Section 1.2—Linear Equations and Rational Equations

Linear Equation in One Variable—an equation that can be written in the form $ax+b=0$ where a and b are real numbers, and $a \neq 0$.

Solving an equation in x involves determining all values of x that result in a true statement when substituted into the original equation; such values are called **solutions** or **roots**.

Solution Set—the set of all solutions that satisfy an equation

Equivalent Equations—two or more equations that have the same solution set

Solving a Linear Equation

1. Simplify the algebraic expression on each side by removing grouping symbols and combining like terms.
2. Collect all the variable terms on one side and the numbers or constant terms on the terms, on the other side.
3. Isolate the variable and solve.
4. Check the proposed solution in the original equation.

Example—Solve and check

a. $4x+5=29$

$-5 \quad -5$

$4x=24$

$\frac{4x}{4}=\frac{24}{4}$

$\boxed{x=6}$

Check:

$4x+5=29$

$4(6)+5=29$

$24+5=29$

$29=29$ ✓

b. $4(2x+1)=29+3(2x-5)$

$8x+4=29+6x-15$

$8x+4=6x+14$

$-6x \quad -6x$

$2x+4=14$

$-4 \quad -4$

$\frac{2x}{2}=\frac{10}{2}$

$\boxed{x=5}$

Equations are easier to solve when they do not contain fractions. To rid the equation of the fraction, begin by multiplying both sides of the equation by the least common denominator of any fractions in the equation. We then "clear the equation of the fractions."

LCD: 28

Example—Solve and check $28\left[\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}\right]$

$7(x-3)=2(5)-4(x+5)$

$7x-21=10-4x-20$

$7x-21=-4x-10$

$+4x \quad +4x$

$11x-21=-10$

$+21 \quad +21$

$\frac{11x}{11}=\frac{11}{11}$

$x=1$

Rational Equation—an equation that contains one or more rational expressions

have to check

Example—Solve the following:

a. $18x\left[\frac{5}{2x}=\frac{17}{18}-\frac{1}{3x}\right]$

$9(5)=17x-1(6)$

$45=17x-16$

$+16 \quad +16$

$\frac{51}{17}=\frac{17x}{17}$

$\boxed{x=3}$

b. LCD: $(x-2)3$

$\left[\frac{x}{x-2}=\frac{2}{x-2}-\frac{2}{3}\right]$

$\boxed{\varnothing}$

$3x=2(3)-2(x-2)$

$3x=6-2x+4$

$3x=-2x+10$

$+2x \quad +2x$

$\frac{5x}{5}=\frac{10}{5}$

$x=2$

~~$x-2$~~

~~$2-2=0$~~

Identity—an equation that is true for all real numbers; both sides must be defined

Conditional Equation—an equation that is not an identity but that is true for at least one real number

Inconsistent Equation—an equation that is not true for even one real number; the lack of a solution is represented by the empty set: $\varnothing$

Example—Solve and determine whether the equation $4x-7=4(x-1)+3$ is an identity, a conditional equation, or an inconsistent equation.

$$4x-7=4(x-1)+3$$

$$4x-7=4x-4+3$$

$$4\cancel{x}-7=4\cancel{x}-1$$

$$-\cancel{4x} \quad -\cancel{4x}$$

$$-7=-1$$

$$\varnothing$$

inconsistent solution