

Section 2.8—Distance & Midpoint Formulas, and Circles

Distance Formula—the distance, d , between the points (x_1, y_1) and (x_2, y_2) in the rectangular coordinate system is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Distance is never negative.

Example—Find the distance between $(-4, 9)$ and $(1, -3)$.

$$d = \sqrt{(-4 - 1)^2 + (9 - (-3))^2}$$

$$d = \sqrt{(-5)^2 + (12)^2}$$

$$d = \sqrt{25 + 144} = \sqrt{169} = 13$$

Midpoint Formula—If a segment has endpoints of (x_1, y_1) and (x_2, y_2) , then the coordinates of the segment's midpoint are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Example—Find the midpoint of the line segment with coordinates $(1, 2)$ and $(7, -3)$.

$$\left(\frac{1 + 7}{2}, \frac{2 + (-3)}{2} \right)$$

$$\left(\frac{8}{2}, -\frac{1}{2} \right) = \left(4, -\frac{1}{2} \right)$$

Circle—a set of all points in a plane that are equidistant from a fixed point

Center—the fixed point in a circle

Radius—the fixed distance from a circle's center to any point that lies on the circle

Standard Form of the Equation of a Circle—

$$(x-h)^2 + (y-k)^2 = r^2$$

with center (h,k) and radius r .

Example—write the standard form of the equation of the circle with:

a. center $(0,0)$ and radius 4
 (h,k) r

$$(x-0)^2 + (y-0)^2 = 4^2$$

$$x^2 + y^2 = 16$$

b. center $(5,-6)$ and radius 10
 (h,k) r

$$(x-5)^2 + (y-(-6))^2 = 10^2$$

$$(x-5)^2 + (y+6)^2 = 100$$

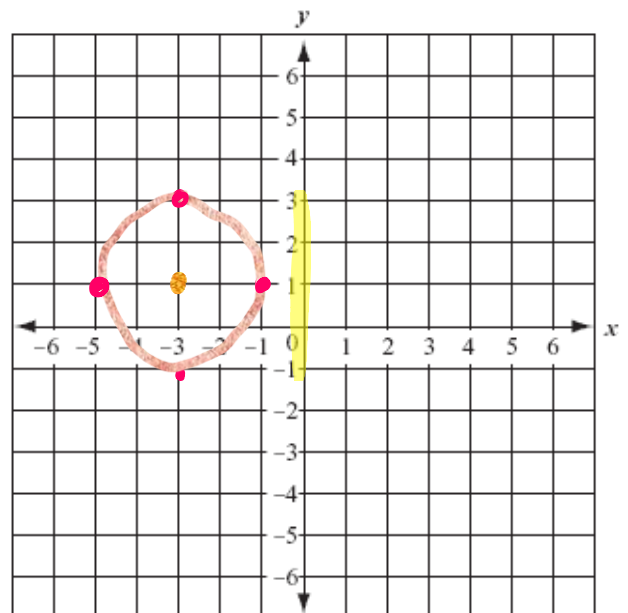
$$(x-h)^2 + (y-k)^2 = r^2$$

Example— center: $(-3,1)$ $r=2$

- Find the center and radius of the circle whose equation is $(x+3)^2 + (y-1)^2 = 4$
- Graph the circle
- Use the graph to identify the relation's domain and range.

domain: $[-5, -1]$

range: $[-1, 3]$



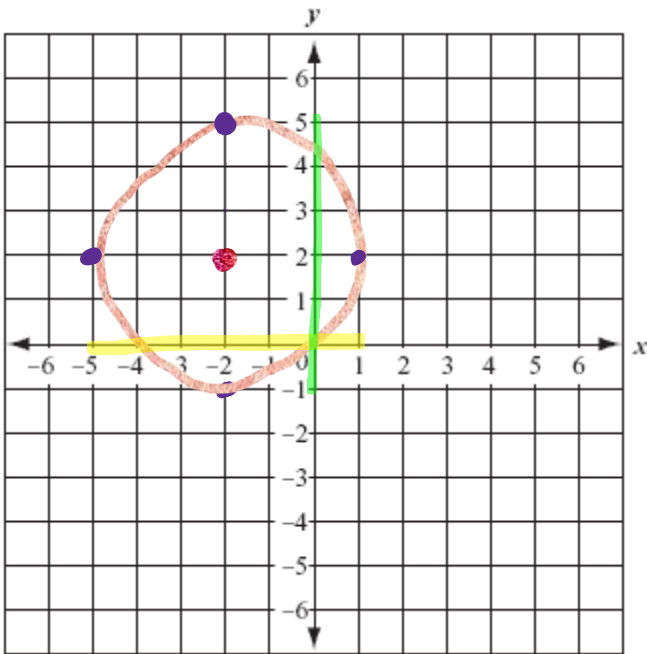
General Form of an Equation—

$$x^2 + y^2 + Dx + Ey + F = 0$$

where D, E, and F are real numbers.

Example—Write each of the following in standard form, graph the circle, and give the domain and range:

a. $x^2 + y^2 + 4x - 4y - 1 = 0$



$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + 4x + 4 + y^2 - 4y + 4 = 1 + 4 + 4$$

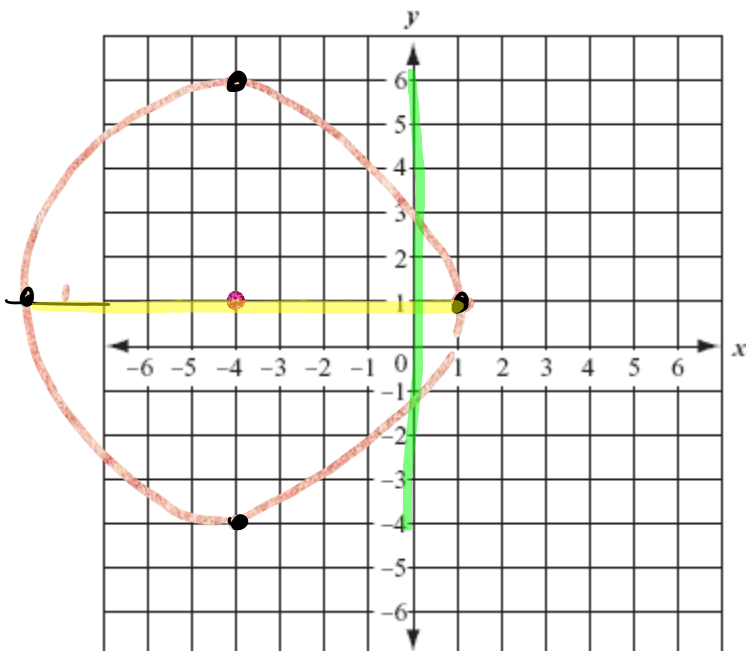
$$(x+2)^2 + (y-2)^2 = 9$$

center: $(-2, 2)$ radius: 3

domain: $[-5, 1]$

range: $[-1, 5]$

b. $x^2 + y^2 + 8x - 2y - 8 = 0$



$$x^2 + 8x + 16 + y^2 - 2y + 1 = 8 + 16 + 1$$

$$(x+4)^2 + (y-1)^2 = 25$$

center: $(-4, 1)$ radius: 5

domain: $[-9, 1]$

range: $[-4, 6]$