Section 5.5—Systems of Inequalities

Solution of an Inequality—an ordered pair of real numbers that gives us a true statement

An ordered pair solution for an inequality is said to **satisfy** the inequality.

Half-plane—set of all points on one side of the line

A solid line is used to show that a line is part of the graph.

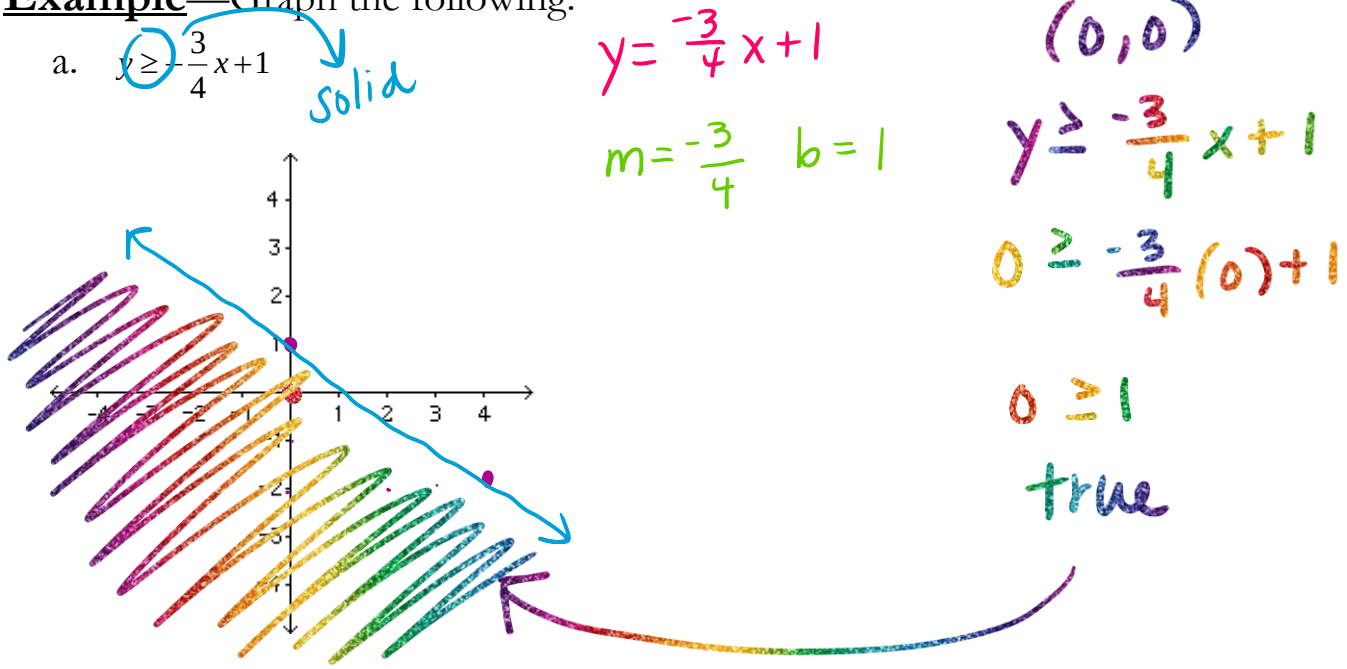
A dashed line is used to show that a line is NOT part of the graph.

Graphing a Linear Inequality in Two Variables

1. Replace the inequality symbol with an equal sign and graph the line. Draw a solid line if the original inequality was ≤ or ≥. Draw a dashed line if the original inequality was < or >.
2. Choose a test point from one of the half planes. Do NOT choose a point that lies on the line. Substitute the test point into the inequality.
3. If a true statement results, shade the half-plane that contains the test point. If a false statement results, shade the half-plane that does not contain the test point.

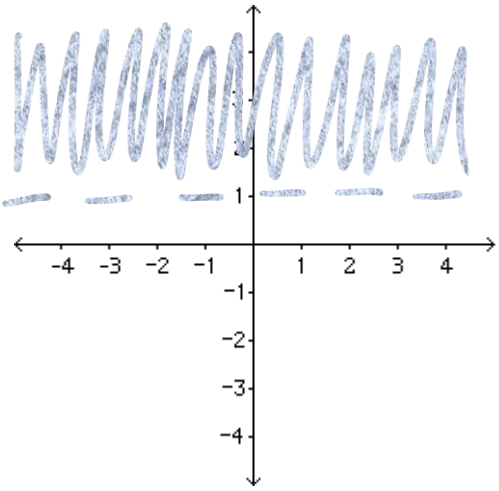
Example—Graph the following:

a. $y \geq -\frac{3}{4}x+1$

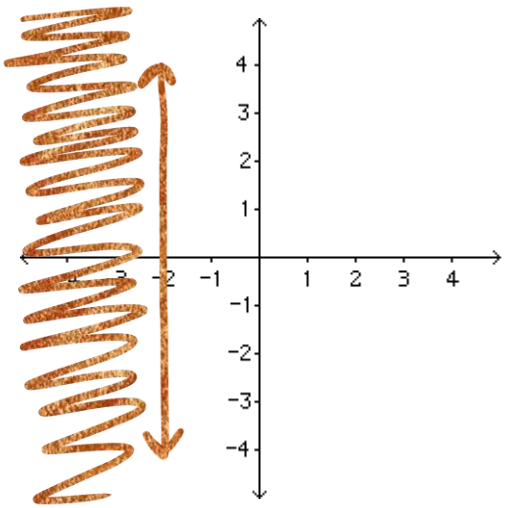


Example—Graph each inequality.

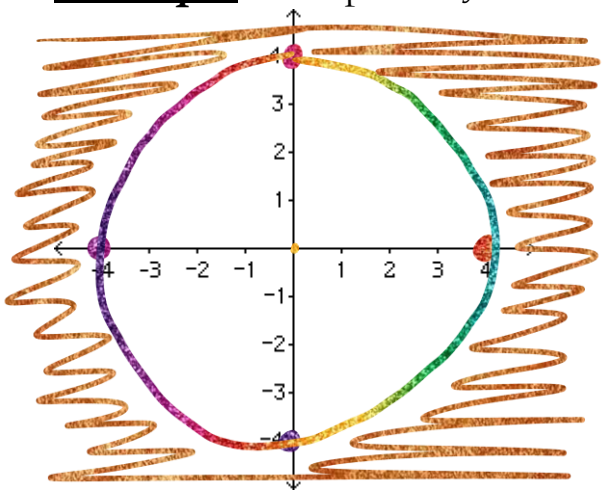
a. $y > 1$ → dashed



b. $x \leq -2$ → solid



Example—Graph $x^2 + y^2 \geq 16$ → solid → circle



$(x-h)^2 + (y-k)^2 = r^2$

center: (h,k) $(0,0)$

radius: $r = 4$

test point: $(0,0)$

$x^2 + y^2 \geq 16$

$0^2 + 0^2 \geq 16$ false

$0 \geq 16$

System of Linear Inequalities—two or more linear inequalities

Solution of a System of Linear Inequalities—an ordered pair that satisfies both inequalities in the system

Solution Set—the set of all solutions to a system of linear inequalities

Example—Graph the solution set of $\begin{cases} x-3y<6 \\ 2x+3y\geq -6 \end{cases}$

dashed

$x-3y<6$

$x-3y=6$

$-x \quad -x$

$-3y=-x+6$

$\frac{-3y}{-3}=\frac{-x}{-3}+\frac{6}{-3}$

$y=\frac{1}{3}x-2$

$m=\frac{1}{3} \quad b=-2$

solid

$2x+3y\geq -6$

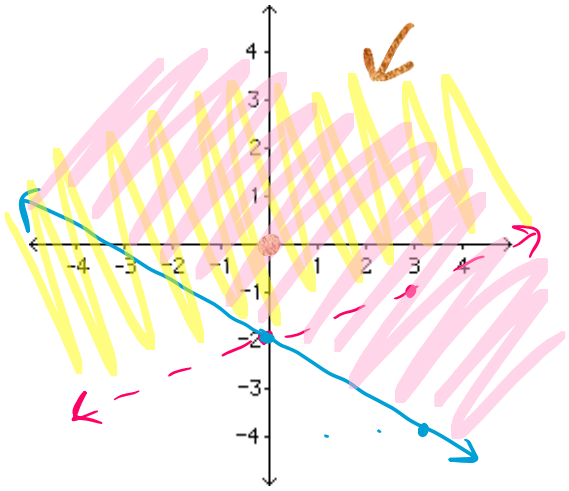
$2x+3y=-6$

$-2x \quad -2x$

$\frac{3y}{3}=\frac{-2x}{3}-\frac{6}{3}$

$y=-\frac{2}{3}x-2$

$m=-\frac{2}{3} \quad b=-2$



$x-3y<6$

$0-3(0)<6$

$0<6$

true

$2x+3y\geq -6$

$2(0)+3(0)\geq -6$

$0\geq -6$

true

$f(x)=a(x-h)^2+k$

Example—Graph the solution set to $\begin{cases} y\geq x^2-4 \\ x+y\leq 2 \end{cases}$

solid

$(h,k)=(0,-4) \rightarrow$ vertex

x-int $f(x)=0$

$x^2-4=0$

$(x+2)(x-2)=0$

$x+2=0 \quad x-2=0$

$-2 \ -2 \quad +2 \ +2$

$x=-2 \quad x=2$

$(-2,0) \quad (2,0)$

y-int $f(0)$

$y=0^2-4$

$y=-4$

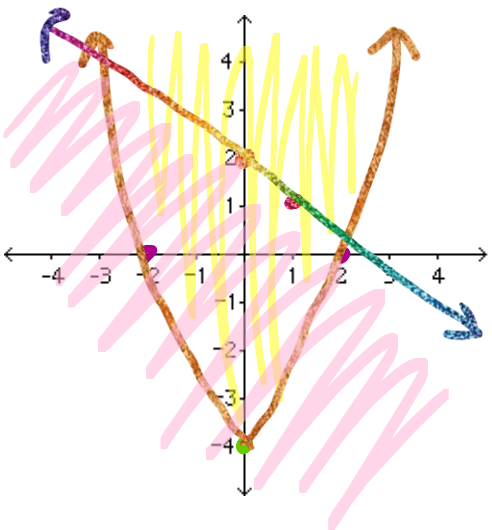
$(0,-4)$

test point

$y\geq x^2-4$

$0\geq 0^2-4$

$0\geq -4$ true



$x+y\leq 2$

solid

$x+y=2$

$-x \quad -x$

$y=-x+2$

$m=-1 \quad b=2$

$x+y\leq 2$

$0+0\leq 2$ true

$0\leq 2$