Section 1.5—Quadratic Equations

Quadratic Equation—an equation written in the general form $ax^2+bx+c=0$ where a, b, & c are real numbers and $a \neq 0$.

Zero-Product Principle—If the product of two algebraic expressions is zero, then at least one of the factors is equal to zero. If AB = 0, then A = 0 or B = 0.

Solving an Equation by Factoring

1. If necessary, rewrite the equation in the general form $ax^2+bx+c=0$ by moving all terms to one side, leaving zero on the other side.
2. Factor completely.
3. Apply the zero-product principle, setting each factor containing a variable equal to zero.
4. Solve the equations from step three.
5. Check the solutions in the original equation.

Example—Solve by factoring

a. $3x^2-9x=0$

$3x(x-3)=0$

$3x=0$, $\frac{3x}{3}=\frac{0}{3}$, $\boxed{x=0}$

$x-3=0$, $+3\ \ +3$, $\boxed{x=3}$

b. $2x^2+x=1$

$-1\ \ -1$

$2(-1)=-2$

$2\ \ \ -1$

$2x^2+x-1=0$

$2x^2+2x-1x-1=0$

$2x(x+1)-1(x+1)=0$

$(x+1)(2x-1)=0$

$x+1=0$, $-1\ \ -1$, $\boxed{x=-1}$

$2x-1=0$, $+1\ \ +1$, $\frac{2x}{2}=\frac{1}{2}$, $\boxed{x=\frac{1}{2}}$

The Square Root Property—If u is an algebraic expression and d is a nonzero real number, then $u^2=d$ has exactly two solutions:

If $u^2=d$, then $u=\sqrt{d}$ or $u=-\sqrt{d}$

Equivalently,

If $u^2=d$, then $u=\pm\sqrt{d}$

isolate the squared variable

Example—Solve using the square root property

a. $3x^2-21=0$

$3x^2-21=0$

$+21\ \ +21$

$\frac{3x^2}{3}=\frac{21}{3}$

$\sqrt{x^2}=\sqrt{7}$

$\boxed{x=\pm\sqrt{7}}$

b. $5x^2+45=0$

$-45\ \ -45$

$\frac{5x^2}{5}=\frac{45}{5}$

$\sqrt{x^2}=\sqrt{9}$

$\boxed{x=\pm 3}$

c. $\sqrt{(x+5)^2} = \sqrt{11}$

$x+5 = \pm\sqrt{11}$

$-5 \quad -5$

$\boxed{x = 5 \pm \sqrt{11}}$

<u>The Quadratic Formula</u>—can be used to find the solutions of a quadratic equation in general form $ax^2 + bx + c = 0$, with $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<u>Example</u>—Solve using the quadratic formula

a. $2x^2 + 2x - 1 = 0$ $\quad a = 2 \quad b = 2 \quad c = -1$

$2 \cdot 2 \cdot 3$

$4 \cdot 3$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4}$$

$$= \boxed{\frac{-1 \pm \sqrt{3}}{2}}$$

b. $2x^2 - 6x + 1 = 0$ $\quad a = 2 \quad b = -6 \quad c = 1$

$2 \cdot 2 \cdot 7$

$4 \cdot 7$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(1)}}{2(2)} = \frac{6 \pm \sqrt{36 - 8}}{4} = \frac{6 \pm \sqrt{28}}{4} = \frac{6 \pm 2\sqrt{7}}{4}$$

$$= \boxed{\frac{3 \pm \sqrt{7}}{2}}$$

c. $3x^2 - 2x + 4 = 0$ $\quad a = 3 \quad b = -2 \quad c = 4$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)} = \frac{2 \pm \sqrt{4 - 21}}{6} = \frac{2 \pm \sqrt{-17}}{6}$$

$$= \boxed{\frac{2 \pm i\sqrt{17}}{6}}$$