Section 5.4—Systems of Nonlinear Equations in Two Variables

System of Nonlinear Equations—contains at least one equation that cannot be expressed in the form Ax + By = C; also called a nonlinear equation

Solution Set—set of all solutions to a system

Unlike linear systems these graphs can be circles, parabolas, or anything other than two lines.

We solve a nonlinear system using either the substitution or addition method.

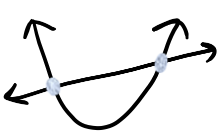
Solving a Nonlinear System Using Substitution

1. Solve one of the equations for one of the variables in terms of the other.
2. Substitute the expression from step 1 into the other equation.
3. Solve the resulting equation containing one variable.
4. Back-substitute the value(s) into the equation from step 1.
5. Check the proposed solutions in both of the given equations.

Example—Solve using substitution

a. $\begin{cases} x^2 = y - 1 \\ 4x - y = -1 \end{cases}$

$4x - y = -1$
$-4x \qquad -4x$
$-y = -4x - 1$
$y = 4x + 1$



$x^2 = y - 1$
$x^2 = (4x+1) - 1$
$x^2 = 4x$
$-4x \quad -4x$
$x^2 - 4x = 0$
$x(x-4) = 0$
$x = 0 \qquad x - 4 = 0$
$+4 \quad +4$
$x = 4$

$y = 4x + 1$
$y = 4(0) + 1$
$y = 1$
$\boxed{(0, 1)}$

$y = 4x + 1$
$y = 4(4) + 1$
$y = 16 + 1$
$y = 17$
$\boxed{(4, 17)}$

b. $\begin{cases} x+2y=0 \\ (x-1)^2+(y-1)^2=5 \end{cases}$

$x+2y=0$
$-2y \quad -2y$
$x=-2y$

$(-2y-1)^2+(y-1)^2=5$

$4y^2+4y+1+y^2-2y+1=5$

$5y^2+2y+2=5$
$\quad -5 \quad -5$

$5y^2+2y-3=0$

$5y^2+5y-3y-3=0$

$5y(y+1)-3(y+1)=0$

$(y+1)(5y-3)=0$

$y+1=0$
$-1 \quad -1$
$y=1$

$5y-3=0$
$+3 \quad +3$
$\frac{5y}{5}=\frac{3}{5}$
$y=\frac{3}{5}$

$5(-3)=-15$
$5 \div -3$

$(-2y-1)(-2y-1)$
$4y^2+2y+2y+1$
$4y^2+4y+1$

$(y-1)(y-1)$
$y^2-y-y+1$
y^2-2y+1

$x=-2y$
$x=-2(-1)$
$x=2$ $\boxed{(2,1)}$

$x=-2y$
$x=-2\left(\frac{3}{5}\right)$
$x=\frac{-6}{5}$ $\boxed{\left(\frac{-6}{5},\frac{3}{5}\right)}$

Steps for Solving a Nonlinear System Using Addition

1. Write both equations in the form $Ax^2 + By^2 = C$
2. Multiply either or both equations so that the sum of one of the coefficients is 0.
3. Add the equations and solve for the remaining variable.
4. Back-substitute to find the remaining variable.
5. Check.

Example—Solve using addition:

a. $\left(\begin{cases} 3x^2 + 2y^2 = 35 \\ 4x^2 + 3y^2 = 48 \end{cases}\right.$ (-3), (2)

$$-9x^2 - \cancel{6y^2} = -105$$
$$8x^2 + \cancel{6y^2} = 96$$
$$-x^2 = -9$$
$$\sqrt{x^2} = \sqrt{9}$$
$$x = \pm 3$$

$$3x^2 + 2y^2 = 35$$
$$3(3)^2 + 2y^2 = 35$$
$$3(9) + 2y^2 = 35$$
$$27 + 2y^2 = 35$$
$$-27 \qquad -27$$
$$\frac{2y^2}{2} = \frac{8}{2}$$
$$\sqrt{y^2} = \sqrt{4}$$
$$y = \pm 2$$

$$3x^2 + 2y^2 = 35$$
$$3(-3)^2 + 2y^2 = 35$$
$$3(9) + 2y^2 = 35$$
$$27 + 2y^2 = 35$$
$$-27 \qquad -27$$
$$\frac{2y^2}{2} = \frac{8}{2}$$
$$\sqrt{y^2} = \sqrt{4}$$
$$y = \pm 2$$

$(3, 2)$ $(3, -2)$ $(-3, 2)$ $(-3, -2)$

b. $\begin{cases} y = x^2 + 5 \\ x^2 + y^2 = 25 \end{cases}$

$y = \cancel{x^2} + 5$

$-x^2 \quad \cancel{-x^2}$

$-x^2 + y = 5$

$\cancel{-x^2} + y = 5$

$\cancel{x^2} + y^2 = 25$

$y^2 + y = 30$

$-30 \quad -30$

$y^2 + y - 30 = 0$

$(y-5)(y+6) = 0$

$y - 5 = 0$

$+5 \quad +5$

$y = 5$

$y + 6 = 0$

$-6 \quad -6$

$y = -6$

$y = x^2 + 5$

$5 = x^2 + \cancel{5}$

$-5 \quad -5$

$0 = x^2$

$\sqrt{x^2} = \sqrt{0}$

$x = 0$

$\boxed{(0, 5)}$

$y = x^2 + 5$

$-6 = x^2 + 5$

$-5 \quad -5$

$-11 = x^2$

$\sqrt{x^2} = \sqrt{-11}$

$x = \pm\sqrt{-11}$

$x = \pm i\sqrt{11}$

imaginary