1

$$\log_2\left(\frac{8\sqrt{2}}{16}\right) + \log_2(32) - 2\log_2(4) =$$

$$= \log_2(8\sqrt{2}) - \log_2 16 + \log_2 2^5 - 2\log_2 2^2 =$$

$$= \log_2 2^3 + \log_2 2^{\frac{1}{2}} - \log_2 2^4 + \log_2 2^5 - 2\log_2 2^2 =$$

$$= 3 + \frac{1}{2} - 4 + 5 - 4 = -8 + 8 + \frac{1}{2} = \frac{1}{2}$$

2

$$\log_3(x-1) + \log_3(x+1) = 2$$

$$\log_3(x-1)(x+1) = 2$$

$$\log_3(x^2 - 1^2) = 2$$

$$x^2 - 1^2 = 3^2 \Rightarrow x^2 = 3^2 + 1 = 10$$

$$x = \pm\sqrt{10} \approx 3{,}16$$

$x = 3,16$ ✓ $x = -3,16$ ✗

$x - 1 > 0$ $x - 1 < 0$

$x + 1 > 0$ $x + 1 < 0$

$x = ,16$

3) 10000 $; 6% per year = 20000 $

$10000\left(1 + \frac{0,06}{4}\right)$ - 1st Quarter

$10000\left(1 + \frac{0,06}{4}\right)^2$ - 2nd Quarter

$10000\left(1 + \frac{0,06}{4}\right)^3$ - 3rd Quarter

$10000\left(1 + \frac{0,06}{4}\right)^4$ - 4 Quarter = 1 year

t - years

$$10000\left(1 + \frac{0,06}{4}\right)^{4t} = 20000$$

$$\left(1+\frac{0,06}{4}\right)^{4t} = \frac{20000}{10000} = 2$$

$$1,015^{4t} = 2$$

$$\ln(1,015)^{4t} = \ln 2$$

$$4t \cdot \ln(1,015) = \ln(2)$$

$$4t = \frac{\ln 2}{\ln(1,015)} \Rightarrow t = \frac{\ln 2}{4 \ln 1,015}$$

$$t = \frac{0,6931}{4 \cdot 0,0149} = 11,64$$

4) $N(t) = N_0 e^{-kt}$

$N_0$ - initial amount; $k$ - const, $t$ - years

half-Life $\lambda$ - 5 years, find $k$.?

$$\frac{N_0}{2} = N_0 e^{-k \cdot 5} \; ; \; \frac{1}{2} = e^{-k5}$$

$$\ln\left(\frac{1}{2}\right) = -k5 \ln(e) \to 1$$

$$-k = \frac{\ln\left(\frac{1}{2}\right)}{5} /$$

$$-k = -0{,}138629 \; ; \Rightarrow k = 0{,}138629$$

$$k \approx 0{,}14$$

5

$$100g \xrightarrow{3 \text{ hours}} 70g$$

$$100g \xrightarrow{?} 20g$$

$$70 = 100 \cdot e^{-k \cdot 3}$$

$$\frac{7}{10} = e^{-k3} \Rightarrow \ln\left(\frac{7}{10}\right) = -3k$$

$$k = \frac{\ln\left(\frac{7}{10}\right)}{-3} = 0{,}11889648$$

$$20 = 100 \cdot e^{-kt}$$

$$\ln\left(\frac{2}{10}\right) = -k \cdot t \;;\Rightarrow t = \frac{\ln\left(\frac{2}{10}\right)}{-k}$$

$$t = \frac{\ln\left(\frac{2}{10}\right)}{-0{,}1189} = \boxed{13{,}54 \text{ hours}}$$

(6) find vector from $A(1,2,3)$ to $B(4,6,9)$

$$\vec{v}(AB) = (4-1, 6-2, 9-3) = (3,4,6)$$

$$|\vec{v}| = \sqrt{3^2+4^2+6^2} = 7{,}81$$

(7) $v = 4\hat{i} - 2\hat{j} + 4\hat{k}$ in matrix and find magnitude

$$\vec{v} = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}$$

$$\vec{v} = \sqrt{4^2+2^2+4^2} = \boxed{8{,}30}$$

(8) $\vec{a} = (2, -1, 3)$ ; $\vec{b} = (-1, 4, 2)$

compute $3\vec{a} - 2\vec{b}$

$3\vec{a} = (6, -3, 9)$ ; $2\vec{b} = (-2, 8, 4)$

$3\vec{a} - 2\vec{b} = (6\hat{i} - 3\hat{j} + 9\hat{k}) - (-2\hat{i} + 8\hat{j} + 4\hat{k})$

$= 8\hat{i} - 11\hat{j} + 5\hat{k} = (8, -11, 5)$

(9) Find the angle between $\vec{p}$ and $\vec{q}$ ; $\vec{p} = (1, 2, 3)$ $\vec{q} = (4, -5, 6)$

$\vec{p} \cdot \vec{q} = \|\vec{p}\| \cdot \|\vec{q}\| \cdot \cos\theta$

$\vec{p} \cdot \vec{q} = 1 \cdot 4 + 2 \cdot (-5) + 3 \cdot 6 = 12$

$\sqrt{1^2 + 2^2 + 3^2} \cdot \sqrt{4^2 - 5^2 + 6^2} \cdot \cos\theta = 12$

$3{,}74 \cdot 8{,}775 \cdot \cos\theta = 12$

$32{,}829$

$$\cos\theta = \frac{12}{32{,}8329} = 0{,}365487$$

$$\cos\theta = 0{,}365487 \Rightarrow \theta \approx 69^\circ$$

(10) $\vec{u} = (2, -1, 4)$ and $\vec{v}(-8, 4, -16)$ orthogonal -?

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos\theta = 0$ if $\theta = 90^\circ$

$\vec{u} \cdot \vec{v} = 2 \cdot (-8) - 1 \cdot 4 + 4 \cdot (-16) =$

$= -16 - 4 - 64 = -84$ - not orthogonal

(11) $2A - 3D$

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix}$$

$$2A = 2 \cdot \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 0 & 6 \end{bmatrix}$$

$$3B = 3 \cdot \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 15 \\ -6 & 3 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} 4 & -2 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 12 & 15 \\ -6 & 3 \end{bmatrix} =$$

$$= \begin{bmatrix} -8 & -17 \\ 6 & 3 \end{bmatrix}$$

(12) $E = CD$; $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $D = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$C \cdot D = \begin{bmatrix} 1 \cdot 5 + 2 \cdot 7 & 1 \cdot 6 + 2 \cdot 8 \\ 3 \cdot 5 + 4 \cdot 7 & 3 \cdot 6 + 4 \cdot 8 \end{bmatrix} =$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

13

$$\begin{cases} x+y+z=6 \\ 2x-y+3z=14 \\ -3x+2y-2z=-10 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{array}\right]$$

$$-2R_1+R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ -3 & 2 & -2 & -10 \end{array}\right]$$

$$R_3+3R_1 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 5 & 1 & 8 \end{array}\right]$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & \frac{8}{3} & \frac{34}{3} \end{bmatrix}$$

$\frac{5}{3}R_2 + R_3$

$\frac{5}{3} \cdot -3 = -5, \quad -5 + 5 = 0$

$\frac{5}{3} \cdot 1 + 1 = 1\frac{5}{3} = \frac{8}{3}$

$\frac{5}{3} \cdot 2 + 8 = \frac{10}{3} + 8 = \frac{34}{3}$

$\frac{8}{3}z = \frac{34}{3}, \Rightarrow z = \frac{34}{3} \cdot \frac{3}{8}$

$z = 4\frac{1}{4}$

$-3y + 4{,}25 = 2$

$y = \frac{2 - 4{,}25}{-3} = 0{,}75$

$x = 6 - 0{,}75 - 4{,}25 = 1$

Answer: $x = 1$, $y = 0{,}75$, $z = 4{,}25$

(14) Reduced row

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_3 + R_1$

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 - 3R_3$

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

**15** $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ find $A^{-1}$

$A \cdot A^{-1} = I$ $\quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$$

$R_2 - 2R_1$ $\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & -2 & 1 \end{array}\right]$

$$R_1 - R_2 \begin{bmatrix} 1 & 0 & | & 3 & -2 \\ 1 & 1 & | & -2 & 1 \end{bmatrix}$$

$$R_2 - R_1 \begin{bmatrix} 1 & 0 & | & 3 & -1 \\ 0 & 1 & | & -5 & 2 \end{bmatrix}$$ ✓

$$\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$