Section 1.7—Linear Inequalities and Absolute Value Inequalities

Solving an Inequality—the process of finding the set of numbers that make the inequality a true statement

Interval Notation—used to represent subsets (small collections) of real numbers

Open Interval—(a, b) represents the set of real numbers between, but not including a & b

$$(a,b) = \{x | a < x < b\}$$

✓ x is greater than a AND less than b

Closed Interval—$[a,b]$ represents the set of real numbers between and including the points a & b

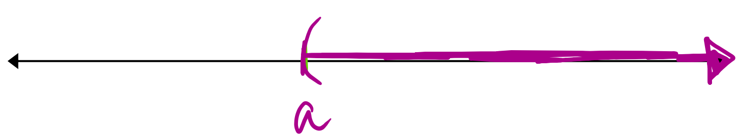
$$[a,b] = \{x | a \le x \le b\}$$

✓ x is greater than or equal to a AND less than or equal to b

Infinite Interval—(a,∞) represents the set of real numbers greater than a

$$(a,\infty) = \{x | x > a\}$$

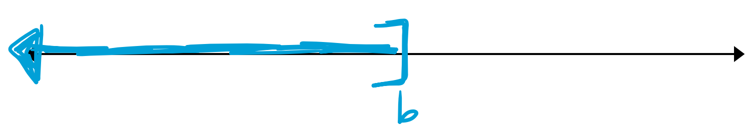
✓ the infinity symbol does not represent a real number; it indicates that the interval extends indefinitely to the right



Infinite Interval—$(-\infty,b]$ represents the set of real numbers that are less than or equal to b

$$(-\infty,b] = \{x | x \le b\}$$

✓ the infinity symbol does not represent a real number; it indicates that the interval extends indefinitely to the left



open circles

Parentheses indicate endpoints that are not included in an interval.

closed circles

Square brackets indicate endpoints that are included in an interval.

The following table lists nine possible types of intervals used to describe the subset of numbers.

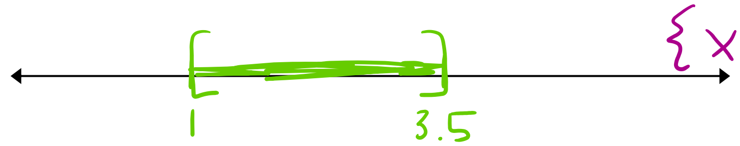
Let a and b be real numbers such that $a < b$.		
Interval Notation	**Set-Builder Notation**	**Graph**
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\{x \mid x$ is a real number$\}$ or $\mathbb{R}$ (set of all real numbers)	

<u>Example</u>—Express each interval in set-builder notation and graph.

a. $[-2,5)$

$\{x \mid -2 \le x < 5\}$

b. $[1,3.5]$



$\{x \mid 1 \le x \le 3.5\}$

c. $(-\infty,-1)$

$\{x \mid x < -1\}$

Intersections and Unions of Intervals

<u>Intersection</u>—set of elements that are common to both set A and set B; denoted $\cap$

Union—set of elements that are in both set A or set B or both sets; denoted $\cup$

Finding Intersections and Unions of Intervals

1. Graph each interval on a number line
2. Determine if you are looking for the intersection or union
   a. **Intersection**—take the portion of the number line that the two graphs have in common
   b. **Union**—take the portion of the number line that represents the total collection of numbers in the two graphs

and

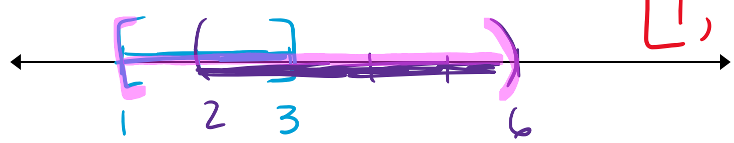
Example—Use graphs to find each set

a. $[1,3]\cap(2,6)$

$(2,3]$

b. $[1,3]\cup(2,6)$

OR



$[1,6)$

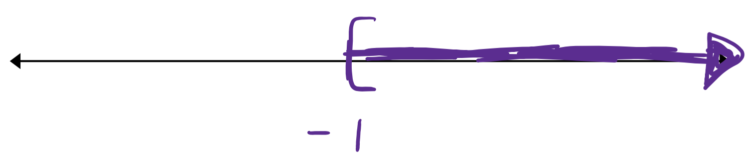
Solving Linear Inequalities

A linear inequality can be written in one of the following forms: $ax+b<0$, $ax+b\le 0$, $ax+b>0$, or $ax+b\ge 0$.

What happens when you add, subtract, multiply or divide each side of an inequality by positive and negative numbers?

When you multiply or divide both sides of an inequality by a negative number, reverse the inequality symbol.

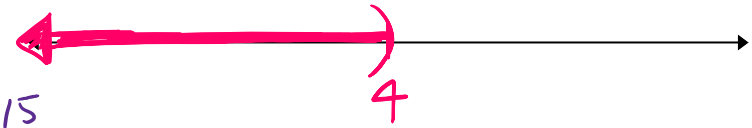
Example—Solve and graph the solution set on a number line $2-3x\le 5$



$$2-3x\le 5$$
$$-2 \qquad -2$$
$$-3x \le 3$$
$$\frac{-3x}{-3} \quad \frac{3}{-3}$$
$$x \ge -1$$

$[-1,\infty)$

Example—Solve and graph the solution set on a number line $3x+1>7x-15$



$$3x+1>\cancel{7x}-15$$
$$-7x \qquad -\cancel{7x}$$
$$-4x+1>-15$$
$$-1 \qquad -1$$
$$\frac{-\cancel{4}x}{-\cancel{4}} > \frac{-16}{-4}$$

$x<4$

$(-\infty,4)$

If there is no solution to an inequality, the solution set is the empty set, $\varnothing$.

If an inequality is true for all real numbers, the solution set is $(-\infty,\infty)$ or $\{x \mid x \text{ is a real number}\}$

Example—Solve each inequality

a. $3(x+1)>3x+2$

$$\cancel{3x}+3>\cancel{3x}+2$$
$$-\cancel{3x} \qquad -\cancel{3x}$$
$$3>2$$

true

$(-\infty,\infty)$

b. $x+1\le x-1$

$$-x \qquad -x$$
$$1\le -1$$

False → no solution

$\varnothing$

Compound Inequality—two inequalities written as one statement

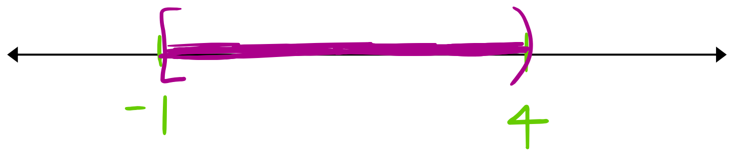
- ✓ For example: $-3<2x+1$ *and* $2x+1\le 3$ can be written as the single compound inequality $-3<2x+1\le 3$

An intersection is implied when you have a compound inequality.

The goal in solving an inequality is to isolate the variable in the middle.

You must perform the same operation on all three parts!!!

Example—Solve and graph the solution set on a number line $1 \le 2x+3 < 11$



$1 \le 2x + \cancel{3} < 11$
$-3 \quad -\cancel{3} \quad -3$

$\frac{-2}{2} \le \frac{2x}{2} < \frac{8}{2}$

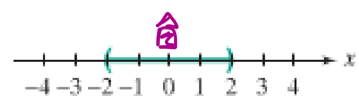
$-1 \le x < 4$

$[-1, 4)$

Solving Inequalities with Absolute Value

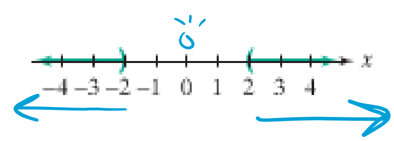
When less than is used in an absolute value inequality, it means the distance from x is less than the value.

For example: $|x| < 2$ means that the distance of x from 0 is less than 2; thus x can lie between -2 and 2. We would write that as (-2, 2) or $\{x \mid -2 < x < 2\}$



When greater than is used in an absolute value inequality, it means the distance from x is greater than the value.

For example: $|x| > 2$ means that the distance of x from 0 is greater than 2; thus x can be less than -2 or greater than 2. We would write $x < -2$ *or* $x > 2$; in interval notation as $(-\infty, -2) \cup (2, \infty)$



If X is an algebraic expression and c is a positive number,

1. The solutions of $|X| < c$ are the numbers that satisfy $-c < X < c$
2. The solutions of $|X| > c$ are the numbers that satisfy $X < -c$ *or* $X > c$

These rules apply for $\le$ *and* $\ge$, when replacing < and > respectively.

Example—Solve and graph the solution set on a number line: $|x-2| < 5$

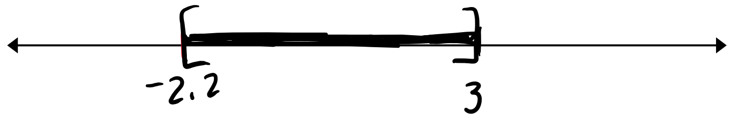
$-5 < x - 2 < 5$
$+2 \quad +2 \quad +2$

$-3 < x < 7$

$(-3, 7)$

isolate the absolute value

Example—Solve and graph the solution set on the number line: $-3|5x-2|+20 \geq -19$



$[-2.2, 3]$

$$-3|5x-2|+20 \geq -19$$
$$\quad -20 \qquad -20$$

$$-3|5x-2| \geq -39$$
$$\div -3 \qquad \div -3$$

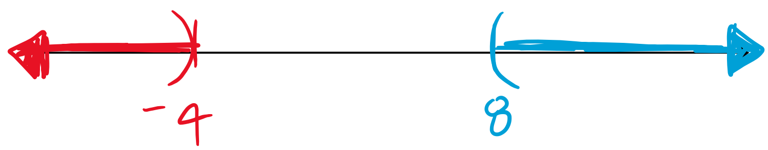
$$|5x-2| \leq 13$$

$$-13 \leq 5x-2 \leq 13$$
$$+2 \qquad +2 \qquad +2$$

$$\frac{-11}{5} \leq \frac{5x}{5} \leq \frac{15}{5}$$

$$-2.2 \leq x \leq 3$$

Example—Solve and graph the solution set on a number line: $18 < |6-3x|$



$$|6-3x| > 18$$

$(-\infty, -4) \cup (8, \infty)$

$$6-3x < -18$$
$$-6 \qquad -6$$
$$\frac{-3x}{-3} < \frac{-24}{-3}$$
$$x > 8$$

$$6-3x > 18$$
$$-6 \qquad -6$$
$$\frac{-3x}{-3} > \frac{12}{-3}$$
$$x < -4$$