Section 2.3—Linear Functions and Slope

Slope: measures how steep a line is; ratio of vertical change to horizontal change; rise over run

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the slope of the line passing through each pair of points.

a. $(-3, 4)$ & $(-4, 2)$

$$m = \frac{y-y}{x-x} = \frac{4-2}{-3+(+4)} = \frac{2}{1}$$

$$= \frac{2-4}{-4+(+3)} = \frac{-2}{-1} = \frac{2}{1}$$

b. $(4, -2)$ & $(-1, 5)$

$$m = \frac{y-y}{x-x} = \frac{-2-5}{4+(+1)} = \frac{-7}{5}$$

Four Possibilities for the Slope of a Line:

- positive slope: line rises from left to right
- negative slope: line that falls from left to right
- zero slope: horizontal line
- undefined slope: vertical line

Point Slope Form of an Equation: $y - y_1 = m(x - x_1)$

- (x_1, y_1) is a point on the line
- m is the slope of the line

Example: Write the point-slope form of the equation of the line with slope 6 that passes through the point $(2, -5)$. Then solve for y.

$y - y_1 = m(x - x_1)$

$y - (-5) = 6(x - 2)$

$y + 5 = 6(x - 2)$ → point-slope form

$y + 5 = 6x - 12$

$-5 \qquad -5$

$y = 6x - 17$

Example: Write the point-slope form of the equation of the line passing through the points $(-2,-1)$ and $(-1,-6)$. Then solve for y. $y-y_1=m(x-x_1)$

$m=\frac{y-y}{x-x}=\frac{-1+(+6)}{-2+(+1)}$

$=\frac{5}{-1}=-5$

$y-(-1)=(-5)(x-(-2))$

$y+1=(-5)(x+2)$

$y+1=-5x-10$

$\underline{-1 \qquad\quad -1}$

$y=-5x-11$

Slope Intercept Form: $y=mx+b$

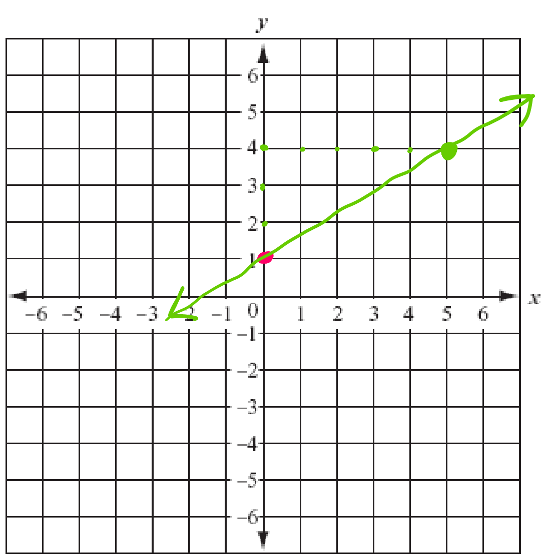
- m represents the slope
- b represents the y-intercept

Graphing y = mx + b using the slope and the y-intercept.

1. Plot the y-intercept; $(0,b)$
2. Get to the second point by using m to determine how much to "rise" and how much to "run;" STARTING AT THE Y-INTERCEPT.
3. Draw a line through the two points.

Example: Graph the linear function $f(x)=\frac{3}{5}x+1$

$m=\frac{3}{5}$ rise / run $\quad b=1$



Equation of a Horizontal Line: $y = b$ → no x-value, does not cross the x-axis

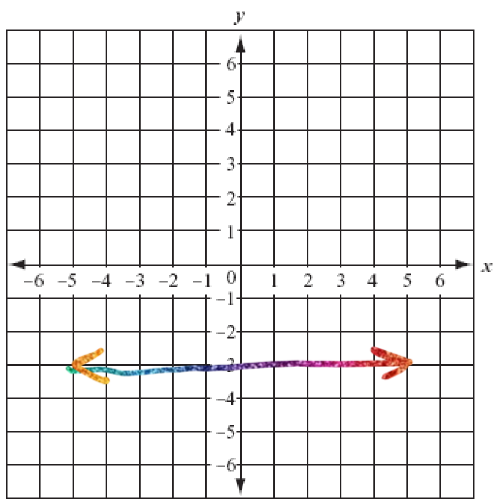
✓ Horizontal lines have a slope of zero.

Equation of a Vertical Line: $x = a$ → no y-value, does not cross the y-axis

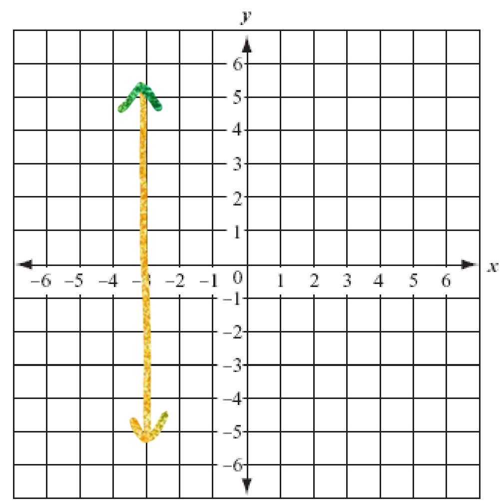
✓ Vertical lines have a slope that is undefined.

Example: Graph the following.

a. $y = -3$

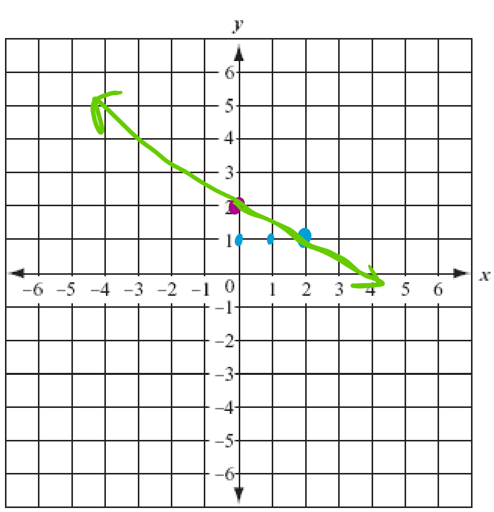


b. $x = -3$



General Form of an Equation: $Ax + By + C = 0$, both A and B cannot be 0

Example: Find the slope and y-intercept of the line whose equation is $3x + 6y - 12 = 0$. Then use those to graph the line.



$y = mx + b$

$3x + 6y - 12 = 0$

$-3x \qquad\qquad -3x$

$6y - 12 = -3x$

$+12 \qquad\qquad +12$

$\frac{6y}{6} = \frac{-3x}{6} + \frac{12}{6}$

$y = \frac{-1}{2}x + 2$

$y = mx + b$

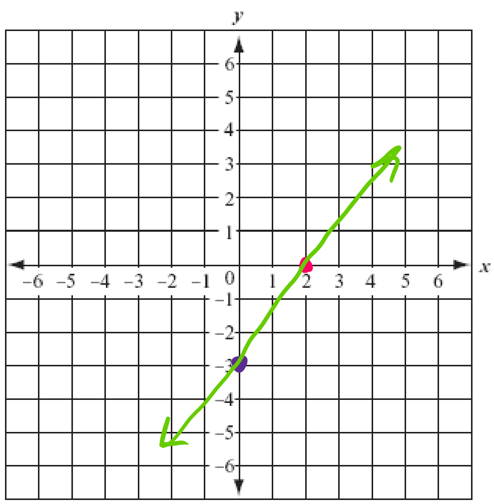
$m = \frac{-1}{2}$ → rise / run

$b = 2$

Using Intercepts to Graph Ax + By + C = 0

1. Find the x-intercept. Let $y = 0$ and solve for x. Plot that point.
2. Find the y-intercept. Let $x = 0$ and solve for y. Plot that point.
3. Draw a line through the two points.

Example: Graph $3x - 2y - 6 = 0$ using intercepts.



x-int $y=0$

$3x-2(0)-6=0$

$3x-\cancel{6}=0$

$+\cancel{6}\quad +6$

$\frac{3x}{3}=\frac{6}{3}$

$x=2$

$(2,0)$

y-int $x=0$

$3(0)-2y-6=0$

$-2y-6=0$

$+6\quad +6$

$\frac{-2y}{-2}=\frac{6}{-2}$

$y=-3$

$(0,-3)$

Summary of Equations of Lines

Point-Slope Form	$y - y_1 = m(x - x_1)$
Slope-Intercept Form	$y = mx + b; \quad f(x) = mx + b$
Horizontal Line	y = b
Vertical Line	x = a
General	Ax + By + C = 0