1

a) $y = x^2 + 1$ is a function

for one input one output

b) $y^2 = x + 1$ is <u>not</u> a function

$y = \pm\sqrt{x+1}$

2

Which functions are surjective

1) $f: \mathbb{Z} \to \mathbb{Z}$ defined by

$f(n) = 3n$

f - is <u>not</u> surjective, because $\mathbb{Z}$ is

int number, and n could be rational

2) $g: \{1, 2, 3\} \to \{a, b, c\}$

defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$

g - is <u>not</u> surjective

"b" is missing

3) $h: \{1, 2, 3\} \to \{1, 2, 3\}$

$$\begin{tikzcd}
1 \arrow[dr] & 2 \arrow[dl] & 3 \arrow[d] \\
1 & 2 & 3
\end{tikzcd}$$

; h - is surjective, because for each output there is at least one input.

---

3 Which of them are injective

f and h are injective

If $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{1}{x} - 2$

is $g = f^{-1}$? $f(x)$ $\frac{1}{x-2}$

Swap

$y = \frac{1}{x-2}$

$x = \frac{1}{y-2}$

$y = \frac{1}{x} - 2$

$f^{-1}(y) = \frac{1}{y} - 2 \Rightarrow g = f^{-1}$

4 Find the inverse of the function

$f(x) = 2 + \sqrt{x-4}$

$y = 2 + \sqrt{x-4}$ Swap

$x = 2 + \sqrt{y-4}$

$\sqrt{y-4} = x - 2$

$y - 4 = (x-2)^2$

$y = (x-2)^2 + 4$; $f^{-1}(x) = (x-2)^2 + 4$

---

5 $C = \frac{5}{9}(F - 32)$

$\frac{9}{5}C = F - 32$

$F = \frac{9}{5}C + 32$

6 Find the domain and range

$g(x) = 2\sqrt{x-4}$

$x-4 \geq 0$

$x \geq 4$

Domain $x \in [4; +\infty)$

Range $g(x) \in [0; +\infty)$

7 $h(x) = -2x^2 + 4x - 9$

$x = -\frac{4}{2(-2)} = 1$

$h(1) = -2 + 4 - 9 = -7$

Domain $x \in (-\infty; +\infty)$

Range $h(x) \in (-\infty; -7]$

8 $f(x) = \frac{x-4}{x^2-2x-15}$

$x^2 - 2x - 15 = 0$; $D = b^2 - 4ac$

$D = 4 - 4(-15) = 64$

$\sqrt{64} = \pm 8$

$$x_1 = \frac{2+8}{2} = 5 \qquad x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$x_2 = \frac{2-8}{2} = -3$$

Domain $x \in (-\infty; -3) \cup (-3; 5) \cup (5; +\infty)$

9

$$f(x) = \begin{cases} -2x+1 & -1 \le x < 0 \\ x^2+2 & 0 \le x \le 2 \end{cases}$$

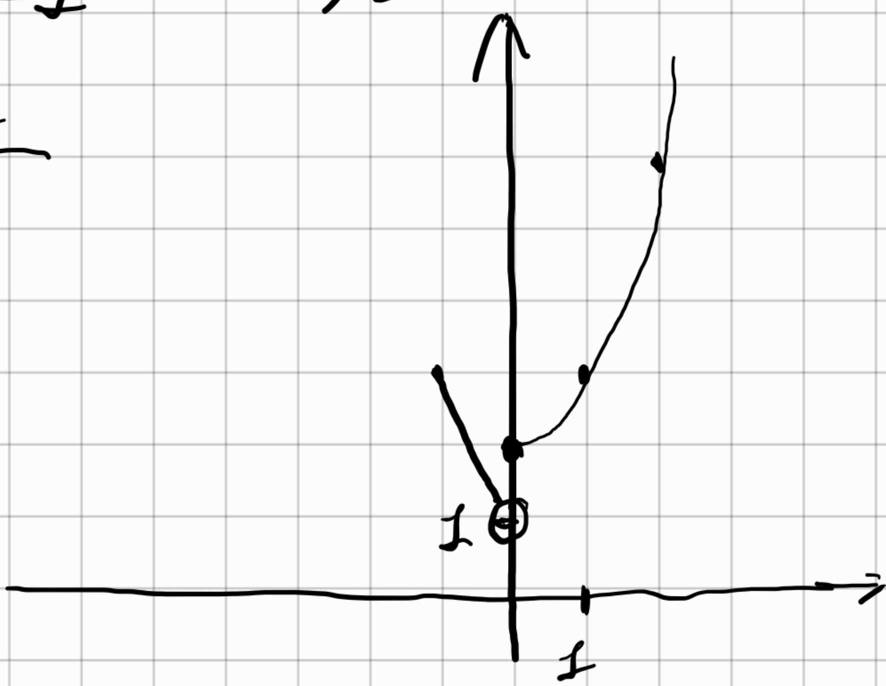
$f(x) = -2x+1$ $\qquad$ $f(x) = x^2+2$

$f(-1) = -2\cdot(-1)+1 = 3$ $\qquad$ $f(0) = 2$

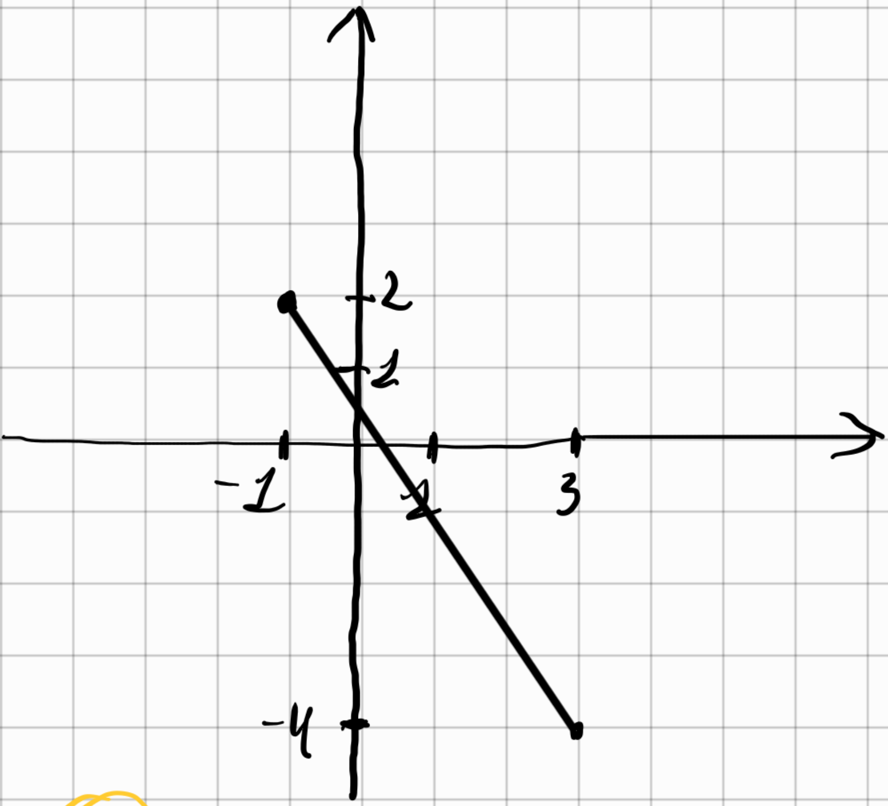
$f(0) = -2\cdot 0 + 1 = 1$ $\qquad$ $f(1) = 3$ $\qquad$ $f(2) = 2^2+2 = 6$

x	y
-1	3
0	1

x	y
0	2
1	3
2	6



(10) Find the Slope of the Line that Passes through the points (-1;2) and (3;-4)

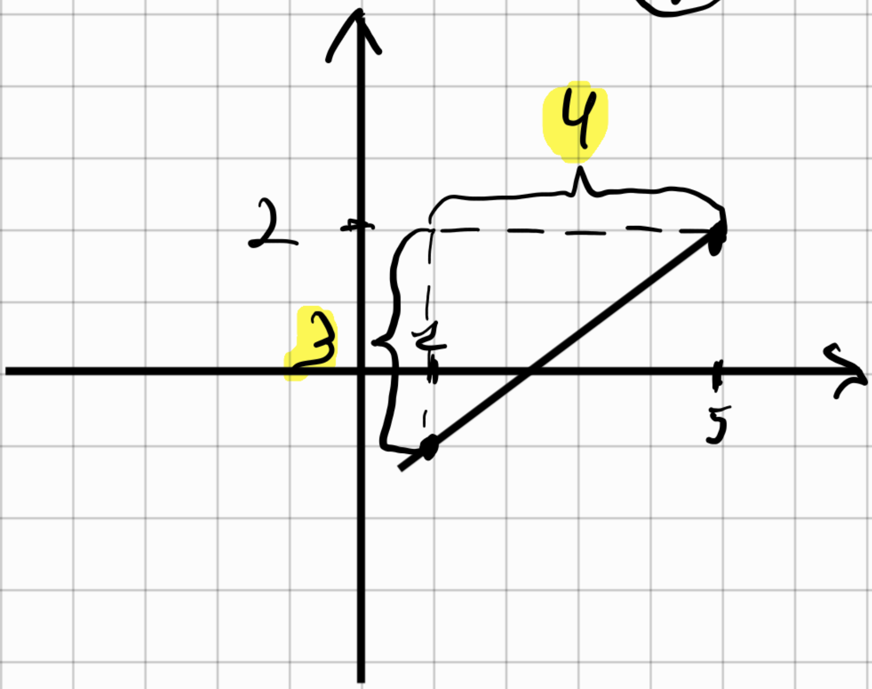


$$\text{Slope} = \frac{-4-2}{3-(-1)} = \frac{-6}{4}$$

$$-\frac{3}{2}$$

(11) Graph the Line Passing point (1,-1)

Slope $m = \frac{3}{4}$ or



$$y = mx + b$$

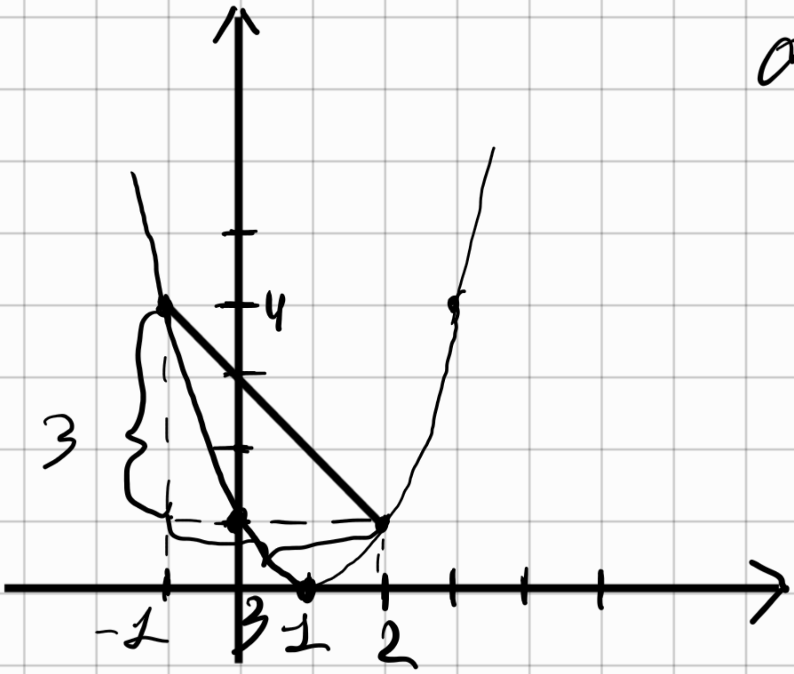
$$\frac{3}{4} \cdot 1 + b = -1$$

$$b = -\frac{3}{4} - 1 = -\frac{7}{4}$$

$$x = 5$$

$$y = \frac{15}{4} - \frac{7}{4} = \frac{8}{4} = 2$$

12) Given $g(t)$, find average rate of change on interval $[-1, 2]$



avr slope at $[-1, 2]$ is -1

13) Compute the average rate of change of $f(x) = x^2 - \frac{1}{x}$; on $x \in [2, 4]$

$f(2) = 2^2 - \frac{1}{2} = 3\frac{1}{2}$; $f(4) = 4^2 - \frac{1}{4} = 15\frac{3}{4}$

$3,5$ $\qquad$ $15,75$

$$m = \frac{15,75 - 3,5}{4-2} = 6,125$$

14 Given $f(x)=t^2-t$ and $h(x)=3x+2$, evaluate $f(h(1))$.

$h(1)=3\cdot1+2=5$

$f(5)=5^2-5=20$

15 Find the domain of $(f\circ g)(x)$ where:

$f(x)=\frac{5}{x-1}$ and $g(x)=\frac{4}{3x-2}$

$(f\circ g)(x)$; $g(x)=\frac{4}{3x-2}$

$f\left(\frac{4}{3x-2}\right)=\frac{5}{\frac{4}{3x-2}-1}$

1) $3x-2\neq0$

$x\neq\frac{2}{3}$

2) $\frac{4}{3x-2}\neq1$

$3x-2\neq4$

$x\neq2$

Answear: $x\in(-\infty;\frac{2}{3})\cup(\frac{2}{3};2)\cup(2;+\infty)$

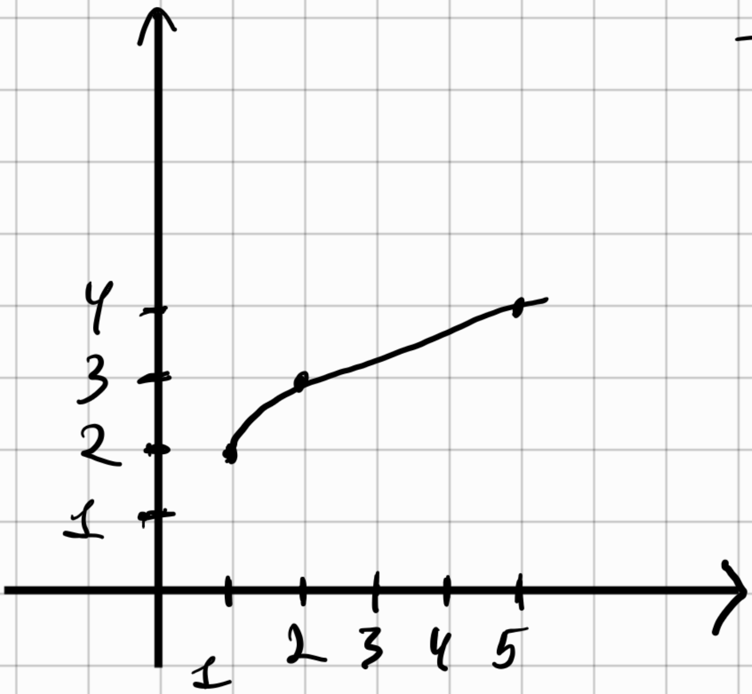
16 Find and simplify $(g-f)(x)$ and $\left(\frac{g}{f}\right)(x)$
$f(x) = x-1$ and $g(x) = x^2-1$. Are they the same?

$(g-f)(x) = g(x) - f(x) = x^2-1-(x-1) =$
$= x^2-1-x+1 = x^2-x = x(x-1)$

$\left(\frac{g}{f}\right)(x) = \frac{x^2-1}{x-1} = \frac{x^2-1^2}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} =$
$= x+1$

$(g-f)(x) = x(x-1) \neq \left(\frac{g}{f}\right)(x) = x+1$

17 Write a formula for sqrt function



$f(x) = \sqrt{x-1}+2$
$f(1) = \sqrt{1-1}+2 = 2$
$f(2) = \sqrt{2-1}+2 = 3$
$f(5)\ \sqrt{5-1}+2 = 4$

$f(x) = \sqrt{x-1}+2$

18 Write a formula for $f(x)=\frac{1}{x}$ that shifts functions graph one unite to right and one unit up

$$f(x)=\frac{1}{x-1}+1$$

19 $f(x)=x^3+2x$; is it even, odd, or neither

even $\Rightarrow f(x)=f(-x)$ ×

odd $\Rightarrow f(-x)=-f(x)$ ✓

$f(1)=1^3+2\cdot1=3$; $f(-1)=-1^3-2\cdot1=-3$

20 $f(s)=s^4+3s^2+7$; even, odd or neither?

even $\Rightarrow f(x)=f(-x)$ ✓

odd $\Rightarrow f(-x)=-f(x)$ ×

$f(1)=1^4+3\cdot1^2+7=11$; $f(-1)=(-1)^4+3\cdot(-1)^2+7=$
$=11$

21 Point - slope form $y - y = m(x - x_1)$

Write point-slope for $(5,1)$ and $(8,7)$ &

write slope-intercept form $y = mx + b$

1) Point-slope $7 - 1 = m(8 - 5) \Rightarrow$

$\Rightarrow 6 = m \cdot 3$; $\Rightarrow m = 2$; $1 = 2 \cdot 5 + b \Rightarrow$

$\Rightarrow b = 1 - 10 = -9$; $y = 2x - 9$

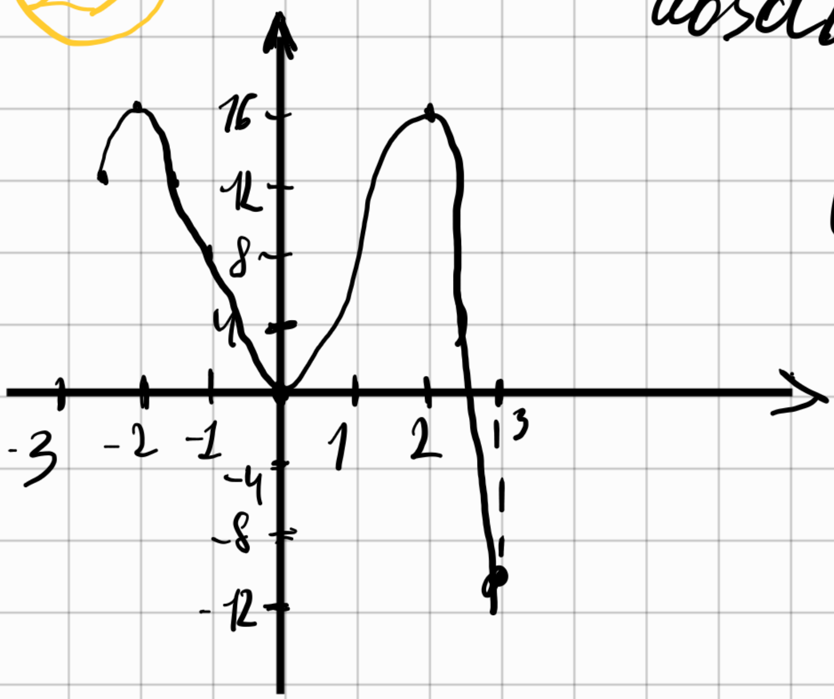
22 $f(x)$ is linear and passes $(3,-2)$ and $(8,1)$

find slope

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}; \quad \frac{1 - (-2)}{8 - 3} = \frac{3}{5} = 0{,}6$$

function is increasing $m > 0$

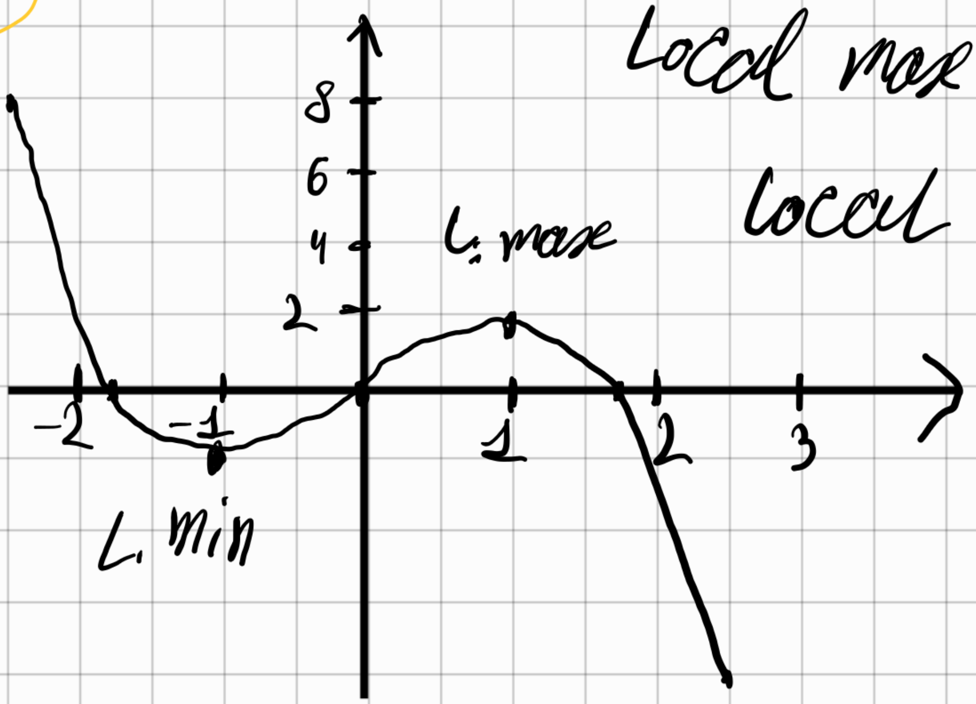
23



absolute max at $x=-2$ and $x=2$ (16)

absolute min at $x=3$ (−10)

24



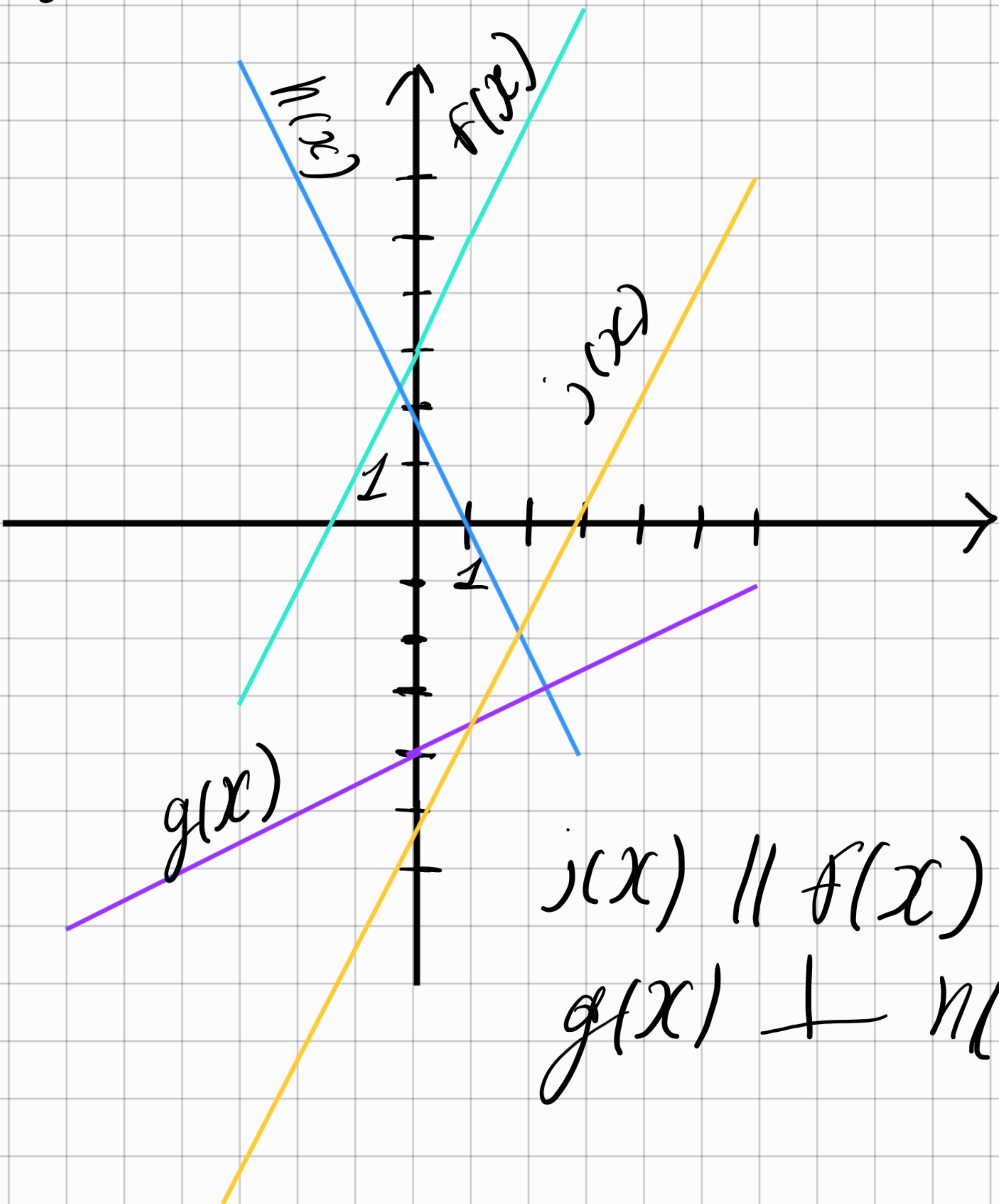
Local max at $x=1$

local min at $x=-1$

25 Which are // and which are ⊥

$f(x) = 2x + 3$ $\qquad$ $h(x) = -2x + 2$

$g(x) = \frac{1}{2}x - 4$ $\qquad$ $j(x) = 2x - 6$



$j(x) \parallel f(x)$

$g(x) \perp h(x)$

26

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \quad | \cdot (-2) \end{cases}$$

$$\begin{cases} 2x + y = 7 \\ -2x + 4y = -12 \end{cases} + \Rightarrow 5y = -5, \; y = -1$$

$$2x - 1 = 7$$

$$2x = 8 \Rightarrow x = 4$$

Answer: intersection is at $(4, -1)$

27

$$\begin{cases} 4x + 2y = 4 \\ 6x - y = 8 \quad | \cdot 2 \end{cases}$$

$$\begin{cases} 4x + 2y = 4 \\ 12x - 2y = 16 \end{cases} + \Rightarrow 16x = 20 \Rightarrow x = \frac{20}{16}$$

$$x = 1{,}25$$

$$6 \cdot 1{,}25 - y = 8$$

$$y = 6 \cdot 1{,}25 - 8 = -0{,}5$$

$(1{,}25, -0{,}5)$

or

$\left(\frac{5}{4}, -\frac{1}{2}\right)$

(28) Find vertex $f(x)=2x^2-6x+7$

rewrite $f(x)$ in standard form

$2x^2-6x+7$

$h=-\frac{b}{2a}$; $h=-\frac{-6}{2\cdot2}=\frac{6}{4}=1\frac{1}{2}$

$k=f(h)=f\left(\frac{-b}{2a}\right)$; $k=2\cdot\left(\frac{6}{4}\right)^2-6\frac{6}{4}+7=$

$=2\frac{1}{2}$

$f(x)=2\left(x-1\frac{1}{2}\right)^2+2\frac{1}{2}$

(29) Find the domain and range of

$f(x)=-5x^2+9x-1$

$h=-\frac{b}{2a}$; $h=-\frac{9}{-2\cdot5}=\frac{9}{10}$ (- that is the value of x when y is max)

$f\left(\frac{9}{10}\right)=-5\cdot\left(\frac{9}{10}\right)^2+9\cdot\frac{9}{10}-1=3,05$

The domain $x\in(-\infty;+\infty)$

The range is $f(x)$ $(-\infty;3,05]$

30 Find y- and x- intercepts $f(x)=3x^2+5x-2$

$f(0)$ - y intercept

$f(0)=3\cdot0+5\cdot0-2=-2$

$f(x)=0$ - x intercept

$3x^2+5x-2=0$

$D=b^2-4ac$

$5^2-4\cdot3\cdot(-2)=49$

$x_{12}=\frac{-b\pm\sqrt{D}}{2a}$

$\sqrt{49}=7$

$x_1=\frac{-5+7}{6}=\frac{2}{6}=\frac{1}{3}$

$x_2=\frac{-5-7}{6}=\frac{-12}{6}=-2$

y- intercept at $(0,-2)$

x- intercept at $(\frac{1}{3},0)$ and $(-2,0)$

31 Solve, graph on number line

a. $-1 \le 2x-5 < 7 \mid +5$

$4 \le 2x < 12 \mid :2$

$2 \le x < 6 \Rightarrow x \in [2, 6)$

б. $x^2+7x+10<0$

$x^2+7x+10=0$

$D=b^2-4ac$

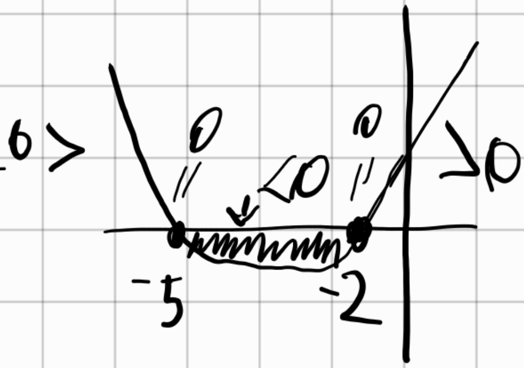
$D=7^2-4\cdot 10=9$

$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$

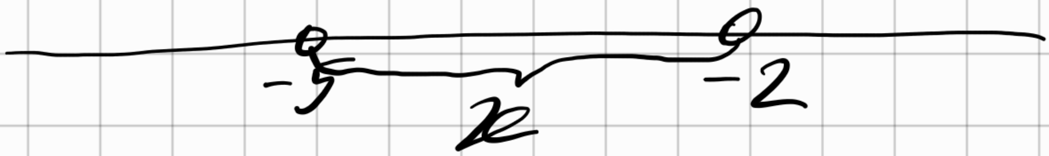
$\sqrt{9}=3$

$x_1=\frac{-7+3}{2}=\frac{-4}{2}=-2$

$x_2=\frac{-7-3}{2}=-\frac{10}{2}=-5$



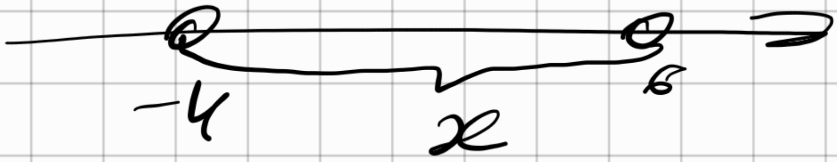
$x \in (-5, -2)$



c. $-6 < x-2 < 4 \quad |+2$

$-4 < x < 6$

$x \in (-4; 6)$



32

$10-(2y+1) \le -4(3y+2)-3$

$10+3 \le -4(3y+2)+2y+1$

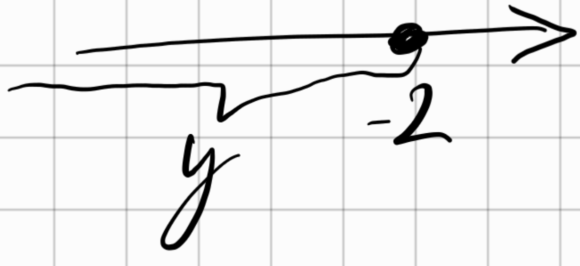
$10+3-1 \le -12y-8+2y$

$10+3-1+8 \le -10y$

$20 \le -10y$

$2 \le -y \Rightarrow y \le -2$

$y \in (-\infty; -2]$



33

$x(x+3)^2(x-4)<0$

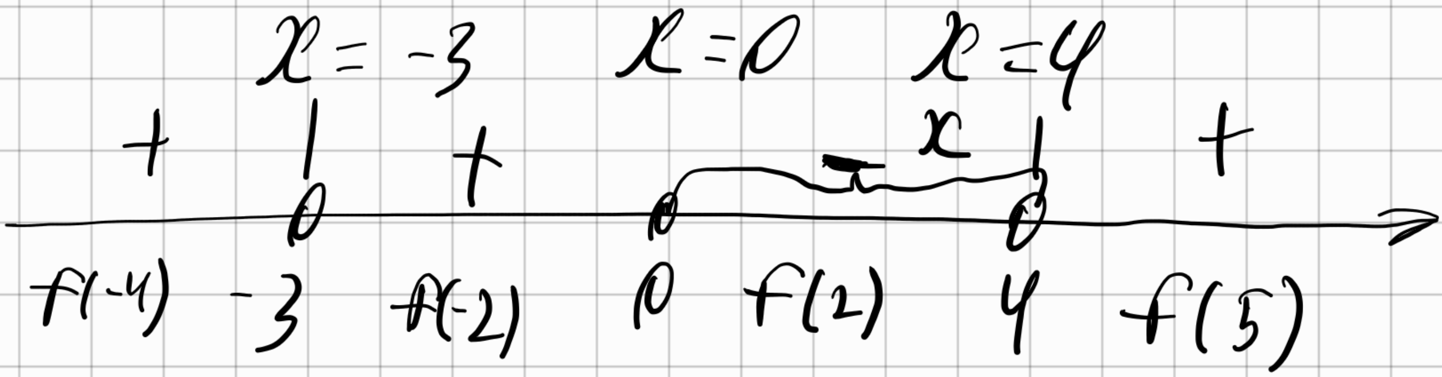
1) $x \neq 0$

2) $x+3 \neq 0$

$x \neq -3$

3) $x-4 \neq 0$

$x \neq 4$



Lets check function in random dots

Betweem -3 0 and 4

$f(-4) = -4(-4+3)^2(-4-4) = 32$ (+)

$f(-2) = -2(-2+3)^2(-2-4) = 12$ (+)

$f(2) = 2(2+3)^2(2-4) = -100$ (−)

$f(5) = \underset{+}{5}\underset{+}{(5+3)^2}\underset{+}{(5-4)} =$ (+)

<0 $x \in (0;4)$

34 ?

$$2x^4 > 3x^3 + 9x^2 \quad | : x^2 \Rightarrow x \neq 0$$

$$2x^2 > 3x + 9$$

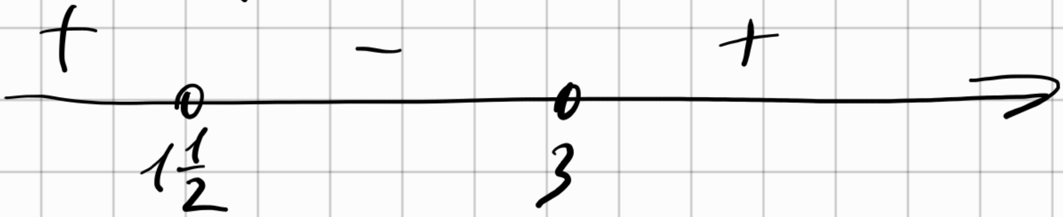
$$2x^2 - 3x - 9 > 0$$

$$2x^2 - 3x - 9 = 0 \qquad D = b^2 - 4ac$$

$$D = 9 + 4 \cdot 9 \cdot 2 = 81; \sqrt{81} = 9 \qquad x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$x_1 = \frac{3+9}{4} = 3$$

$$x_2 = \frac{3-9}{4} = -\frac{6}{4} = -1\frac{1}{2}$$



$$x \in \left(-\infty; -1\frac{1}{2}\right) \cup (3; +\infty)$$

35 $f(x) = -\frac{1}{2}|4x-5| + 3$; determine x for which $f(x)$ is negative

$$-\frac{1}{2}|4x-5| + 3 < 0$$

$$-\frac{1}{2}|4x-5| < -3 \quad | \cdot -2$$

$$|4x-5| > 6$$

$$|4x-5| = 6$$

$4x-5 = 6$ $4x-5=-6$

$4x = 11$ $4x = -1$

$x_1 = \frac{11}{4} = 2\frac{3}{4}$ $x_2 = -\frac{1}{4}$

$$f(-1) = -\frac{1}{2}|-4-5| + 3 = -4{,}5+3 = -1{,}5 \; (-)$$

$$f(0) = -\frac{1}{2}\cdot 5 + 3 = -2{,}5+3 = 0{,}5 \; (+)$$

$$f(3) = -\frac{1}{2}|4\cdot 3-5| + 3 = -3{,}5+3 = -0{,}5 \; (-)$$

$x \in (-\infty; -\frac{1}{4}) \cup (2\frac{3}{4}; +\infty)$

36

$13-2|4x-7|\leq 3$

$13-3-2|4x-7|\leq 0$

$10-2|4x-7|\leq 0 \quad |:2$

$5-|4x-7|\leq 0$

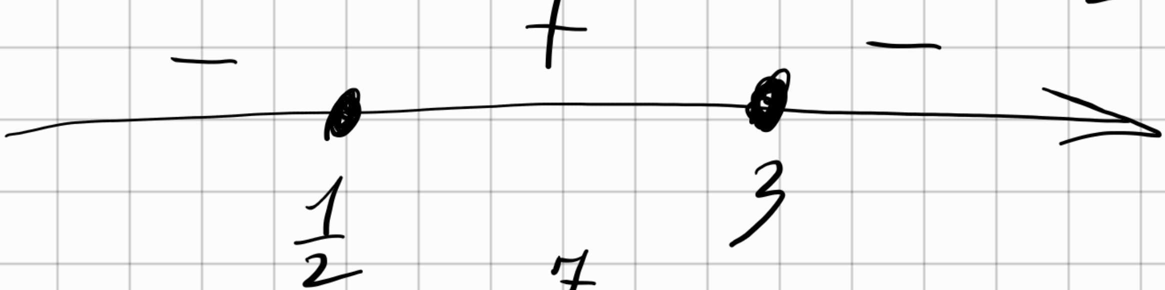
$|4x-7|\geq 5$

$$|4x-7|=5$$

$4x-7=5 \qquad\qquad 4x-7=-5$

$4x=12 \qquad\qquad 4x=2$

$x=3 \qquad\qquad x=\frac{1}{2}$



$f(0)=10-2\overset{7}{|-7|}=-4\,(-)$

$f(1)=10-2|\overset{3}{4-7}|=4\,(+)$

$f(4)=10-2\overset{9}{|16-7|}=-8\,(-)$

$x\in(-\infty;\frac{1}{2}]\cup[3;+\infty)$