Section 4.4—Exponential and Logarithmic Equations

Exponential Equation—an equation containing a variable in an exponent

All exponential functions are one-to-one; that is no two different ordered pairs have the same second component.

Solving Exponential Equations by Expressing Each Side as a Power of the Same Base $7^{3} = 7^{\times}$

If $b^M = b^N$, then M = N.

- 1. Rewrite the equation in the form $b^{M} = b^{N}$
- 2. Set M = N.
- 3. Solve for the variable.

Example—Solve

a.
$$5^{3x-6} = 125$$
 $5 \cdot 5 \cdot 5 = 5^{3}$
 $3x - 4 = 3$
 $16 + 4$
 $3x = 9$
 $3x = 3$
 $4x = 3$

b.
$$8^{x+2} = 4^{x-3}$$

 $(2^{3(x+2)} = 2^{2(x-3)})$
 $3(x+2) = 2(x-3)$
 $3x+6 = 2x-6$
 $-2x$ $-2x$
 $x = -12$

Most exponential equations cannot be rewritten so that each side has the same base.

Using Natural Logarithms to Solve Exponential Equations

- 1. Isolate the exponential expression
- 2. Take the natural logarithm on both sides
- 3. Simplify using one of the following

$$\ln b^x = x \ln b$$
 or $\ln e^x = x$

4. Solve for the variable

Example—Solve the following

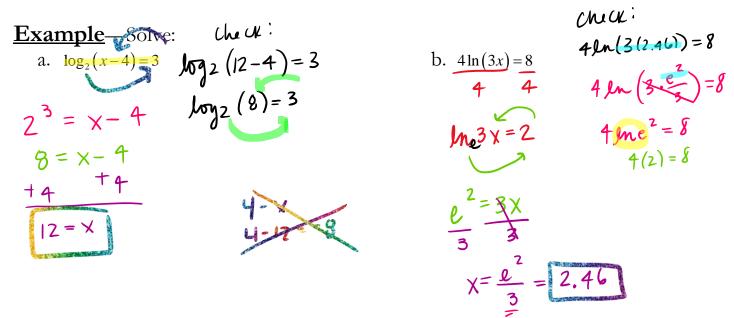
a.
$$5^{x} = 134$$
 $105^{x} = 10134$
 $105^{x} = 10134$

b.
$$10^{x} = 8000$$
 $\ln 10^{x} = \ln 8000$
 $\times \ln 10 = \ln 8000$
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<u>Logarithmic Equation</u>—an equation containing a variable in a logarithmic expression

Using the Definition of a Logarithm to Solve Logarithmic Equations

- 1. Express the equation in the form $\log_b M = c$
- 2. Use the definition of a logarithm to rewrite the equation in exponential form: $\log_b M = c$ means $b^c = M$
- 3. Solve for the variable
- 4. Check proposed solutions in the original equation. Include in the solution set only values for which M > 0.



Logarithmic expressions are defined only for logarithms of positive real numbers.

Always check proposed solutions of logarithmic equation in the original equation. Exclude values that give you the log of a negative number or a log of 0

To rewrite $\log_b M = c$ as $b^c = M$, we have to have a single logarithm whose coefficient is one.

Example Solve
$$\log x + \log(x-3) = 1$$
. (Well: $\log (5) + \log (5-3) = 1$) $\log (5) + \log (5-3) = 1$ $\log (5) + \log (5) = 1$

<u>Using the One-to-One Property of Logarithms to Solve Logarithmic Equations</u>

- 1. Express the equation in the form $\log_b M = \log_b N$. (Remember: you must have a single logarithm whose coefficient is 1 on each side of the equation.)
- 2. Use the one-to-one property to rewrite the equation without logarithms: If $\log_b M = \log_b N$ then M = N.
- 3. Solve for the variable.
- 4. Check in the original equation. Include only the values for which M > 0 and N > 0.

Example—Solve
$$\ln(x-3) = \ln(7x-23) \ln(x+1)$$
 (McK:

 $\ln(x-3) = \ln(\frac{7x-23}{x+1})$ $\chi^2 - 9\chi - 3 = -23$ $\ln(4-3) = \ln(7(4)-23) - \ln(4+1)$
 $\chi^2 - 9\chi + 20 = 0$ $\ln(1) = \ln 5 - \ln 5$
 $(\chi - 4) (\chi - 5) = 0$
 $\chi^2 - 3\chi + \chi - 3 = 7\chi - 23$ $\chi^2 - 2\chi - 3 = 7\chi - 23$
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