    double h = hypot(a, b);
    this.a = a / h;
    this.b = b / h;
    this.c = c / h;
  }

  Point intersect(Line other) {
    if (parallel(other)) return null;
    double determinant = b * other.a - a * other.b;
    double x = (c * other.b - b * other.c) / determinant;
    double y = (a * other.c - c * other.a) / determinant;
    return new Point(x, y);
  }

  boolean parallel(Line other) {
    return abs(a * other.b - b * other.a) < epsilon;
  }

  boolean contains(Point point) {
    return abs(value(point)) < epsilon;
  }

  Line perpendicular(Point point) {
    return new Line(-b, a, b * point.x - a * point.y);
  }

  double value(Point point) {
    return a * point.x + b * point.y + c;
  }

  Point[] intersect(Circle circle) {
    double distance = distance(circle.center);
    if (distance > circle.radius + epsilon)
        return new Point[0];
    Point intersection = intersect(
      perpendicular(circle.center));
    if (abs(distance - circle.radius) < epsilon)
        return new Point[]{intersection};
    double shift = sqrt(
      circle.radius * circle.radius -
      distance * distance);
    return new Point[]{
      new Point(
        intersection.x + shift * b,
        intersection.y - shift * a),
      new Point(
        intersection.x - shift * b,
        intersection.y + shift * a)
    };
  }

  double distance(Point center) { return abs(value(center)); }

  double angle() { return atan2(-a, b); }
```

```
  @Override
  public boolean equals(Object o) {
    Line line = (Line) o;
    if (!parallel(line)) return false;
    if (abs(a * line.c - c * line.a) > epsilon ||
        abs(b * line.c - c * line.b) > epsilon)
      return false;
    return true;
  }
}
```

Ray.java

Description: Ray class.

c5fbb5, 28 lines

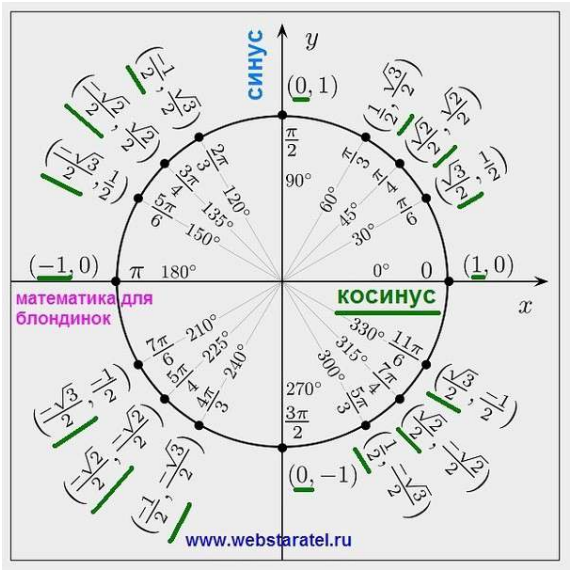
```
class Ray {
  Point base;
  Line line;
  Line perpendicular;

  Ray(Point base, double direction) {
    this.base = base;
    line = new Line(base, direction);
    this.perpendicular = line.perpendicular(base);
  }

  Ray(Point base, Point other) {
    this.base = base;
    line = base.line(other);
    this.perpendicular = line.perpendicular(base);
  }

  boolean contains(Point point) {
    return line.contains(point) &&
           perpendicular.value(point) > -epsilon;
  }

  double distance(Point point) {
    if (perpendicular.value(point) > -epsilon)
      return line.distance(point);
    return base.distance(point);
  }
}
```

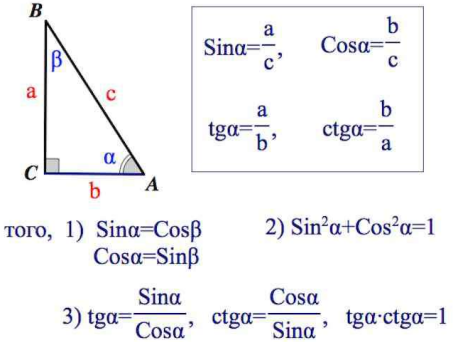
Vector.java

Description: Vector class (do we need it?).

71c9cd, 15 lines

```
class Vector {
  Point point;

  Vector(Point from, Point to) {
    this(to.x - from.x, to.y - from.y);
  }

  double angleTo(Vector other) {
    return canonicalAngle(other.point.angle() - point.angle());
  }

  double length() { return point.value(); }

  double angle() { return point.angle(); }
}
```



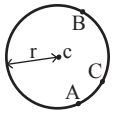


8.2 Circles

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

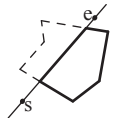
"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

8.3 Polygons

PolygonCut.h

Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));



"Point.h", "lineIntersection.h" f2b7d4, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no colinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(N+Q\log n)$

"Point.h" 758f22, 39 lines

```
typedef array<P, 2> Line;
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(line[0].cross(poly[i], line[1]))
array<int, 2> lineHull(Line line, vector<P> poly) {
  int endA = extrVertex(poly, (line[0] - line[1]).perp());
  int endB = extrVertex(poly, (line[1] - line[0]).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n\log n)$

"Point.h" d31bbf, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  trav(p, v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if the box is wider than high (not best
      // heuristic...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n\log n)$

"Point.h" bf87ec, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
};

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q q[] = {new Quad{0,0,0,orig}, new Quad{0,0,0,arb},
           new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
  rep(i,0,4)
    q[i]->o = q[-i & 3], q[i]->rot = q[(i+1) & 3];
  return *q;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
```

```
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

1ec4d3, 6 lines

```
template<class V, class L>
double signed_poly_volume(const V& p, const L& trilist) {
  double v = 0;
  trav(i, trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}\left(n^2\right)$

"Point3D.h" c172e9, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  trav(it, FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).

Time: $\mathcal{O}\left(N\right)$

e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: `rotate(v.begin(), v.begin()+min_rotation(v), v.end());`

Time: $\mathcal{O}\left(N\right)$

358164, 8 lines

```
int min_rotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(i,0,N) {
    if (a+i == b || s[a+i] < s[b+i]) {b += max(0, i-1); break;}
    if (s[a+i] > s[b+i]) { a = b; break; }
  }
  return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.

Time: $\mathcal{O}\left(n \log n\right)$

38db9f, 23 lines

```
struct SuffixArray {
```

```
  vi sa, lcp;
  SuffixArray(string& s, int lim=256) { // or basic_string<int>
    int n = sz(s) + 1, k = 0, a, b;
    vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
    sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
      p = j, iota(all(y), n - j);
      rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i,0,n) ws[x[i]]++;
      rep(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
        (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
    }
    rep(i,1,n) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
      for (k && k--, j = sa[rank[i] - 1];
          s[i + k] == s[j + k]; k++);
  }
};
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$

aae0b8, 50 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
  }

  // example: find longest common substring (uses ALPHA = 28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```

Hashing.h

Description: Self-explanatory methods for string hashing.

acb5db, 44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
struct H {
  typedef uint64_t ull;
  ull x; H(ull x=0) : x(x) {}
#define OP(O,A,B) H operator O(H o) { ull r = x; asm \
  (A "addq %%rdx, %0\n adcq $0,%0" : "+a"(r) : B); return r; }
  OP(+,,"d"(o.x)) OP(*,"mul %1\n", "r"(o.x) : "rdx")
  H operator-(H o) { return *this + ~o.x; }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};

vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return {};
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;
  vector<H> ret = {h};
  rep(i,length,sz(str)) {
    ret.push_back(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;
}

H hashString(string& s) { H h{}; trav(c,s) h=h*C+c; return h; }
```

AhoCorasick.h

Description: Aho-Corasick tree is used for multiple pattern matching. Initialize the tree with create(patterns). find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(_, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input.
Time: create is $\mathcal{O}(26N)$ where N is the sum of length of patterns. find is $\mathcal{O}(M)$ where M is the length of the word. findAll is $\mathcal{O}(NM)$.

716ac4, 67 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'};
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1, nmatches = 0;
    Node(int v) { memset(next, v, sizeof(next)); }
  };
  vector<Node> N;
  vector<int> backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    trav(c, s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) {
    N.emplace_back(-1);
    rep(i,0,sz(pat)) insert(pat[i], i);
    N[0].back = sz(N);
    N.emplace_back(0);

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      rep(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[ed].start])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  vi find(string word) {
    int n = 0;
    vi res; // ll count = 0;
    trav(c, word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vi> findAll(vector<string>& pat, string word) {
    vi r = find(word);
    vector<vi> res(sz(word));
    rep(i,0,sz(word)) {
      int ind = r[i];
      while (ind != -1) {
```

```
        res[i - sz(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

StringAlgorithms.java

Description: Z- and Phi- string functions.

2a1dd0, 35 lines

```
class StringAlgorithms {
  int[] zFunction(char[] s) {
    int n = s.length;
    int[] z = new int[n];
    for (int i = 1, l = 0, r = 0; i < n; i++) {
      if (i <= r) z[i] = min(r - i + 1, z[i - l]);
      while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
      if (i + z[i] - 1 > r) {
        l = i;
        r = i + z[i] - 1;
      }
    }
    return z;
  }

  int[] prefixFunction(char[] s) {
    int n = s.length;
    int[] pi = new int[n];
    int k = 0;
    for (int i = 1; i < n; i++) {
      while ((k > 0) && (s[k] != s[i])) k = pi[k - 1];
      if (s[k] == s[i]) k++;
      pi[i] = k;
    }
    return pi;
  }

  boolean contains(String s, String target) {
    int[] z = zFunction((target + s).toCharArray());
    for (int i = target.length(); i < z.length; i++)
      if (z[i] >= target.length())
        return true;
    return false;
  }
}
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).

Time: $\mathcal{O}(\log N)$

edce47, 23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).

Time: $\mathcal{O}(N \log N)$

9e9d8d, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.

Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`

Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

753a4c, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \ldots < f(i) \geq \cdots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f, change it to $>$, also at (B).

Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`

Time: $\mathcal{O}(\log(b-a))$

9155b4, 13 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) // (A)
      a = mid;
    else
      b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LongestIncreasingSubsequenceWithBinarySearch.java

Description: LIS with binary search.

89b488, 27 lines

```
class LongestIncreasingSubsequenceWithBinarySearch {
  int[] find(int[] a) {
    int n = a.length;
    int[] d = new int[n + 1];
    int[] parent = new int[n];
    int maxLen = 0;
    for (int i = 0; i < n; i++) {
      int l = 0;
      int r = maxLen + 1;
      while (r - l > 1) {
        int m = l + r >> 1;
        if (a[d[m]] < a[i]) l = m;
        else r = m;
      }
      if (r == maxLen + 1 || a[d[r]] > a[i])
        d[r] = i;
      parent[i] = l == 0 ? -1 : d[l];
      maxLen = max(maxLen, r);
    }
    int[] res = new int[maxLen];
    for (int i = 0, at = d[maxLen]; i < maxLen; i++) {
      res[maxLen - i - 1] = at;
      at = parent[at];
    }
    return res;
  }
}
```

10.3 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

Time: $\mathcal{O}(N^2)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \le k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}\left((N + (hi - lo))\log N\right)$

d38d2b, 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.5 Optimization tricks

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastMod.h

Description: Compute $a\%b$ about 4 times faster than usual, where b is constant but not known at compile time. Fails for $b = 1$.

c977c5, 10 lines

```
typedef unsigned long long ull;
typedef __uint128_t L;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(ull((L(1) << 64) / b)) {}
  ull reduce(ull a) {
    ull q = (ull)((L(m) * a) >> 64), r = a - q * b;
    return r >= b ? r - b : r;
  }
};
```

BumpAllocator.h

Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

745db2, 8 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

SmallPtr.h

Description: A 32-bit pointer that points into BumpAllocator memory.

"BumpAllocator.h" 2dd6c9, 10 lines

```
template<class T> struct ptr {
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
    assert(ind < sizeof buf);
  }
  T& operator*() const { return *(T*)(buf + ind); }
  T* operator->() const { return &**this; }
  T& operator[](int a) const { return (&**this)[a]; }
  explicit operator bool() const { return ind; }
};
```

BumpAllocatorSTL.h

Description: BumpAllocator for STL containers.
Usage: `vector<vector<int, small<int>>> ed(N);`

bb66d4, 14 lines

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;

template<class T> struct small {
  typedef T value_type;
  small() {}
  template<class U> small(const U&) {}
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);
  }
  void deallocate(T*, size_t) {}
};
```

Unrolling.h

520e76, 5 lines

```
#define F {...; ++i;}
int i = from;
while (i&3 && i < to) F // for alignment, if needed
while (i + 4 <= to) { F F F F }
while (i < to) F
```

SIMD.h

Description: Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern `"_mm(256)?_name_(si(128|256)|epi(8|16|32|64)|pd|ps)"`. Not all are described here; grep for `_mm_` in `/usr/lib/gcc/*/4.9/include/` for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and `#define __SSE__` and `__MMX__` before including it. For aligned memory use `_mm_malloc(size, 32)` or `int buf[N] alignas(32)`, but prefer loadu/storeu.

551b82, 43 lines

```
#pragma GCC target ("avx2") // or sse4.1
#include "immintrin.h"

typedef __m256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))

// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256, _mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts of x
// sad_epu8: sum of absolute differences of u8, outputs 4xi64
// maddubs_epi16: dot product of unsigned i7's, outputs 16xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g. _epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|hi)

int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m;
  int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; }
mi zero() { return _mm256_setzero_si256(); }
mi one() { return _mm256_set1_epi32(-1); }
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }

ll example_filteredDotProduct(int n, short* a, short* b) {
  int i = 0; ll r = 0;
  mi zero = _mm256_setzero_si256(), acc = zero;
  while (i + 16 <= n) {
    mi va = L(a[i]), vb = L(b[i]); i += 16;
    va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
    mi vp = _mm256_madd_epi16(va, vb);
    acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
      _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)));
  }
  union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[i];
  for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; // <- equiv
  return r;
}
```

Techniques (A)

techniques.txt 159 lines

```
Recursion
Divide and conquer
  Finding interesting points in N log N
Algorithm analysis
  Master theorem
  Amortized time complexity
Greedy algorithm
  Scheduling
  Max contiguous subvector sum
  Invariants
  Huffman encoding
Graph theory
  Dynamic graphs (extra book-keeping)
  Breadth first search
  Depth first search
  * Normal trees / DFS trees
  Dijkstra's algorithm
  MST: Prim's algorithm
  Bellman-Ford
  Konig's theorem and vertex cover
  Min-cost max flow
  Lovasz toggle
  Matrix tree theorem
  Maximal matching, general graphs
  Hopcroft-Karp
  Hall's marriage theorem
  Graphical sequences
  Floyd-Warshall
  Euler cycles
  Flow networks
  * Augmenting paths
  * Edmonds-Karp
  Bipartite matching
  Min. path cover
  Topological sorting
  Strongly connected components
  2-SAT
  Cut vertices, cut-edges and biconnected components
  Edge coloring
  * Trees
  Vertex coloring
  * Bipartite graphs (=> trees)
  * 3^n (special case of set cover)
  Diameter and centroid
  K'th shortest path
  Shortest cycle
Dynamic programming
  Knapsack
  Coin change
  Longest common subsequence
  Longest increasing subsequence
  Number of paths in a dag
  Shortest path in a dag
  Dynprog over intervals
  Dynprog over subsets
  Dynprog over probabilities
  Dynprog over trees
  3^n set cover
  Divide and conquer
  Knuth optimization
  Convex hull optimizations
  RMQ (sparse table a.k.a 2^k-jumps)
  Bitonic cycle
  Log partitioning (loop over most restricted)
Combinatorics
  Computation of binomial coefficients
  Pigeon-hole principle
  Inclusion/exclusion
  Catalan number
  Pick's theorem
Number theory
  Integer parts
  Divisibility
  Euclidean algorithm
  Modular arithmetic
  * Modular multiplication
  * Modular inverses
  * Modular exponentiation by squaring
  Chinese remainder theorem
  Fermat's little theorem
  Euler's theorem
  Phi function
  Frobenius number
  Quadratic reciprocity
  Pollard-Rho
  Miller-Rabin
  Hensel lifting
  Vieta root jumping
Game theory
  Combinatorial games
  Game trees
  Mini-max
  Nim
  Games on graphs
  Games on graphs with loops
  Grundy numbers
  Bipartite games without repetition
  General games without repetition
  Alpha-beta pruning
Probability theory
Optimization
  Binary search
  Ternary search
  Unimodality and convex functions
  Binary search on derivative
Numerical methods
  Numeric integration
  Newton's method
  Root-finding with binary/ternary search
  Golden section search
Matrices
  Gaussian elimination
  Exponentiation by squaring
Sorting
  Radix sort
Geometry
  Coordinates and vectors
  * Cross product
  * Scalar product
  Convex hull
  Polygon cut
  Closest pair
  Coordinate-compression
  Quadtrees
  KD-trees
  All segment-segment intersection
Sweeping
  Discretization (convert to events and sweep)
  Angle sweeping
  Line sweeping
  Discrete second derivatives
Strings
  Longest common substring
  Palindrome subsequences
  Knuth-Morris-Pratt
  Tries
  Rolling polynomial hashes
  Suffix array
  Suffix tree
  Aho-Corasick
  Manacher's algorithm
  Letter position lists
Combinatorial search
  Meet in the middle
  Brute-force with pruning
  Best-first (A*)
  Bidirectional search
  Iterative deepening DFS / A*
Data structures
  LCA (2^k-jumps in trees in general)
  Pull/push-technique on trees
  Heavy-light decomposition
  Centroid decomposition
  Lazy propagation
  Self-balancing trees
  Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
  Monotone queues / monotone stacks / sliding queues
  Sliding queue using 2 stacks
  Persistent segment tree
```