**May 5, 2024**

**GMT312 - GLOBAL NAVIGATION SATELLITE SYSTEMS**

**SPRING 2023/24**

**ASSIGNMENT III**

**Due: May 12, 2024 – 23:59**

## Computation of Final Satellite Coordinates for Signal Emission Time

### Theoretical Background

*a) Computation of signal emission time*

The measurements collected at the receiver and represented by the time tags in the observation file, i.e. RINEX file, are directly linked to the signal reception time. However, to obtain the coordinates of the receiver, it is required to acquire the satellite position at signal emission time, which represents the actual geometry between the receiver and satellite. After the determination of signal emission time, the satellite coordinates can be computed.

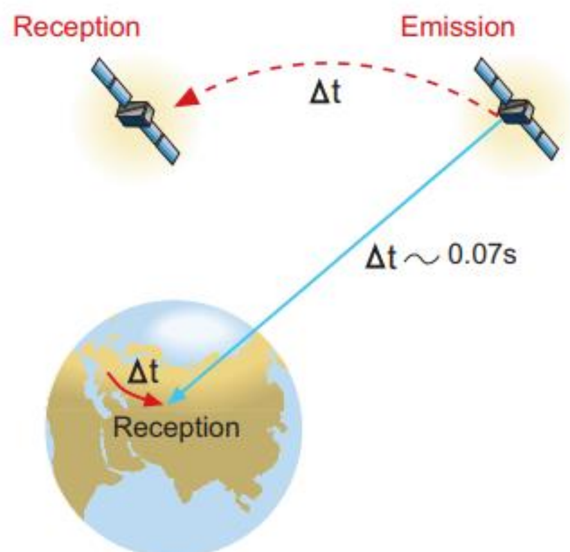


Figure: Satellite coordinates at emission and reception time.

The relation between signal reception time and emission time can be represented as follows:

$$P = c(t_{rcv}^{reception} - t_{sat}^{emission})$$

$$t_{sat}^{emission} = t_{rcv}^{reception} - \Delta t \quad \text{where} \quad \Delta t = P/c$$

$$t^{emission} = t_{sat}^{emission} - \delta t^{sat} = t_{rcv}^{reception} - \frac{P}{c} - \delta t^{sat}$$

Note: The detailed explanation can be found in Section 5.1.1 of the book "*ESA GNSS DATA PROCESSING Volume I: Fundamentals and Algorithms*".

So, the signal emission time can be computed using the satellite clock error and code pseudorange measurements for the related satellite. While the code pseudorange measurements and reception time (time tag of the measurement) can be found in the observation file (RINEX file), the satellite clock error should be calculated for the related satellites. In general, there are three different sources to compute the receiver clock error (or correction): 1) Using the polynomial coefficients given in the navigation file 2) Interpolating the receiver clock corrections given in the precise ephemeris (SP3 file) 3) Finally, using the more specific clock file generated by the International GNSS Service (IGS).

*b) Computation of satellite coordinates*

Once the signal emission time is determined, the satellite coordinates can be computed at this epoch in an Earth-centered Earth-fixed (ECEF) coordinate system (e.g. ITRS or WGS84). Still, it must be considered that any ECEF coordinate system rotates with the Earth during the travel of the signal from the satellite to the receiver, as it is an "Earth-fixed" system. Since the signal reception time is common for all measurements, the ECEF coordinates of the satellites must be computed at the signal reception time (in the related GNSS time scale). However, for the coordinates of the satellite which are computed at the signal emission time, the related ECEF system is attached to the signal emission epoch. Therefore, these coordinates must be corrected to ECEF coordinates which is attached to the signal reception time taking the rotation of the Earth into account. If we name the satellite coordinates at the signal emission time as approximate satellite coordinates ($\boldsymbol{r}_{apr}^{sat}$), the final coordinates of satellite in the ECEF system at the signal reception time can be computed as follows:

$$\boldsymbol{r}_{final}^{sat} = \mathbf{R}_3(w_E \Delta t)\, \boldsymbol{r}_{apr}^{sat}$$

where $w_E$ is Earth's rotation rate, $\mathbf{R}_3$ is the rotation matrix and $\Delta t$ is the signal travel time which can be computed as:

$$\Delta t = [\![\boldsymbol{r}_{apr}^{sat} - \boldsymbol{r}_0^{rcv}]\!]/c$$

where $\boldsymbol{r}_0^{rcv}$ is an approximate position of the receiver, $c$ is the speed of the light and $[\![\boldsymbol{r}_{apr}^{sat} - \boldsymbol{r}_0^{rcv}]\!]$ denotes the geometric distance between related two points.

**ASSIGNMENT:**

Please compute the final coordinates ($\boldsymbol{r}_{final}^{sat}$) of <u>the first two GPS satellites</u> in the observation file of MERS station for March 1, 2024, for the reception epoch specifically assigned to you. You need to compute the signal emission time and the approximate satellite position ($\boldsymbol{r}_{apr}^{sat}$) for the signal emission time from the related SP3 file. Then, you can compute the final coordinates of these satellites as described above.

<u>Steps:</u>

1) Compute the observation epoch (reception time) which will be used for the calculations (seconds of the day):

$t_{rcv}^{reception}$ = (*the sum of all digits of your school ID*) *930 sec. (between 0-86400).

If the reception time is a common multiple of 900, add 720 seconds to it.

2) Application day is March 1, 2024. Download following two files from any of the IGS ftp servers: observation file in RINEX2 or RINEX3 format and precise ephemeris (SP3 file). Note that you will need to uncompress Hatanaka compressed RINEX observation files.

3) Write a function (with *Python or MATLAB*) to compute the emission time of a given code pseudorange with precise ephemerides (SP3) as follows:

The design of the function should be as follows:

[*tems*]=emist(*trec*, *pc*, *clk*)

| | | | |
|---|---|---|---|
| Inputs: | *trec* | Reception time | (Seconds of the day) |
| | *pc* | Code pseudorange observation from the observation file | (Meter) |
| | *clk* | A matrix (or corresponding array) composed of the satellite clock corrections for ten consecutive epochs (five epochs after and before the reception time) which are obtained from the related precise ephemeris as well as the time tags (t) for these epochs to be used with Lagrange interpolation | 10x2 matrix |
| Outputs | *tems* | The signal emission time computed using precise ephemerides | (Seconds of the day) |

4) Write a function (with *Python or MATLAB*) to compute the final satellite coordinates considering the Earth's rotation exploiting the function you write in the previous step. The function should be designed to compute the final coordinates for only one satellite. Therefore, you will need to run your code 2 times to acquire the final coordinates of the related two GPS satellites. Please use C/A (C1) observations to compute the signal emission time. Also, you should use the approximate coordinates of the receiver provided in the header part of observation file (APPROX POSITION XYZ) as the approximate position of the receiver ($\boldsymbol{r}_0^{rcv}$).

The design of the function should be as follows:

[*fpos*]=sat_pos(*trec*, *pc*, *sp3*, *r_apr*)

| | | | |
|---|---|---|---|
| Inputs: | *trec* | Reception time | (seconds of the day) |
| | *pc* | Code pseudorange observation from the observation file | (meter) |
| | *sp3* | A matrix (or corresponding array) composed of the time tags (t); corresponding satellite coordinates (X, Y, Z); and the satellite clock corrections for ten consecutive epochs (five epochs after and before the reception time) which are obtained from the related precise ephemeris | 10x5 matrix |
| | *r_apr* | Approximate receiver coordinates in ECEF | [3x1] (meter) |
| Output: | *fpos* | Final satellite coordinates in ECEF ($\boldsymbol{r}_{final}^{sat}$) | [3x1] (meter) |

Notes:

- Please be careful about the function structures (inputs, outputs etc.). Try to avoid or minimize repetitions of similar codes in your functions.
- Be careful about the units for all steps.
- At the very beginning of each function (after the first line) place comment lines to introduce its functionality besides inputs and outputs. Also put comment lines within your codes to explain main steps.
- The observation files with observation sampling rate of 30 seconds "*mers0610.24o*" or "*MERS00TUR_R_20240610000_01D_30S_MO.rnx*" besides combined IGS precise ephemeris file "*IGS0OPSFIN_20240610000_01D_15M_ORB.SP3*" should be used for the assignment. You will need to uncompress the unix compressed (.Z or .gz) and the Hatanaka compressed files.
- Refer to related file formats (RINEX navigation and SP3c) whenever needed.
- As a part of this assignment, please deliver a report providing the details and results of the calculations in addition to the functions you developed.
- Assignments should be submitted via HUZEM only.
- Collaboration for the homework is strictly disallowed. Please submit your own work. Collaborated submissions will be downrated significantly.
- Constants:
  Velocity of light $c = 299792458\ m/s$
  Earth's gravitational constant (WGS84) $\mu = 3.986005.10^{14}\ m^3/s^2$
  Earth's rotation rate (WGS84) $w_E = 7.2921151467.10^{-5}\ rad/s$