Section 5.1—Systems of Linear Equations with Two Variables

System of Linear Equations—two linear equations where one ordered pair is a solution to both equations in the system

Example—Determine if each ordered pair is a solution for the system $\begin{cases} 2x-3y=-4 \\ 2x+y=4 \end{cases}$

a. (1, 2)

$2(1)-3(2)=-4$

$2-6=-4$

$-4=-4$

✓

$2(1)+(2)=4$

$2+2=4$

$4=4$

✓

yes

b. (7, 6)

$2(7)-3(6)=-4$

$14-18=-4$

$-4=-4$

✓

$2(7)+(6)=4$

$14+6=4$

$20=4$

X

no

We cannot solve one equation that has two variables.

The solution of a system of linear equations can be found by graphing both equations on the same plane.

- ✓ The point of intersection is the solution for the system.
- ✓ This requires very exact and precise graphing—so it is generally not the most reliable method.

Solving a System by Substitution

1. Solve either equation for one variable in terms of the other.
2. Substitute the result from Step 1 into the other equation. This will give you one equation with one variable.
3. Solve that equation for the variable.
4. Substitute the value from Step 3 into one of the original equations and solve for remaining variable.
5. Check the solution in both given equations.

Example—Solve by substitution

a. $\begin{cases} 3x+2y=4 \\ 2x+y=1 \end{cases}$

$2x+y=1$
$-2x \quad -2x$
$y=1-2x$

$y=1-2(-2)$
$y=1+4$
$y=5$

$3x+2y=4$
$3x+2(1-2x)=4$
$3x+2-4x=4$
$-x+2=4$
$-2 \quad -2$
$-x=2$
$x=-2$

$(-2,5)$

b. $\begin{cases} x+2y=2 \\ -4x+3y=25 \end{cases}$

$x+2y=2$
$-2y \quad -2y$
$x=-2y+2$

$x=-2(3)+2$
$x=-6+2$
$x=-4$

$-4(-2y+2)+3y=25$
$8y-8+3y=25$
$11y-8=25$
$+8 \quad +8$
$11y=33$
$\frac{11y}{11}=\frac{33}{11}$
$y=3$

$(-4,3)$

Substitution is easy for variables that are easily isolated.

Addition (or Elimination) Method—our goal is to obtain coefficients for one variable that differ only in sign

Solving Systems by Addition (Elimination)

1. Rewrite both equations in the form $Ax+By=C$
2. Multiply one or both equations by the appropriate number to make the sum of one of the coefficients zero. create opposite coefficients
3. Add the equations from Step 2. The result should be one equation with one variable.
4. Solve the equation with one variable.
5. Substitute the result from Step 4 into one of the original equations and solve for the other variable.
6. Check the solution in both of the original equations.

Example—Solve by addition

a. $\begin{cases} 4x+5y=3 \\ 2x-3y=7 \end{cases}(-2)$

$4\cancel{x}+5y=3$
$-\cancel{4x}+6y=-14$

$\frac{11y}{11}=\frac{-11}{11}$

$y=-1$

$4x+5y=3$
$4x+5(-1)=3$
$4x-5=3$
$+5\quad+5$
$\frac{4x}{4}=\frac{8}{4}$

$x=2$

$(2,-1)$

b. $\begin{cases} 2x=9+3y \\ 4y=8-3x \end{cases}$

$(-3)(2x-3y=9)4$
$(2)(3x+4y=8)3$

$8x-\cancel{12y}=36$
$9x+\cancel{12y}=24$

$\frac{\cancel{17}x}{\cancel{17}}=\frac{60}{17}$

$x=\frac{60}{17}$

$-\cancel{6x}+9y=-27$
$\cancel{6x}+8y=16$

$\frac{\cancel{17}y}{\cancel{17}}=\frac{-11}{17}$

$y=\frac{-11}{17}$

$\left(\frac{60}{17},\frac{-11}{17}\right)$

The Number of Solutions to a System of Two Linear Equations

Number of Solutions	Graphical Representation
Exactly One	The lines intersect at one point.
No Solution	The lines are parallel.
Infinitely Many	The lines are identical.

Consistent System—a linear system that has at least one solution

Inconsistent System—a linear system with no solution

- ✓ when this happens, you will eliminate both variables and a false statement will result

Example—Solve the system $\begin{cases} 5x-2y=4 \\ -10x+4y=7 \end{cases}$ 2

$10\cancel{x}-4\cancel{y}=8$
$-10x+4y=7$

$0=15$

False → no solution

(b/c parallel)

Dependent—equations in a linear system with infinitely many solutions

✓ when this happens you will eliminate both variables and a true statement will result

Example—Solve $\begin{cases} x=4y-8 \\ 5x-20y=-40 \end{cases}$

$5x-20y=-40$

$5(4y-8)-20y=-40$

$20y-40-20y=-40$

$-40=-40$

true → infinite number of solutions

(b/c lines are identical)