Section 1.4—Complex Numbers

What are square roots? How do we get them? $2^2=4$ $\sqrt{4}=2$

The imaginary unit i is defined as $i=\sqrt{-1}$, where $i^2=-1$.

Using the imaginary unit i, we can express the square root of any negative number as a real multiple of i.

For example:
$$\sqrt{-25}$$
$$\sqrt{-1}\sqrt{25}$$
$$i\sqrt{25}$$
$$5i$$

<u>Complex Numbers</u>—the set of all numbers in the form $a+bi$ where a & b are real numbers and i is the imaginary unit. $3+2i$

<u>Real Part</u>—the real number a in $a+bi$

<u>Imaginary Part</u>—the real number b in $a+bi$

<u>Pure Imaginary Number</u>—an imaginary number in the form bi $2i$

<u>Notes</u>: $\cancel{1+3}+2i = 4+2i$

- ✓ A complex number is said to be simplified if it is in the standard form $a+bi$
- ✓ If b is a radical, we write the i before the b; because $\sqrt{5i}$ and $\sqrt{5}i$ can be easily confused. $i\sqrt{5}$
- ✓ Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. For instance, $a+bi=c+di$ if and only if a = c and b = d.

<u>Adding and Subtracting Complex Numbers</u>

- ✓ **Adding**: $(a+bi)+(c+di)=(a+c)+(b+d)i$
- ✓ **Subtracting**: $(a+bi)-(c+di)=(a-c)+(b-d)i$

<u>Example</u>—Add or subtract as indicated

a. $(5-2i)+(3+3i)$

$8+i$

b. $(2+6i)-(12-i)$

$2+6i-12+i$

$\boxed{-10+7i}$

Multiplication of complex numbers is performed the same way as multiplication of polynomials, either by using the distributive property or the FOIL method.

First Outside Inside Last

Example—Find the products

a. $7i(2-9i)$

$14i-63i^2$

$14i-63(-1)$

$14i+63$

$\boxed{63+14i}$

b. $(5+4i)(6-7i)$

$30-35i+24i-28i^2$ (−1)

$30-11i+28$

$\boxed{58-11i}$

Complex Conjugate—of a complex number has the same real and imaginary part as the complex number but with different signs. For example $a+bi$ and $a-bi$ are complex conjugates of one another.

$(2+3i)(2-3i)$

$4-6i+6i-9i^2$

$4-9(-1)$

$4+9=13$

The multiplication of complex conjugates gives a real number:

$(a+bi)(a-bi)=a^2+b^2$

$(a-bi)(a+bi)=a^2+b^2$

2^2+3^2

$4+9=13$

Complex conjugates are used to divide complex numbers. By multiplying the numerator and denominator by the complex conjugate of the denominator, you will obtain a real number in the denominator.

You **cannot** leave an i in the denominator of a fraction.

Example—Divide and express the result in standard form.

a. $\frac{3i}{4+i}$

$\frac{3i}{4+i}\cdot\frac{(4-i)}{(4-i)}$

$\frac{12i-3i^2}{4^2+1^2}$ (−1) $=\frac{12i+3}{16+1}$

$\boxed{\frac{3+12i}{17}}=\boxed{\frac{3}{17}+\frac{12}{17}i}$

b. $\frac{5+4i}{4-i}\cdot\frac{(4+i)}{(4+i)}$

$\frac{20+5i+16i+4i^2}{4^2+1^2}$ (−1)

$=\frac{20+21i-4}{16+1}=\frac{16+21i}{17}$

$\boxed{\frac{16}{17}+\frac{21}{17}i}$

Principal Square Root—defined by $\sqrt{-b} = i\sqrt{b}$

When performing operations with square roots of negative numbers, begin by expressing all square roots in terms of i; then perform the indicated operation.

CORRECT:
$$\sqrt{-25}\cdot\sqrt{-4}$$
$$i\sqrt{25}\cdot i\sqrt{4}$$
$$5i\cdot 2i = 10i^2$$
$$10(-1) = -10$$

INCORRECT:
$$\sqrt{-25}\cdot\sqrt{-4}$$
$$\sqrt{100}$$
$$10$$

Example—Perform the indicated operations and write the results in standard form

a. $\sqrt{-27}+\sqrt{-48}$

$3\cdot 3\cdot 3$ $\quad 2\cdot 2\cdot 2\cdot 2\cdot 3$

$2\cdot 2$

$i\sqrt{27}+i\sqrt{48}$

$3i\sqrt{3}+4i\sqrt{3}$

$7i\sqrt{3}$

b. $\left(-2+\sqrt{-3}\right)^2$

$(-2+\sqrt{-3})(-2+\sqrt{-3}) = (-2+i\sqrt{3})(-2+i\sqrt{3})$

$4-2i\sqrt{3}-2i\sqrt{3}+i^2\sqrt{9}$

$4-4i\sqrt{3}+(-1)(3) = 4-4i\sqrt{3}-3 = 1-4i\sqrt{3}$

12

4 · 3

2 · 2 · 3

c. $\dfrac{-14+\sqrt{-12}}{2}$

$\dfrac{-14+i\sqrt{12}}{2}$

$\dfrac{-14+2i\sqrt{3}}{2} = \dfrac{-14}{2}+\dfrac{2i\sqrt{3}}{2}$

$= -7+i\sqrt{3}$