Section 2.1—Basics of Functions and Their Graphs

Relation: any set of ordered pairs

Domain: set of all first values of the ordered pairs—the x-values

Range: set of all second values of the ordered pairs—the y-values

Example: Find the domain and range of the relation:

$$\{(0, 9.1), (10, 6.7), (20, 10.7), (30, 13.2), (36, 17.4)\}$$

domain = {0, 10, 20, 30, 36}   range = {6.7, 9.1, 10.7, 13.2, 17.4}

Function: a relation in which each value in the domain corresponds to exactly one value in the range.

- ✓ a function can have a y-value repeated as long as it doesn't have an x-value repeated

Example: Determine whether each is a function or not.

a. $\{(1, 2), (3, 4), (5, 6), (5, 8)\}$

No - not a function

b. $\{(1, 2), (3, 4), (6, 5), (8, 5)\}$

yes - function

Functions are usually given in the form of an equation, not as a list of ordered pairs.

Independent Variable: x; it can be assigned any value from the domain

Dependent Variable: y; it depends on the x-value

For example: In the equation, $y = -.016x^2 + .93x + 85$, x represents the number of years worked for a company and y represents the number of sick days earned. Because the number of sick days depends on how many years worked it is the dependent variable.

Not all equations are functions. If more than one value of y is found for x, then it is not a function.

Example: Determine whether each is a function.

a. 2x + y = 6

$-2x \quad -2x$

$y = -2x + 6$

yes

b. $x^2 + y^2 = 1$

$-x^2 \quad -x^2$

$\sqrt{y^2} = \sqrt{1 - x^2}$

$y = \pm\sqrt{1 - x^2}$

NO

Functions are generally named with the letters f, g, h, F, G, or H; however any letter can be used. "f" is the most commonly used.

$f(x)$ is functional notation.

- ✓ It represents the value of the function at that particular x-value.
- ✓ ~~It does NOT mean f times x~~.
- ✓ f(x) and y represent the same thing, they are just different ways of writing the same thing. Like 2(2) and 4.

$(x+4)(x+4)$

Example: Evaluate $f(x) = x^2 - 2x + 7$ for each of the following:

a. $f(-5)$

$f(-5) = (-5)^2 - 2(-5) + 7$

$f(-5) = 25 + 10 + 7$

$\boxed{f(-5) = 42}$

$(-5, 42)$

b. $f(x+4)$

$f(x+4) = (x+4)^2 - 2(x+4) + 7$

$f(x+4) = x^2 + 4x + 4x + 16 - 2x - 8 + 7$

$\boxed{f(x+4) = x^2 + 6x + 15}$

c. $f(-x)$

$f(-x) = (-x)^2 - 2(-x) + 7$

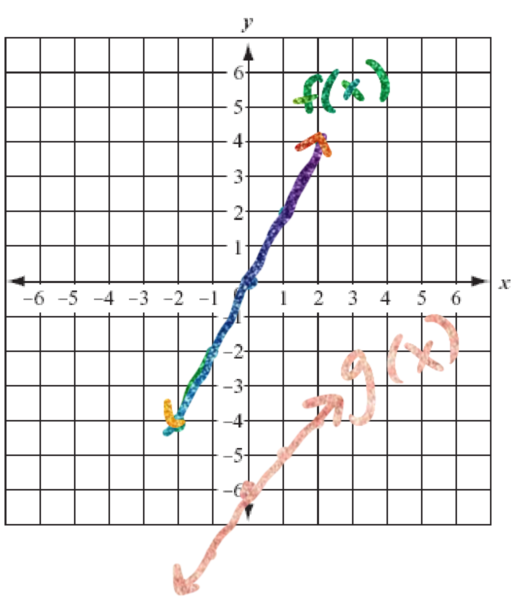
$\boxed{f(-x) = x^2 + 2x + 7}$

Linear Functions: all functions whose graphs are straight lines.

- They are in the form $f(x) = mx + b$

$y = mx + b$

Example: Graph the functions $f(x) = 2x$ and $g(x) = 2x - 3$ on the same graph. Select integers from -2 to 2.



$f(x) = 2x$

-2	2(-2)	-4	(-2,-4)
-1	2(-1)	-2	(-1,-2)
0	2(0)	0	(0,0)
1	2(1)	2	(1,2)
2	2(2)	4	(2,4)

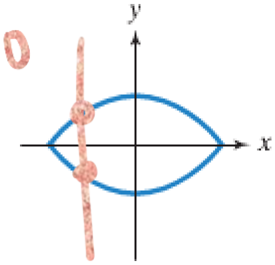
$g(x) = 2(x-3)$

-2	2(-2-3)	-10	(-2,-10)
-1	2(-1-3)	-8	(-1,-8)
0	2(0-3)	-6	(0,-6)
1	2(1-3)	-4	(1,-4)
2	2(2-3)	-2	(2,-2)

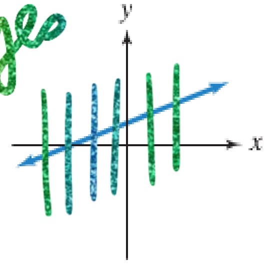
Vertical Line Test: If a vertical line intersects a graph in more than one place, it is not a function.

Example: Use the vertical line test to determine if the graphs are functions.

NO



yes



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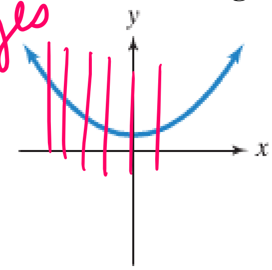

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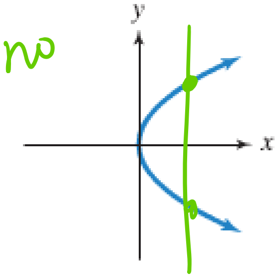
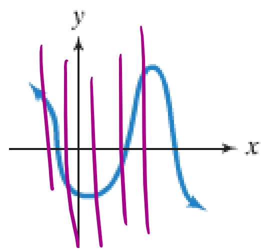
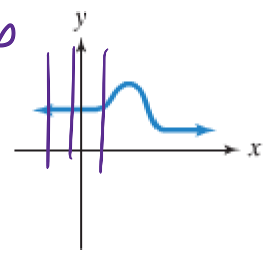


yes



yes



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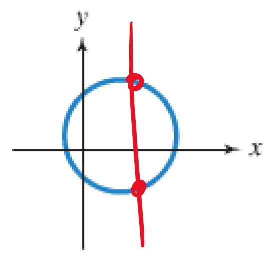


Remember the following about graphs:

- A closed dot indicates that the graph does not extend beyond this point and the point is included in the graph.
- An open dot indicates that the graph does not extend beyond this point and the point is NOT included in the graph.
- An arrow indicates that the graph extends indefinitely in that direction.

Set-Builder Notation:

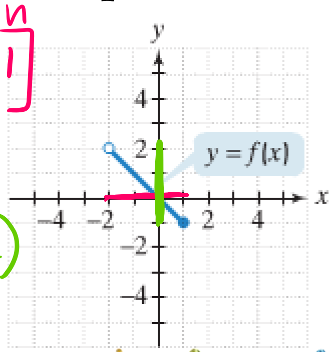
[-3, -2] is interval notation. The brackets tell us the both -3 and -2 are included. The same written in set-builder notation looks like $\{x \mid -3 \leq x \leq -2\}$.
What would we use if ( ) were used instead of [ ]?

↳ <

Example: Use the graph of each function to identify the domain and the range.

domain
(-2, 1]

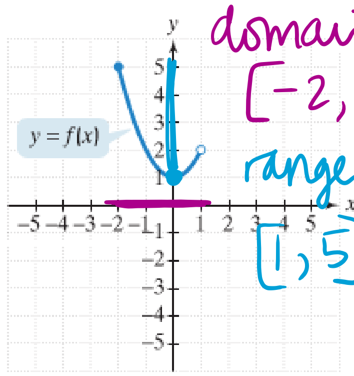
range
[-1, 2)



open circle → ( )

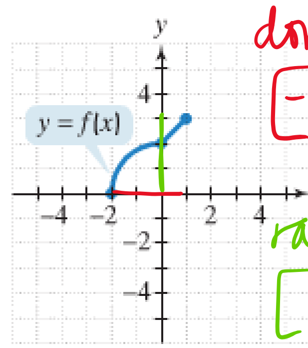
domain
[-2, 1)

range
[1, 5]



domain
[-2, 1]

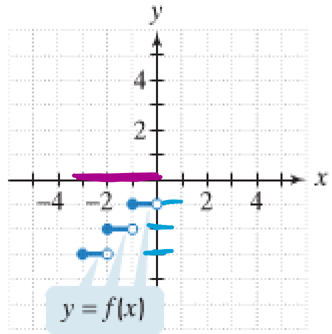
range
[0, 3]

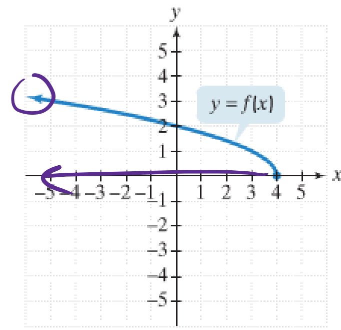


closed circle → [ ]

domain
$[-3, 0)$

range
$\{-3, -2, -1\}$





domain
$(-\infty, 4]$

range
$[0, \infty)$

<u>Zeros of a Function</u>: the x-values for which $f(x) = 0$.

x-intercept

<u>y-intercept</u>: where a graph crosses the y-axis. If the function has a y-intercept at 3, it is written $f(0) = 3$. (0 , 3)

(x , y)

A function can have more than one x-intercept but at most one y-intercept.