Section 1.6—Other Types of Equations

Polynomial Equation—the result of setting two polynomials equal to each other

General Form—one side is 0 and the polynomial is on the other side in descending powers of the variable

Example—Solve by factoring:

a. $4x^4 = 12x^2$

$4x^4 - 12x^2 = 0$

$4x^2(x^2 - 3) = 0$

$\frac{4x^2}{4} = \frac{0}{4}$ $x^2 = 0$ $\boxed{x = 0}$

$x^2 - 3 = 0$
$\quad +3 \quad +3$
$x^2 = 3$
$\boxed{x = \pm\sqrt{3}}$

b. $x^3 + x^2 = 4x + 4$

$x^3 + x^2 - 4x - 4 = 0$

$x^2(x+1) - 4(x+1) = 0$

$(x+1)(x^2 - 4) = 0$

$x + 1 = 0$
$\;\; -1 \quad -1$
$\boxed{x = 1}$

$x^2 - 4 = 0$
$\quad +4 \quad +4$
$\sqrt{x^2} = \sqrt{4}$
$\boxed{x = \pm 2}$

Radical Equation—an equation in which the variable occurs in a square root, cube root, or any higher root

When raising both sides of an equation to an even power, always check proposed solutions in the original equation.

Solving Radical Equations Containing nth Roots

1. If necessary, arrange terms so that one radical is isolated on one side of the equation.
2. Raise both sides of the equation to the nth power to eliminate the nth root.
3. Solve the resulting equation. If this equation still contains radicals, repeat steps 1 and 2.
4. Check all proposed solutions in the original equation.

$(x-3)(x-3)$

Extraneous Solutions—extra solutions that you may get when you raise both sides of a radical equation to an even power; also called extraneous roots

Example—Solve $\sqrt{x+3} + 3 = x$
$\qquad\qquad\qquad -3 \quad -3$

$(\sqrt{x+3})^2 = (x-3)^2$

$x + 3 = x^2 - 3x - 3x + 9$

$x + 3 = x^2 - 6x + 9$
$-x \qquad\qquad -x$

$3 = x^2 - 7x + 9$
$-3 \qquad\qquad -3$

$x^2 - 7x + 6 = 0$

$(x-1)(x-6) = 0$

$x - 1 = 0$ $x - 6 = 0$

$\cancel{x = 1}$ $\boxed{x = 6}$

Check: $\sqrt{1+3} + 3 = 1$

$\sqrt{4} + 3 = 1$

$2 + 3 = 1$ ✗

$5 \neq 1$

$\sqrt{6+3} + 3 = 6$

$\sqrt{9} + 3 = 6$

$3 + 3 = 6$

$6 = 6$ ✓

Solving Radical Equations of Form: $x^{\frac{m}{n}} = k$

Assume that m and n are positive integers, $\frac{m}{n}$ is in lowest terms, and k is a real number.

1. Isolate the expression with the rational exponent.
2. Raise both sides of the equation to the $\frac{n}{m}$ power.

$\sqrt{4} = \pm 2$ $(2)(2)=4$ $(-2)(-2)=4$

$\sqrt[3]{8} = 2$ $(-2)(-2)(-2) = -8$

If m is even:

$$x^{\frac{m}{n}} = k$$

$$\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = \pm k^{\frac{n}{m}}$$

$$x = \pm k^{\frac{n}{m}}$$

If m is odd:

$$x^{\frac{m}{n}} = k$$

$$\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = k^{\frac{n}{m}}$$

$$x = k^{\frac{n}{m}}$$

It is **wrong** to insert the ± symbol when the numerator of the exponent is odd. An odd index has only one root.

3. Check all proposed solutions in the original equation to find out if they are actual solutions or extraneous solutions.

Example—Solve

3 → odd

a. $5x^{\frac{3}{2}} - 25 = 0$

$5x^{3/2} = 25$

$\left(x^{3/2}\right)^{2/3} = (5)^{2/3}$

$x = 5^{2/3}$

$x = \sqrt[3]{5^2}$

$x = \sqrt[3]{25}$

2 → even

b. $x^{\frac{2}{3}} - 8 = -4$

$\left(x^{2/3}\right)^{3/2} = (4)^{3/2}$

$x = \pm 4^{3/2}$

$x = \pm\sqrt{4^3}$

$x = \pm\sqrt{64}$

$x = \pm 8$

Absolute Value—denotes the distance a value is from zero on the number line

always positive

Absolute Value Equation—an equation that contains an absolute value expression

Rewriting an Absolute Value Equation without Absolute Value Bars—If c is a positive real number and X represents any algebraic expression, then $|X| = c$ is equivalent to $X = c$ *or* $X = -c$.

isolate the absolute value

Example—Solve

a. $|2x-1|=5$

$2x-1=5$
$+1 \quad +1$
$2x=6$
$\frac{2x}{2}=\frac{6}{2}$
$\boxed{x=3}$

$2x-1=-5$
$+1 \quad +1$
$2x=-4$
$\frac{2x}{2}=\frac{-4}{2}$
$\boxed{x=-2}$

b. $4|1-2x|-20=0$
$+20 \quad +20$

$\frac{4|1-2x|}{4}=\frac{20}{4}$

$|1-2x|=5$

$1-2x=5$
$-1 \quad -1$
$-2x=4$
$\frac{-2x}{-2}=\frac{4}{-2}$
$\boxed{x=-2}$

$1-2x=-5$
$-1 \quad -1$
$-2x=-6$
$\frac{-2x}{-2}=\frac{-6}{-2}$
$\boxed{x=3}$