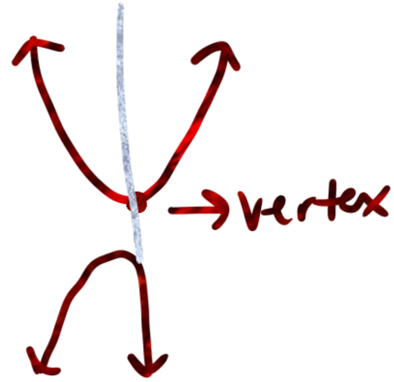
Section 3.1—Quadratic Functions



Quadratic Function—any function in the form

$$f(x) = ax^2 + bx + c$$

Parabola—the graph of any quadratic function

Vertex of a Parabola—turning point; lowest point when it opens upward and highest point when it opens downward

Axis of Symmetry—the line about which a parabola is symmetric

Standard Form of a Quadratic Equation

$$f(x) = a(x-h)^2 + k, \quad a \neq 0$$

- ✓ The vertex is the point (h,k).
- ✓ The parabola is symmetric about the line $x = h$.
- ✓ If $a > 0$, the parabola opens upward; if $a < 0$ the parabola opens downward.

Graphing Quadratic Equations in Standard Form

1. Determine whether the parabola opens upward or downward. a
2. Determine the vertex of the parabola. (h,k)
3. Find any x-intercepts by solving $f(x) = 0$ set equal to 0
4. Find the y-intercept by computing $f(0)$ plug in 0
5. Plot the intercepts, the vertex, and any additional points as needed. Connect the points with a smooth curve.

$f(x)=a(x-h)^2+k$

Example—Graph the quadratic function

$f(x) = -(x-1)^2 + 4$.

step 1 open downward b/c $a<0$

step 2 vertex $(1,4)$

step 4 y-int

$f(0) = -(0-1)^2 + 4$

$f(0) = -(-1)^2 + 4$

$f(0) = -1 + 4$

$f(0) = 3$ $(0,3)$

step 3 x-int

$-(x-1)^2 + 4 = 0$

$\quad\quad -4 \quad -4$

$-(x-1)^2 = -4$

$\sqrt{(x-1)^2} = \sqrt{4}$

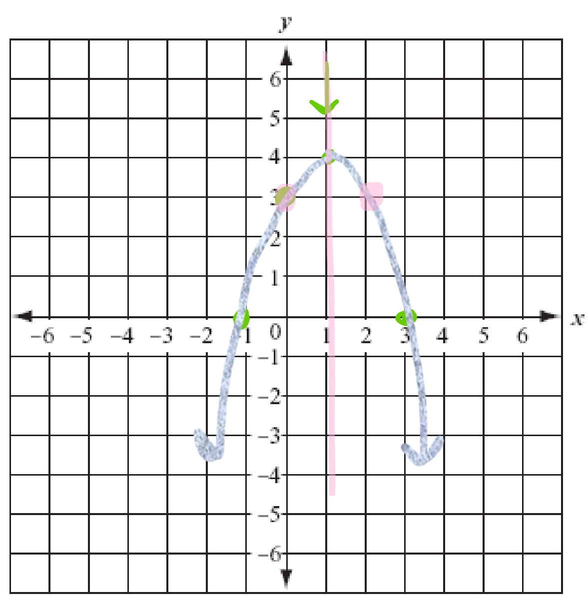
$x - 1 = \pm 2$

$x-1=2$ $\quad$ $x-1=-2$

$+1 \quad +1$ $\quad$ $+1 \quad +1$

$x=3$ $\quad$ $x=-1$

$(3,0)$ $(-1,0)$



$f(x) = a(x-h)^2 + k$

Example—Graph the quadratic function

$f(x) = (x-2)^2 + 1$

step 1 upward $a > 0$

step 2 vertex $(2, 1)$

step 3 x-int

$(x-2)^2 + 1 = 0$

$\quad\quad\;\; -1 \;\; -1$

$\sqrt{(x-2)^2} = \sqrt{-1}$

$x - 2 = \pm\sqrt{-1}$

$x - 2 = \pm i$

$+2 \quad +2$

$x = 2 \pm i$ no real solution

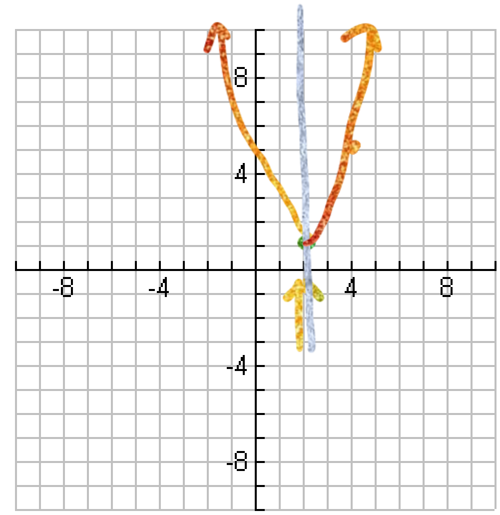
Step 4 y-int

$f(0) = (0-2)^2 + 1$

$f(0) = (-2)^2 + 1$

$f(0) = 4 + 1$

$f(0) = 5 \quad (0, 5)$



Vertex of a Parabola with Equation $f(x) = ax^2 + bx + c$

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

to find x

plug that in to find y

We can still use the same 5 steps to graph a parabola in this form. The only difference is how we find the vertex.

Example—Graph the quadratic function $f(x) = -x^2 + 4x + 1$. Use the graph to determine the domain and range.

$a = -1 \quad b = 4 \quad c = 1$

step 1 $a < 0$; downward

Step 2 $x = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

$f(2) = -(2)^2 + 4(2) + 1 \quad (2, 5)$

$= -4 + 8 + 1 = 5$

step 3 x-int $-x^2 + 4x + 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(-1)(1)}}{2(-1)}$

$= \frac{-4 \pm \sqrt{16+4}}{-2} = \frac{-4 \pm \sqrt{20}}{-2} = \frac{-4+\sqrt{20}}{2} \;\&\; \frac{-4-\sqrt{20}}{2}$

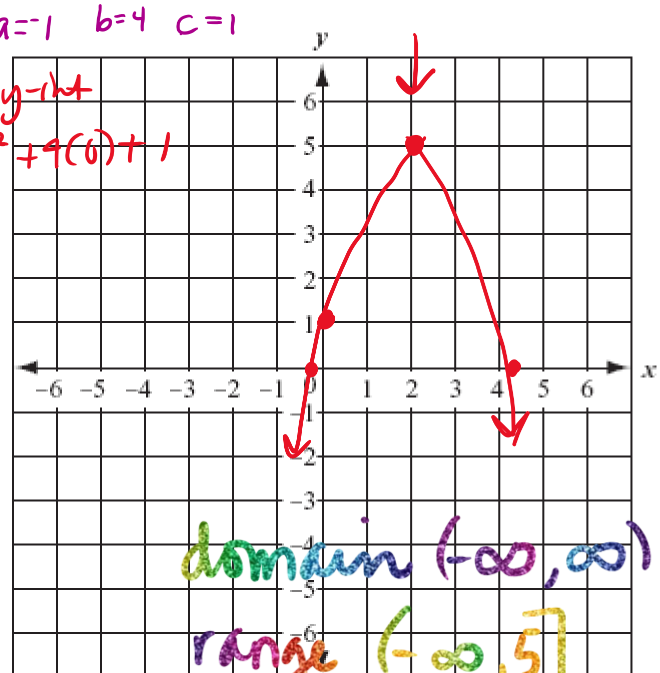
$(-.24, 0) \quad (4.24, 0)$

Step 4 y-int

$f(0) = -(0)^2 + 4(0) + 1$

$f(0) = 1$

$(0, 1)$



Minimum and Maximum: Quadratic Functions

Consider the quadratic function $f(x) = ax^2 + bx + c$

1. If $a > 0$, then f has a minimum that occurs at $x = -\frac{b}{2a}$. This minimum value is at $f\left(-\frac{b}{2a}\right)$.

2. If $a < 0$, then f has a maximum that occurs at $x = -\frac{b}{2a}$. This minimum value is at $f\left(-\frac{b}{2a}\right)$.

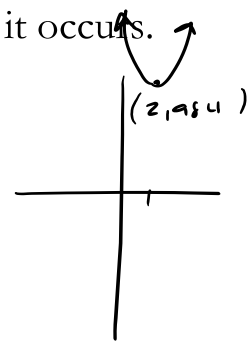
In each case, the value of x gives the location of the minimum or maximum value.

Example—Using $f(x) = 4x^2 - 16x + 1000$, complete the following.

a. Determine, without graphing, whether the function has a minimum value or maximum value. $a = 4 \rightarrow a > 0$ ∇ minimum

b. Find the maximum or minimum value and determine where it occurs.

c. Identify the domain and range.



$x = \frac{-b}{2a} = \frac{-(-16)}{2(4)} = \frac{16}{8} = 2$

domain: $(-\infty, \infty)$

range: $[984, \infty)$

$f(2) = 4(2)^2 - 16(2) + 1000$

$f(2) = 16 - 32 + 1000$

$f(2) = 984$

$(2, 984)$

Example—The function $f(x) = .4x^2 - 36x + 1000$ models the number of accidents, $f(x)$, per 50 million miles driven, for drivers x years old, where $16 \le x \le 74$. What is the age of a driver having the least number of car accidents? What is the minimum number of car accidents per 50 million miles driven?

$x = \frac{-b}{2a} = \frac{-(-36)}{2(.4)} = \frac{36}{.8} = 45 \rightarrow$ age w/ least # of accidents

$f(45) = .4(45)^2 - 36(45) + 1000$

$f(45) = .4(2025) - 1620 + 1000$

$f(45) = 810 - 1620 + 1000$

$f(45) = 190$