1 $S = 5 + 9 + 13 + \dots + 89$

$a_1 = 5$; $d = 4$

$a_n = a_1 + (n-1)d$

$89 = 5 + (n-1)4$

$84 = (n-1)4$

$84 : 4 = n-1 \Rightarrow n-1 = 21$; $n = 22$

$$S = \sum_{k=1}^{n=22} 5 + (k-1)4$$

2

$$\sum_{k=3}^{n=15} (2k+1)$$ Shift to start at $k = 1$

246

$k = 3$; $j = k-2$; $k = j+2$

$$\sum_{k=3}^{15} (2k+1) \Rightarrow \sum_{j=1}^{13} (2(j+2)+1) \Rightarrow$$

$$\Rightarrow \sum_{j=1}^{13} (2j+5)$$

③ $a_1 = 12$; $a_n = a_{n-1} + d$ if $a_{10} = 57$

find d and a_{25}

$$\frac{a_{10} - a_1}{9} = d = \frac{57 - 12}{9} = 5 \rightarrow d$$

45

or

$$a_{10} = 12 + (10-1)d = 12 + 9d = 57$$

$$9d = 45 \Rightarrow \boxed{d = 5}$$

$$a_{25} = 24 \cdot 5 + 12 = \boxed{132}$$

④ Find sum of multiples of 7 between 100 and 1000

1) $100 : 7 = 14{,}3 \Rightarrow 7 \cdot 15 = 105$

2) $1000 : 7 = 142{,}9 \Rightarrow 7 \cdot 142 = 994$

$$n = \frac{a_n \; a_1}{d} + 1 = \frac{994 - 105}{7} + 1 = 128$$

$$S = \frac{a_1 + a_n}{2} \cdot n = \frac{105 + 994}{2} \cdot 128 = \boxed{70336}$$

5 $S = \sum_{k=1}^{n} (3k+2)$ find n if $S = 2650$

$S_n = \frac{a_1 + a_n}{2} \cdot n$; $a_1 = 3 \cdot 1 + 2 = 5$

$a_2 = 3 \cdot 2 + 2 = 8$ $\downarrow 3$

$a_n = a_1 + (n-1) \cdot 3$ $a_3 = 3 \cdot 3 + 2 = 11$ $\downarrow 3$

$$S_n = \frac{a_1 + a_1 + (n-1) \cdot 3}{2} \cdot \frac{n}{1} = 2650$$

$$\frac{2a_1 + 3n - 3}{2} \cdot n = 2650$$

$$\frac{10 - 3 + 3n}{2} \cdot n = 2650$$

$$\frac{7 + 3n}{2} \cdot n = 2650 \Rightarrow (7 + 3n) n = 5300$$

$$3n^2 + 7n = 5300 \Rightarrow 3n^2 + 7n - 5300 = 0$$

$D = b^2 - 4ac$; $D = 49 - 4 \cdot 3 \cdot (-5300) = 63649$

$\sqrt{63649} = 252,3$

$$x_1 = \frac{-7+252{,}3}{6} = 40{,}88 \quad x_{1,2} = \frac{-b \pm \sqrt{D}}{2a};$$

$$x_2 = \frac{-7-252{,}3}{6} = -43{,}22$$

$n \approx 40{,}88 \Rightarrow n = 41$

6 5_{th} term is 20 and 15_{th} is 60
What is 10_{th} term

$\frac{60+20}{2} = 40$; since 10 is between 5 and 15

7 20 steps, first is 5 cm each is higher 6 0,5 cm, whats the hight?

$a_n = a_1 + (n-1)d \quad a_1 = 5;\ n = 20;\ d = 0{,}5$

$S_n = \frac{a_1 + a_n}{2} \cdot n;\ S_n = \frac{a_1 + a_1 + (n-1)d}{2} \cdot n$

$$\frac{10+19\cdot 0{,}5}{2}\cdot 20 = 195\ \text{cm}$$

8

$a_1 = 11 \quad d = 3 \quad S_n = 1000 \quad n?$

$a_n = a_1 + (n-1)d \qquad S_n = \frac{a_1 + a_n}{2}\cdot n$

$$\frac{a_1 + a_1 + (n-1)d}{2}\cdot n = 1000$$

$$\frac{2a_1 + nd - d}{2}\cdot n = 1000$$

$$\frac{22 + 3n - 3}{2}\cdot n = 1000$$

$$3n^2 + 19n - 2000 = 0$$

$D = 19^2 + 4\cdot 3\cdot 2000 = 24361 \qquad D = b^2 - 4ac$

$\sqrt{24361} = 156{,}08$

$n_1 = \frac{-19 + 156{,}08}{6} = 22{,}85 \qquad x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

$n_2 = \ominus$

$n_1 \approx 22{,}85 \Rightarrow n = 23$

9 Rewrite $\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k$ as if $k=0$

$k=3$; $j=k-3$; $k=j+3$

$a_1 = 4\left(\frac{1}{2}\right)^3 = \frac{4}{8} = \frac{1}{2}$ $\quad a_n = a_1 \cdot r^{n-1}$

$a_2 = 4\left(\frac{1}{2}\right)^4 = \frac{4}{16} = \frac{1}{4}$ $\quad \frac{1}{4} = \frac{1}{2} \cdot r^1$ $\quad r = \frac{1}{2}$

$\sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3}$; $\sum_{j=0}^{9} 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)^j$

$\sum_{j=0}^{9} \left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^j = \sum_{j=0}^{9} \left(\frac{1}{2}\right)^{j+1}$

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10 Find 10th term if $a_2 = -6$; $a_5 = 48$

$a_2 = a_1 \cdot r^1$; $a_5 = a_1 \cdot r^4$

$a_1 = \frac{a_2}{r} \Rightarrow a_5 = \frac{a_2}{\cancel{r}} \cdot r^{\cancel{4}3} \Rightarrow a_5 = a_2 r^3$

$r^3 = \frac{a_5}{a_2} = -8$ $\quad r = \sqrt[3]{8}$; $r = -2$

$a_1 = \frac{-6}{-2} = 3 \Rightarrow a_{10} = 3 \cdot (-2)^9 = -1536$

11 $a_4 = 54$; $a_7 = 1458$ $r - ?$

$a_4 = a_1 \cdot r^3$; $a_7 = a_1 \cdot r^6$

$a_1 = \frac{a_4}{r^3} \Rightarrow a_7 = \frac{a_4}{\cancel{r^3}} \cdot r^{\cancel{6}\,3}$

$a_7 = a_4 \cdot r^3 \Rightarrow r^3 = \frac{a_7}{a_4}$

$r^3 = \frac{1458}{54} = 27$; $\sqrt[3]{27} = 3$

$r = 3$

12 $S_{15} - ?$ if $a_1 = 8$; $r = \frac{3}{4}$

$S_n = \frac{a_1(r^n - 1)}{r - 1}$

$S_{15} = \frac{8((\frac{3}{4})^{15} - 1)}{\frac{3}{4} - 1} \approx 31{,}57 \approx 32$

13 $P(x)=x^5-4x^3+x^2-7$ Classify by degree and number of terms

Degree: 5 ; Number of terms: 4

14 Simplify $(2x^4-3x^3+x-5)+$

$+(x^3-2x^2+4x+7)=$

$=2x^4-2x^3-2x^2+5x+2$

15 $(x^2-x+2)(x^2+x+1)$

?

$x^4+\cancel{x^3}+\cancel{x^2}-\cancel{x^3}-\cancel{x^2}-x+2x^2+2x+2=$

$=x^4+2x^2+x+2$

17 Factor x^4-13x^2+36

$n=x^2 \Rightarrow n^2-13n+36=0$

$n \quad 9$

$n \quad 4$

$(n-9)(n-4)=0$

$(x^2-9)(x^2-4)=(x-3)(x+3)(x-2)(x+2)$

18 Expand $(2x+3y)^5$

$C_5^5 \; C_5^4 \; C_5^3 \; C_5^2 \; C_5^1 \; C_5^0$

$32x^5+240x^4y+720x^3y^2+1080x^2y^3+810xy^4+$

$5\cdot2^4\cdot3 \quad 10\cdot2^3\cdot3^2 \quad 10\cdot2^2\cdot3^3 \quad 5\cdot2\cdot3^4$

$+243y^5$

19 Divide $6x^3+11x^2-31x+15$ by $3x-2$

$\Downarrow$

$$6x^3+11x^2-31x+15 \;\Big|\; \underline{3x-2}$$
$$ 2x^2+5x-7$$

$$-\underline{(6x^3-4x^2)}$$

$$15x^2-31x$$

$$-\underline{(15x^2-10x)}$$

$$-21x+15$$

$$-\underline{(-21x+14)}$$

$$\underline{1}$$

$$(2x^2+5x-7)(3x-2)+1=$$

$$2x^2+5x-7=\frac{6x^3+11x^2-31x+\overset{(15-1)}{14}}{3x-2}$$