

Section 5.2—Systems of Linear Equations in Three Variables

Linear Equation in Three Variables $Ax + By + Cz = D$

Solution of a System of Linear Equations in Three Variables—an ordered triple of real numbers that satisfies all equations in the system

Example—Show that the ordered triple $(-1, -4, 5)$ is a solution of the system

$$\begin{cases} x - 2y + 3z = 22 & \checkmark \\ 2x - 3y - z = 5 & \checkmark \\ 3x + y - 5z = -32 & \checkmark \end{cases}$$

$$2x - 3y - z = 5$$

$$2(-1) - 3(-4) - (5) = 5$$

$$-2 + 12 - 5 = 5$$

$$5 = 5$$

✓

$$3x + y - 5z = -32$$

$$3(-1) + (-4) - 5(5) = -32$$

$$-3 - 4 - 25 = -32$$

$$-32 = -32$$

✓

$$\begin{aligned} x - 2y + 3z &= 22 \\ (-1) - 2(-4) + 3(5) &= 22 \\ -1 + 8 + 15 &= 22 \end{aligned}$$

$$22 = 22$$

✓

yes

To be a solution for the system it must work for all three equations.

Solving Linear Systems in Three Variables by Eliminating Variables

1. Reduce the system to two equations with two variables. Take two different pairs of equations and use the addition method to get rid of the same variable from each pair.
2. Solve the resulting two equations (with two variables) using addition or substitution. This will give you the value of one variable.
3. Substitute the value for that variable into one of the equations containing only two variables to find the value of the second variable.
4. Use the value of the two variables from steps two and three to find the value of the third variable by substituting into one of the original equations.
5. Check the solution in all three of the original equations.

Example

Solve the system

$$\begin{cases} x+4y-z=20 \\ 3x+2y+z=8 \\ 2x-3y+2z=-16 \end{cases} \quad (2)$$

$$\begin{array}{r} x+4y-z=20 \\ 3x+2y+z=8 \\ \hline \end{array}$$

$$4x+6y=28$$

$$\begin{array}{r} 4x+6y=28 \\ -4x-5y=-24 \\ \hline \end{array}$$

$$y=4$$

$$\begin{array}{r} 2x+8y-2z=40 \\ 2x-3y+2z=-16 \\ \hline \end{array}$$

$$4x+5y=24 \quad (-1)$$

$$4x+5y=24$$

$$4x+5(4)=24$$

$$4x+20=24$$

$$\begin{array}{r} -20 \quad -20 \\ \hline \end{array}$$

$$\begin{array}{r} 4x=4 \\ \hline x=1 \end{array}$$

$$x=1$$

$$3x+2y+z=8$$

$$3(1)+2(4)+z=8$$

$$3+8+z=8$$

$$11+z=8$$

$$\begin{array}{r} -11 \quad -11 \\ \hline \end{array}$$

$$z=-3$$

$$(1, 4, -3)$$

Example—Solve the system $\begin{cases} 2y - z = 7 \\ x + 2y + z = 17 \quad (-2) \\ 2x - 3y + 2z = -1 \end{cases}$

$$2y - z = 7$$

$$2(5) - z = 7$$

$$\begin{array}{r} 10 - z = 7 \\ -10 \quad -10 \\ \hline \end{array}$$

$$-z = -3$$

$$z = 3$$

$$x + 2y + z = 17$$

$$x + 2(5) + (3) = 17$$

$$x + 10 + 3 = 17$$

$$x + 13 = 17$$

$$\begin{array}{r} -13 \quad -13 \\ \hline \end{array}$$

$$x = 4$$

$$\begin{array}{r} -2x - 4y - 2z = -34 \\ 2x - 3y + 2z = -1 \\ \hline \end{array}$$

$$\begin{array}{r} -7y = -35 \\ \hline -7 \quad -7 \end{array}$$

$$y = 5$$

$$(4, 5, 3)$$