Section 4.1—Exponential Functions

Exponential Functions—any function whose equation contains a variable in the exponent

Exponential Function—defined by

$$f(x) = b^x \quad or \quad y = b^x$$

where b is a positive constant other than 1 $(b > 0$ and $b \neq 1)$ and x is any real number.

Examples:

Exponential Functions		NOT Exponential Functions	
$f(x) = 2^x$	base of 2	$F(x) = x^2$	variable in base, not in exponent
$g(x) = 10^x$	base of 10	$G(x) = 1^x$	base must be positive constant other than 1
$h(x) = 3^{x+1}$	base of 3	$H(x) = (-1)^x$	base must be positive
$j(x) = \left(\frac{1}{2}\right)^{x-1}$	base of $\frac{1}{2}$	$J(x) = x^x$	variable in both base and exponent

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Example—The exponential function $f(x) = 42.2(1.56)^x$ models the average amount spent, $f(x)$, in dollars, at a shopping mall after x hours. Find the average amount spent to the nearest dollar, after three hours at a shopping mall.

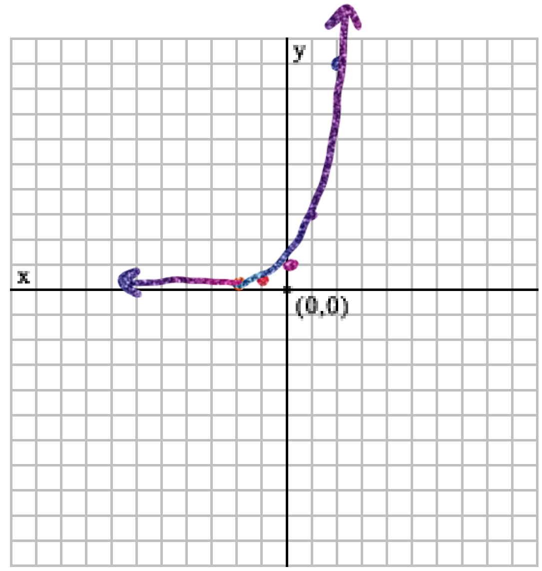
$f(3) = 42.2(1.56)^3$ ^ - exponents

$= 160.208$

$\boxed{\$160}$

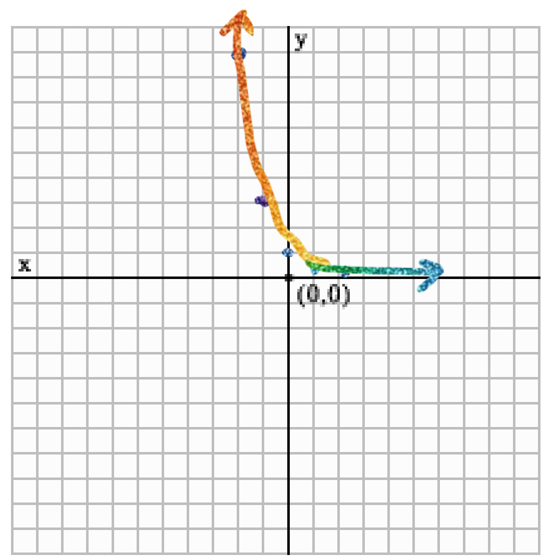
Example—Graph $f(x) = 3^x$.

x	$f(x) = 3^x$	(x, y)
-2	$f(-2) = 3^{-2} = \frac{1}{3^2} = \frac{1}{9} = .11$	$(-2, .11)$
-1	$f(-1) = 3^{-1} = \frac{1}{3} = .33$	$(-1, .33)$
0	$f(0) = 3^0 = 1$	$(0, 1)$
1	$f(1) = 3^1 = 3$	$(1, 3)$
2	$f(2) = 3^2 = 9$	$(2, 9)$



Example—Graph $f(x) = \left(\frac{1}{3}\right)^x$

x	$f(x) = \left(\frac{1}{3}\right)^x$	(x, y)
-2	$f(-2) = \left(\frac{1}{3}\right)^{-2} = 3^2 = 9$	$(-2, 9)$
-1	$f(-1) = \left(\frac{1}{3}\right)^{-1} = 3^1 = 3$	$(-1, 3)$
0	$f(0) = \left(\frac{1}{3}\right)^0 = 1$	$(0, 1)$
1	$f(1) = \left(\frac{1}{3}\right)^1 = .3$	$(1, .3)$
2	$f(2) = \left(\frac{1}{3}\right)^2 = \frac{1}{9} = .11$	$(2, .11)$



Characteristics of Exponential Functions of the Form $f(x) = b^x$

1. The domain of $f(x) = b^x$ consists of all real numbers: $(-\infty, \infty)$. The range of $f(x) = b^x$ consists of all positive real numbers: $(0, \infty)$.
2. The graphs of all exponential functions of the form $f(x) = b^x$ pass through the point (0, 1) because $f(0) = b^0 = 1,\quad (b \neq 0)$. The y-intercept is 1. There is no x intercept.
3. If b > 1, $f(x) = b^x$ has a graph that goes up to the right and is an increasing function. The greater the value of b, the steeper the increase.
4. If $0 < b < 1$, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function. The smaller the value of b, the steeper the decrease.
5. $f(x) = b^x$ is one-to-one and has an inverse that is a function.
6. The graph of $f(x) = b^x$ approaches, but does not touch, the x-axis. The x-axis, or y = 0, is a horizontal asymptote.

An irrational number, symbolized by the letter e, appears as the base in many applied exponential functions.

e^x

$$e \approx 2.718281827$$

<u>Example</u>—The exponential function $f(x) = 1066e^{0.042x}$ models the gray wolf population of the Western Great Lakes x years after 1978. If trends continue, project the gray wolf's population in the recovery area in 2012. $x = 34$

$f(34) = 1066e^{0.042(34)} \rightarrow$ 1066 * e^(.042 * 34)

$f(34) = 1066e^{1.428}$

$f(34) = 4445.59$

$\boxed{4446}$

<u>Compound Interest</u>—interest computed on an original investment as well as on any accumulated interest

<u>Principal</u>—the amount invested

<u>Compounded Semiannually</u>—when compound interest is paid twice a year; the compounding period is 6 months

<u>Compounded Quarterly</u>—when compound interest is paid four times a year; the compounding period is three months

<u>Continuous Compounding</u>—where the number of compounding periods increases infinitely

<u>Formula for Compound Interest</u>—After t years, the balance, A, in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. for n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. for continuous compounding: $A = Pe^{rt}$

Example—A sum of $10,000 is invested at an annual rate of 8%. Find the balance in the account after 5 years subject to:

$P = 10,000$

$r = .08$

$n = 4$

$t = 5$

a. quarterly compounding

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 10,000\left(1+\frac{.08}{4}\right)^{(4(5))}$$

$$A = 10,000(1+.02)^{20}$$

$$A = 10,000(1.02)^{20}$$

$$A = 10,000(1.486)$$

$$\boxed{A = \$14,859.47}$$

b. continuous compounding

$$A = Pe^{rt}$$

$$A = 10,000e^{((.08)(5))}$$

$$A = 10,000e^{.4}$$

$$\boxed{A = \$14,918.25}$$