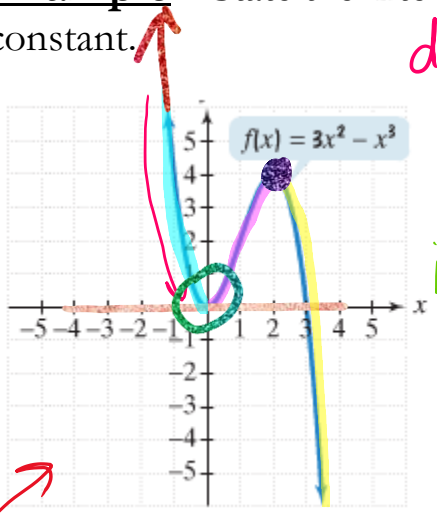
Section 2.2—More on Functions and Their Graphs

A function is increasing if it generally rises from left to right.
A function is decreasing if it generally falls from left to right.
A function is constant if it does not change from left to right.

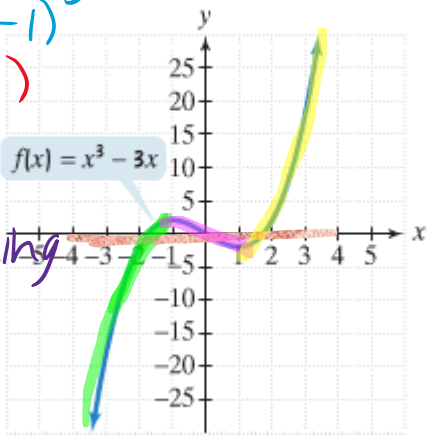
A function can increase over an interval and decrease over another interval, meaning that one function can be both increasing and decreasing.

Example—State the intervals on which the given function is increasing, decreasing, or constant.



decreasing
$(-\infty, 0)$
$(2, \infty)$

increasing
$(0, 2)$



increasing
$(-\infty, -1)$
$(1, \infty)$

decreasing
$(-1, 1)$

The points where a function changes from increasing to decreasing and vice versa are called the relative maximum and relative minimum.

Example—Find the relative minimum and relative maximum in the first graph from the example above.

minimum: $(0, 0)$
maximum: $(2, 4)$

A function is an **even function** if $f(-x) = f(x)$ for all x in the domain of f.

- ✓ The right side of the equation of an even function does not change if x is replaced with –x.

A function is an **odd function** if $f(-x) = -f(x)$ for all x in the domain of f.

- ✓ Every term on the right side of the equation of an odd function changes its sign if x is replaced with –x.

Example—Determine whether each of the following functions is even, odd, or neither.

a. $f(x) = x^2 + 6$

$f(-x) = (-x)^2 + 6$

$f(-x) = x^2 + 6$

even

b. $g(x) = 7x^3 - x$

$g(-x) = 7(-x)^3 - (-x)$

$g(-x) = 7(-x)^3 + x$

$g(-x) = -7x^3 + x$

odd

c. $h(x) = x^5 + 1$

$h(-x) = (-x)^5 + 1$

$h(-x) = -x^5 + 1$

neither

Piecewise Function: A function that is defined by two (or more) equations over a specified domain

Example: A cell phone company has the following plan: $20 per month for 60 minutes and $0.40 for each additional minute. To write an equation, let C represent the monthly cost and t the number of minutes.

$$C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$$

minutes under 60

minutes over 60

Find each of the following:

a. $C(30)$

$C(30) = 20$

b. $C(100)$

$C(100) = 20 + .40(100 - 60)$

$C(100) = 20 + .4(40)$

$C(100) = 20 + 16$

$C(100) = 36$

c. $C(40)$

$C(40) = 20$

d. $C(80)$

$C(80) = 20 + .40(80 - 60)$

$C(80) = 20 + .40(20)$

$C(80) = 20 + 8$

$C(80) = 28$