Section 4.2—Logarithmic Functions

No horizontal line can be drawn that intersects the graph of an exponential function at more than one point. This means that the exponential function is one-to-one and has an inverse.

The inverse function of the exponential function with base b is called the logarithmic function with base b.

Logarithmic Function

For x > 0 & b > 0, b ≠ 1

$y = \log_b x$ is equivalent to $b^y = x$.

The function $f(x) = \log_b x$ is the logarithmic function with base b.

$y = \log_b x$ and $b^y = x$ are two ~~was~~ ways of expressing the same thing.

- $y = \log_b x$ is the **logarithmic form**
- $b^y = x$ is the **exponential form**

Example—Write each equation in its equivalent exponential form.

a. $3 = \log_7 x$ b. $2 = \log_b 25$ c. $\log_4 26 = y$

$7^3 = x$ $b^2 = 25$ $4^y = 26$

Example—Write each equation in its equivalent logarithmic form.

a. $2^5 = x$ b. $b^3 = 27$ c. $e^y = 33$

$\log_2 x = 5$ $\log_b 27 = 3$ $\log_e 33 = y$

Remember logarithms another way to write exponents. So if we want to evaluate $\log_b x$ we have to ask "what exponent must we raise b to in order to get x."

Example—Evaluate.

a. $\log_{10} 100 = 2$ b. $\log_5 \frac{1}{125} = -3$ c. $\log_{36} 6 = \frac{1}{2}$

Basic Logarithmic Properties Involving One

- $\log_b b = 1$; 1 is the exponent to which b must be raised to obtain b $(b^1 = b)$
- $\log_b 1 = 0$; 0 is the exponent to which be must be raised to obtain 1 $(b^0 = 1)$

Example—Evaluate.

a. $\log_9 9 = 1$

b. $\log_8 1 = 0$

Inverse Properties of Logarithms

For b > 0 & b ≠ 1

$\log_b b^x = x$ — The logarithm with base b of b raised to a power that equals that power.

$b^{\log_b x} = x$ — b raised to the logarithm with base b of a number equals that number.

Example—Evaluate

a. $\log_7 7^8 = 8$

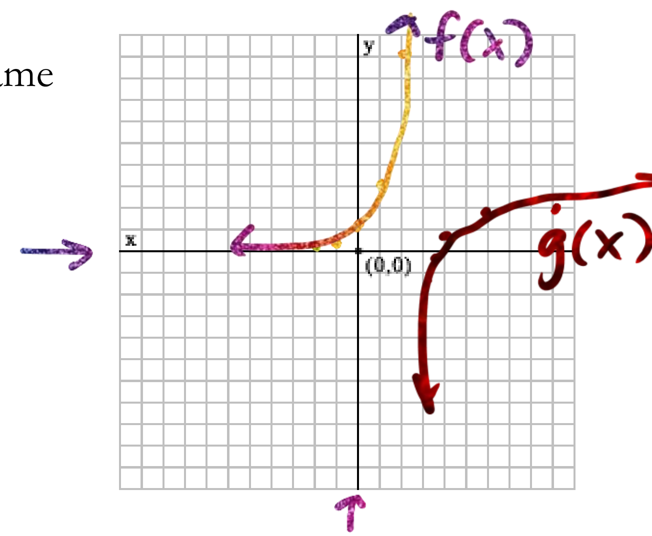
b. $3^{\log_3 17} = 17$

Graphing a Logarithmic Function—Remember that log functions are inverses of exponential functions; meaning that the log function reverses the coordinates of the exponential function. It also means the graph of the log is a reflection of the graph of the exponential function about the line y = x.

Example—Graph $f(x) = 3^x$ *and* $g(x) = \log_3 x$ on the same rectangular coordinate system.

x	3^x	(x, y)
-2	$3^{-2} = \frac{1}{3^2} = .11$	(-2, .11)
-1	$3^{-1} = \frac{1}{3^{-1}} = .33$	(-1, .33)
0	$3^0 = 1$	(0, 1)
1	$3^1 = 3$	(1, 3)
2	$3^2 = 9$	(2, 9)

g(x)
(.11, -2)
(.33, -1)
(1, 0)
(3, 1)
(9, 2)



The domain of a logarithmic function in the form of $f(x) = \log_b x$ is the set of all positive real numbers.

The domain of $f(x) = \log_b g(x)$ consists of all x for which g(x) > 0.

Example—Find the domain of $f(x) = \log_4 (x-5)$.

$x - 5 > 0$
$+5 \quad +5$
$x > 5$

$(5, \infty)$

Common Logarithmic Function—logarithmic function with base 10

The function $f(x) = \log_{10} x$ is usually expressed as $f(x) = \log x$.

Many real-life phenomena start with rapid growth and then the growth begins to level off.

Example—The percentage of adult height attained by a boy who is x years old can be modeled by $f(x) = 29 + 48.8\log(x+1)$ where x represents the boys' age and $f(x)$ represents the percentage of his adult height. Approximately what percentage of his adult height has a boy attained at age 10.

$f(10) = 29 + 48.8\log(10+1)$

$f(10) = 29 + 48.8\log(11)$

$f(10) = 79.82$

Properties of Common Logarithms	
General Properties	**Common Log Properties**
$\log_b 1 = 0$	$\log 1 = 0$
$\log_b b = 1$	$\log 10 = 1$
$\log_b b^x = x$	$\log 10^x = x$
$b^{\log_b x} = x$	$10^{\log x} = x$

The property $\log 10^x = x$ can be used to evaluate common logarithms involving powers of 10. For example:

- $\log 100 = \log 10^2 = 2$
- $\log 1000 = \log 10^3 = 3$
- $\log 10^{7.1} = 7.1$

Natural Logarithmic Function—the logarithmic function with base e. The function $f(x) = \log_e x$ is usually expressed as $f(x) = \ln x$.

Like the domain of all logarithmic functions, the domain of the natural logarithmic function $f(x) = \ln x$ is the set of all positive real numbers. Thus, the domain consists of all x for which $g(x) > 0$

Example—Find the domain of each function.

a) $f(x) = \ln(4 - x)$

$4 - x > 0$

$-4 \quad -4$

$-x > -4$

$x < 4$

$(-\infty, 4)$

b) $h(x) = \ln x^2$

$(-\infty, 0) \cup (0, \infty)$

Properties of Common Logarithms	
General Properties	**Natural Log Properties**
$\log_b 1 = 0$	$\ln 1 = 0$
$\log_b b = 1$	$\ln e = 1$
$\log_b b^x = x$	$\ln e^x = x$
$b^{\log_b x} = x$	$e^{\ln x} = x$

Example—When the outside air temperature is anywhere from 72° to 96° Fahrenheit, the temperature in an enclosed vehicle climbs by 43° in the first hour. The function $f(x) = 13.4 \ln x - 11.6$ models the temperature increase, $f(x)$, in degrees Fahrenheit, after x minutes. Use the function to find the temperature increase, to the nearest degree, after 30 minutes.

$f(x) = 13.4 \ln x - 11.6$

$f(30) = 13.4 \ln(30) - 11.6$

$f(30) = 33.976$

34°