Section 2.6—Combinations of Functions; Composite Functions

Finding a Function's Domain—If a function f does not model data or verbal conditions, its domain is the largest set of real numbers for which the value of f(x) is a real number. Exclude from a function's domain real numbers that cause division by zero and real numbers that make a square root negative.

Example: Find the domain of each function.

a. $f(x) = x^2 + 3x - 17$

$(-\infty, \infty)$

b. $g(x) = \dfrac{5x}{x^2 - 49}$

$x^2 - 49 = 0$

$+49 \quad +49$

$\sqrt{x^2} = \sqrt{49}$

$x = \pm 7$

$(-\infty, -7) \cup (-7, 7) \cup (7, \infty)$

c. $h(x) = \sqrt{9x - 27}$

$9x - 27 \geq 0$

$+27 \quad +27$

$\dfrac{9x}{9} \geq \dfrac{27}{9}$

$x \geq 3$

$[3, \infty)$

The Algebra of Functions: **Sum, Difference, Product, and Quotient**

Let f & g be two functions. The sum, difference, product and quotient are defined as follows:

- ✓ **Sum**: $(f+g)(x) = f(x) + g(x)$
- ✓ **Difference**: $(f-g)(x) = f(x) - g(x)$
- ✓ **Product**: $(f \cdot g)(x) = f(x) \cdot g(x)$
- ✓ **Quotient**: $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$; given $g(x) \neq 0$

Example: Let f(x) = x – 5 and $g(x) = x^2 - 1$. Find the following.

a. (f + g)(x) $= (x-5) + (x^2 - 1)$

$= x^2 + x - 6$

b. (f – g)(x) $= (x-5) - (x^2 - 1)$

$= x - 5 - x^2 + 1$

$= -x^2 + x - 4$

c. (f·g)(x) $= (x-5)(x^2 - 1)$

$= x^3 - x - 5x^2 + 5$

$= x^3 - 5x^2 - x + 5$

d. $\left(\dfrac{f}{g}\right)(x)$ $\dfrac{x-5}{x^2-1}$, $x \neq \pm 1$

Composite Function: the composition of the function f with g

$$(f \circ g)(x) = f(g(x))$$

The domain of the composite function $f \circ g$ is the set of all x-values such that:

1. x is in the domain of g and
2. g(x) is in the domain of f

Example: Given f(x) = 5x + 6 and $g(x) = 2x^2 - x - 1$, find each of the composite functions:

a. $(f \circ g)(x) = f(g(x))$

$f(g(x)) = 5(2x^2 - x - 1) + 6$

$f(g(x)) = 10x^2 - 5x - 5 + 6$

$f(g(x)) = 10x^2 - 5x + 1$

c. $(f \circ g)(-1)$

$f(g(x)) = 10x^2 - 5x + 1$

$f(g(-1)) = 10(-1)^2 - 5(-1) + 1$

$f(g(-1)) = 10 + 5 + 1$

$f(g(-1)) = 16$

$g(x) = 2x^2 - x - 1$

b. $(g \circ f)(x) = g(f(x))$

$(5x+6)(5x+6)$

$g(f(x)) = 2(5x+6)^2 - (5x+6) - 1$

$g(f(x)) = 2(25x^2 + 30x + 30x + 36) - 5x - 6 - 1$

$g(f(x)) = 50x^2 + 60x + 60x + 72 - 5x - 6 - 1$

$g(f(x)) = 50x^2 + 115x + 65$

d. $(f \circ g)(2)$ $f(g(x)) = 10x^2 - 5x + 1$

$f(g(2)) = 10(2)^2 - 5(2) + 1$

$f(g(2)) = 10(4) - 10 + 1$

$f(g(2)) = 40 - 10 + 1$

$f(g(2)) = 31$