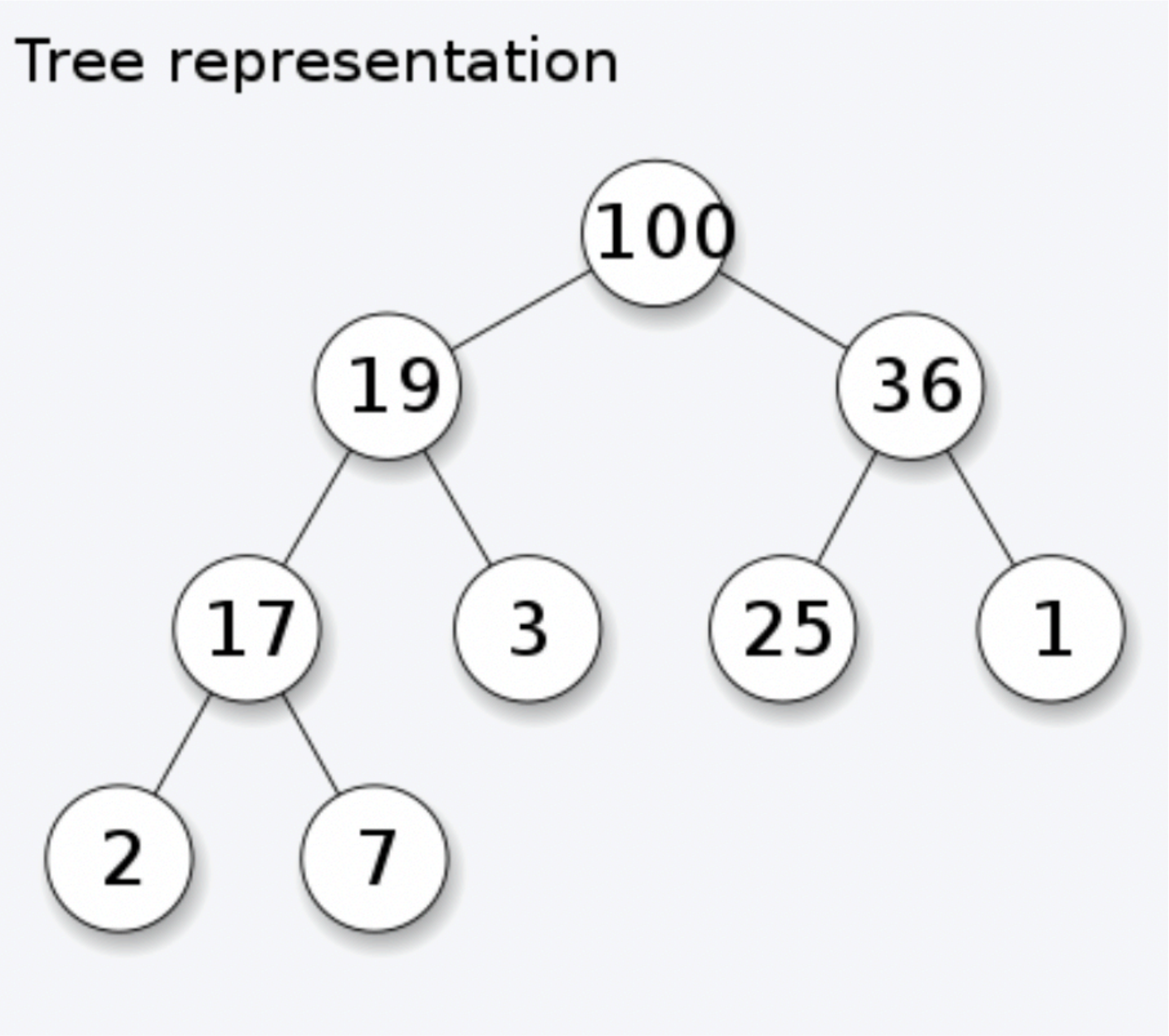
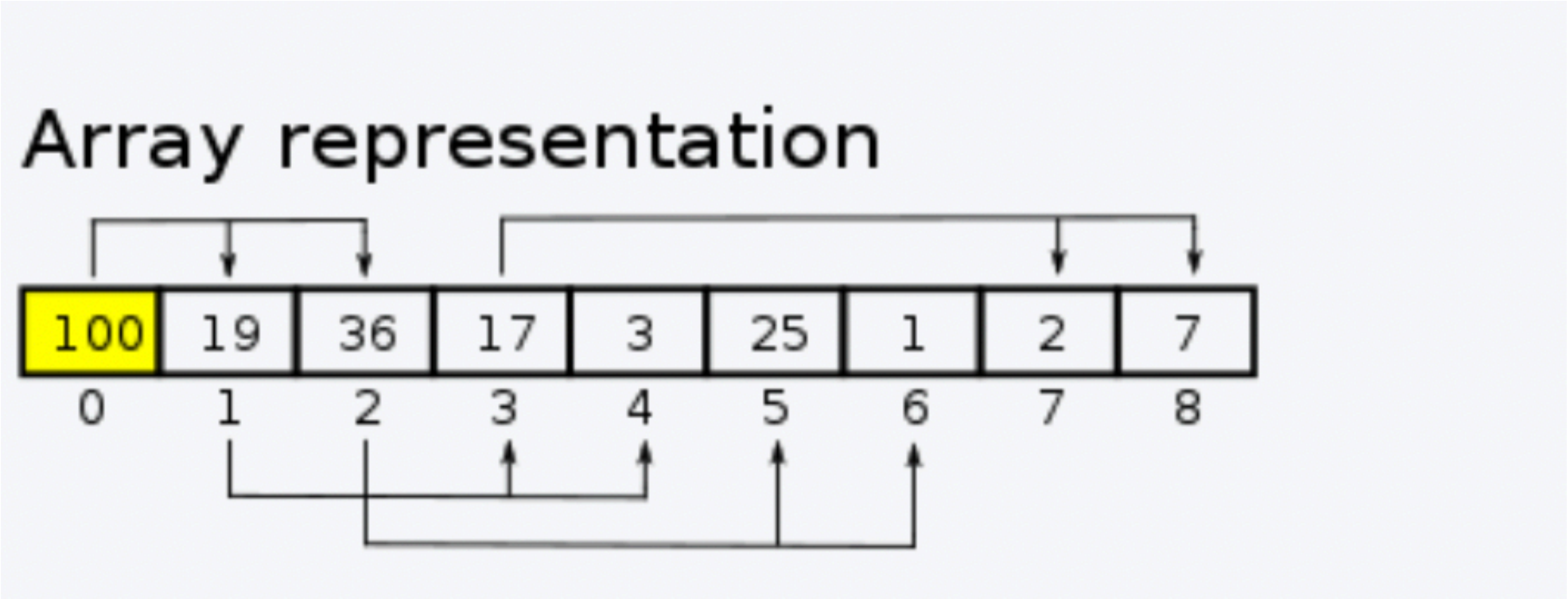
Module 2-3 Heap and its applications





Topic' s Outlines

- ★ Heap
- ★ Heap Sort
- ★ Huffman's Code

Heap ADT

- Heap data structure is a complete binary tree that satisfies the heap property, where any given node is
  - always greater than its child node/s and the key of the root node is the largest among all other nodes. This property is also called max heap property.
  - always smaller than the child node/s and the key of the root node is the smallest among all other nodes. This property is also called min heap property.

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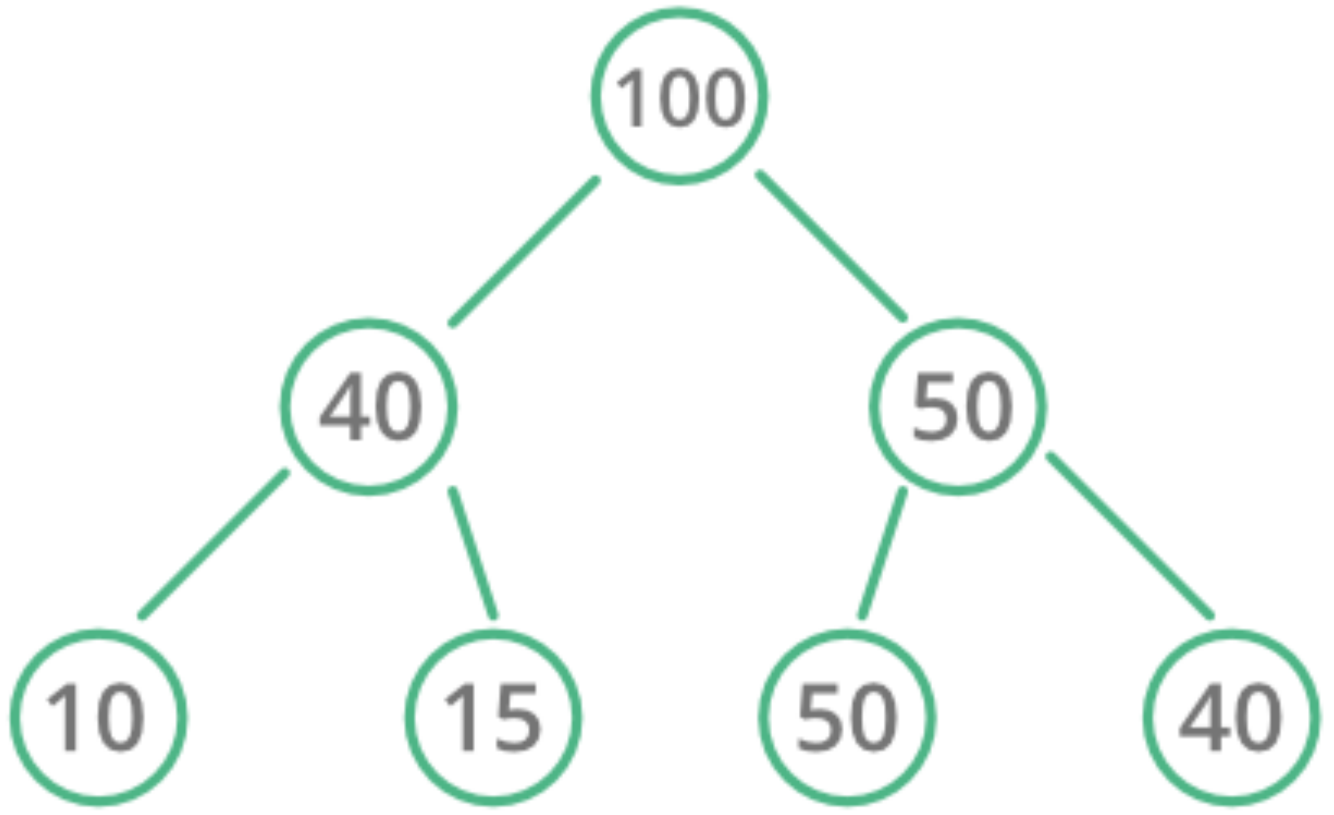

Max Heap

Heap ADT

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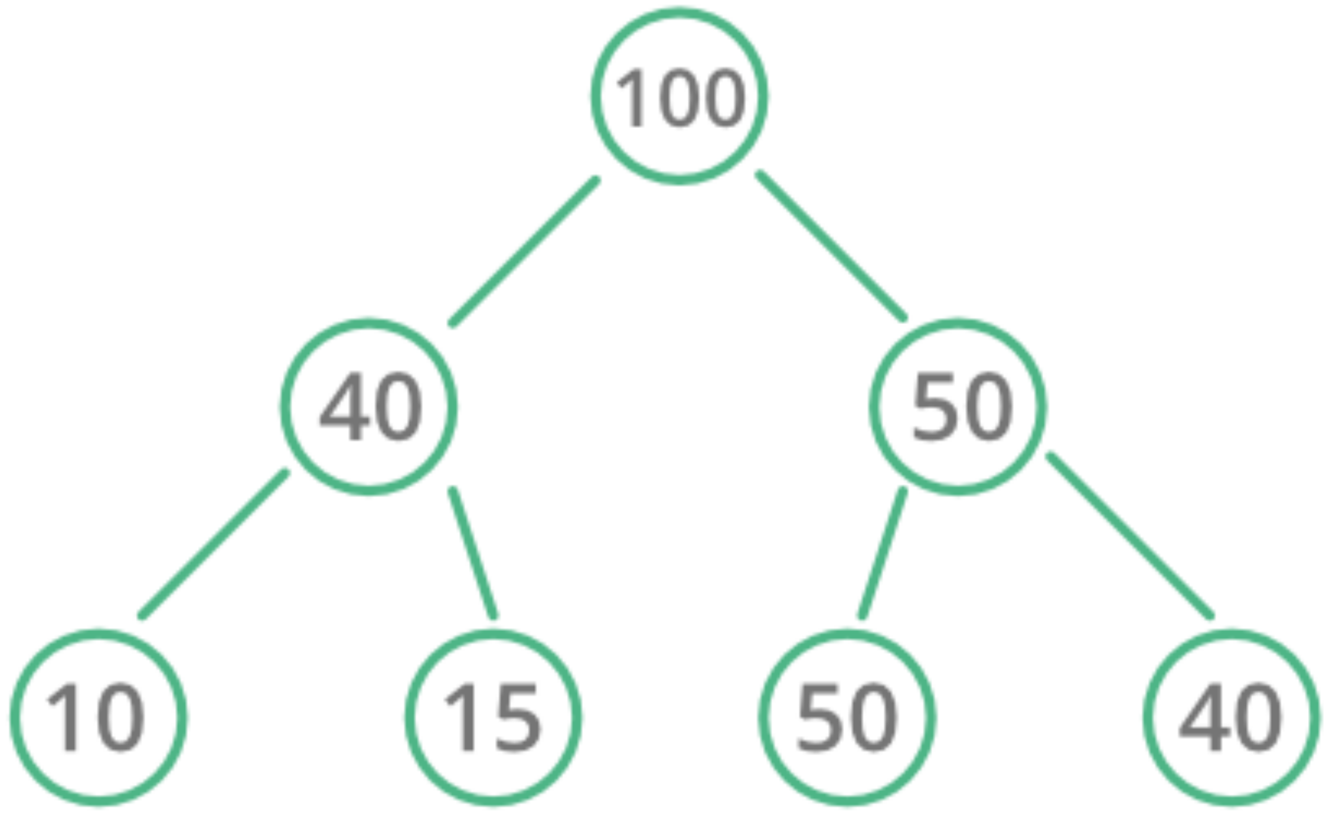
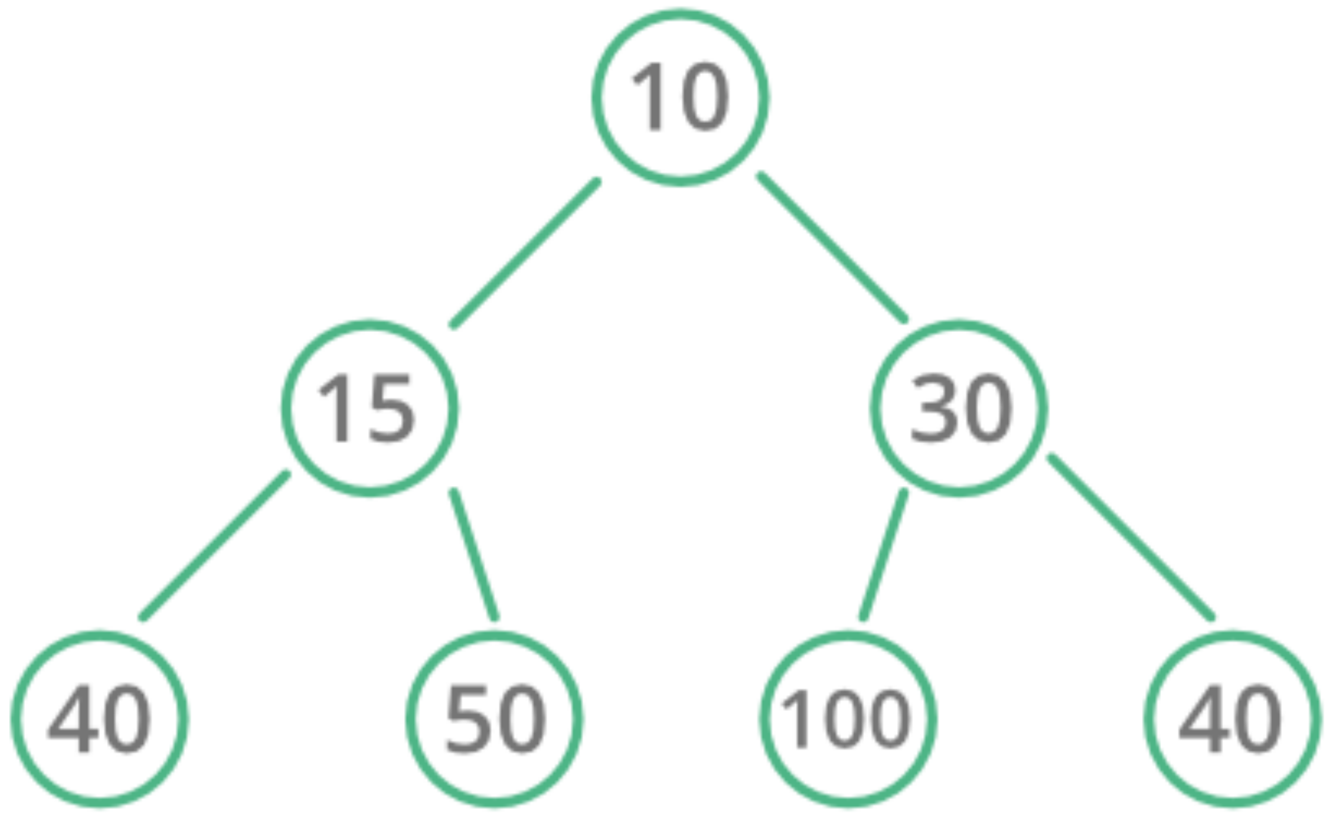
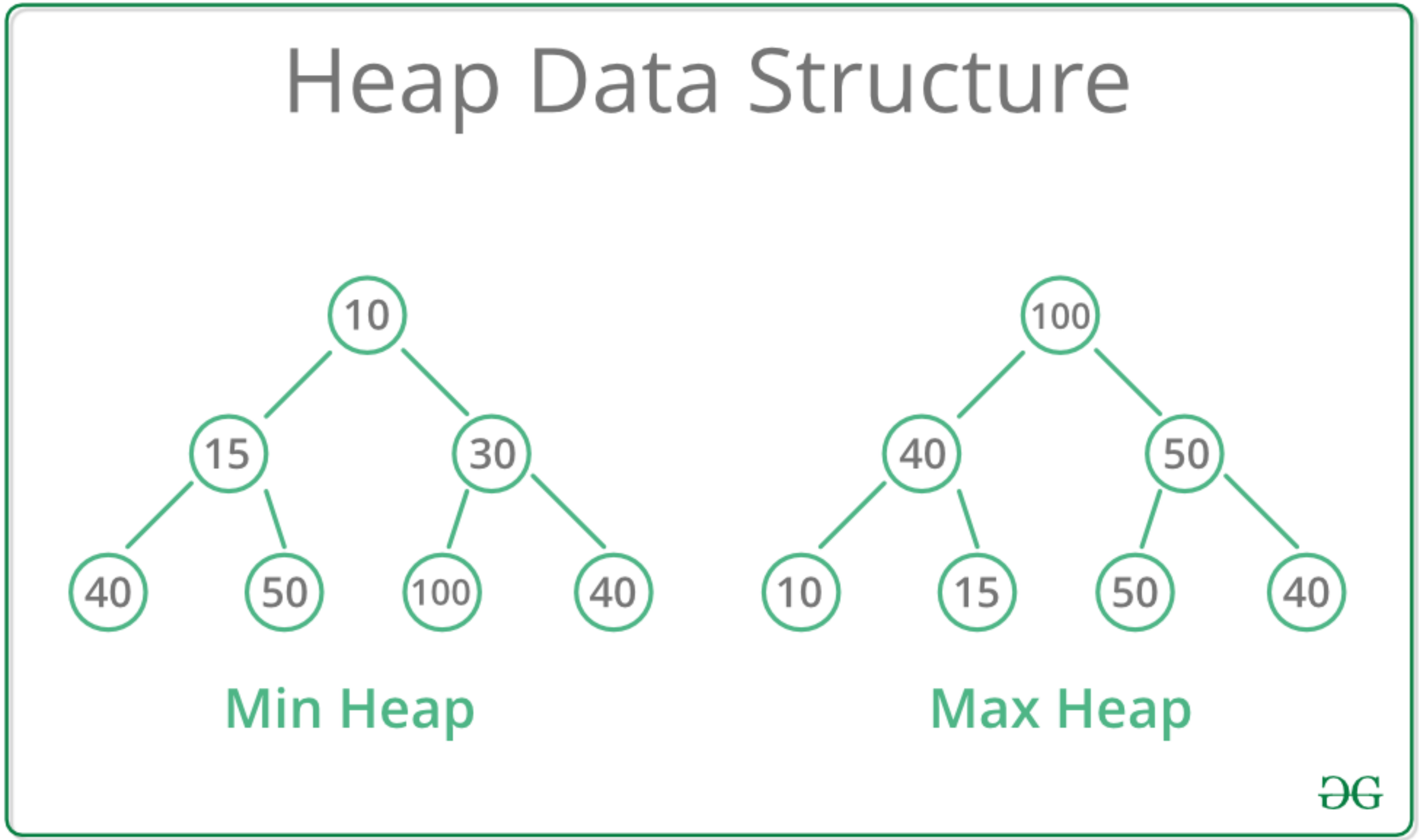


Max Heap



Min Heap

Heap ADT


Heap Data Structure
10
15
30
40
50
100
40
Min Heap
100
40
50
10
15
50
40
Max Heap

Heap Construction

- ★ Initialize an essentially complete binary tree from the keys given
- ★ Heapify the tree as following:
  - Start with the last (rightmost) parental node,
  - Exchange key K with its children if the parental dominance does not hold.
  - Check the parental dominance for the new position of K and repeat until the key K goes to right location.
  - Continue with other parental nodes up to the root.

Heap Construction

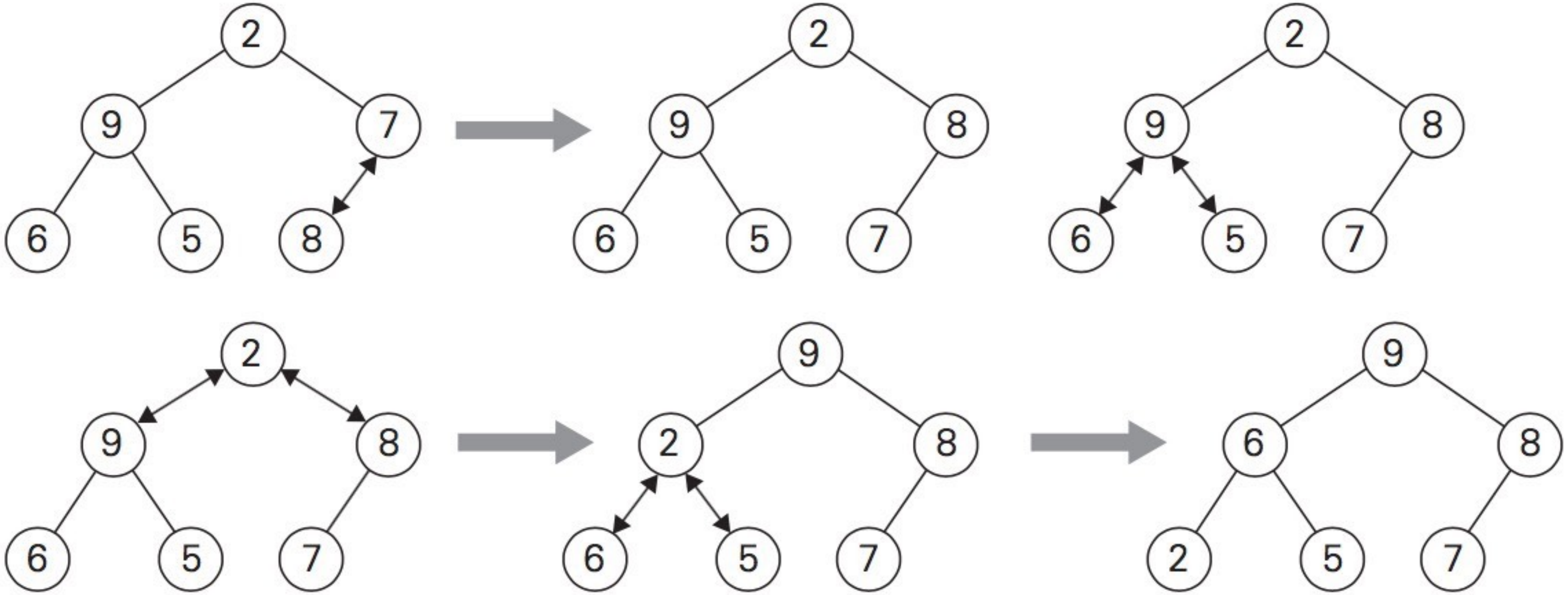
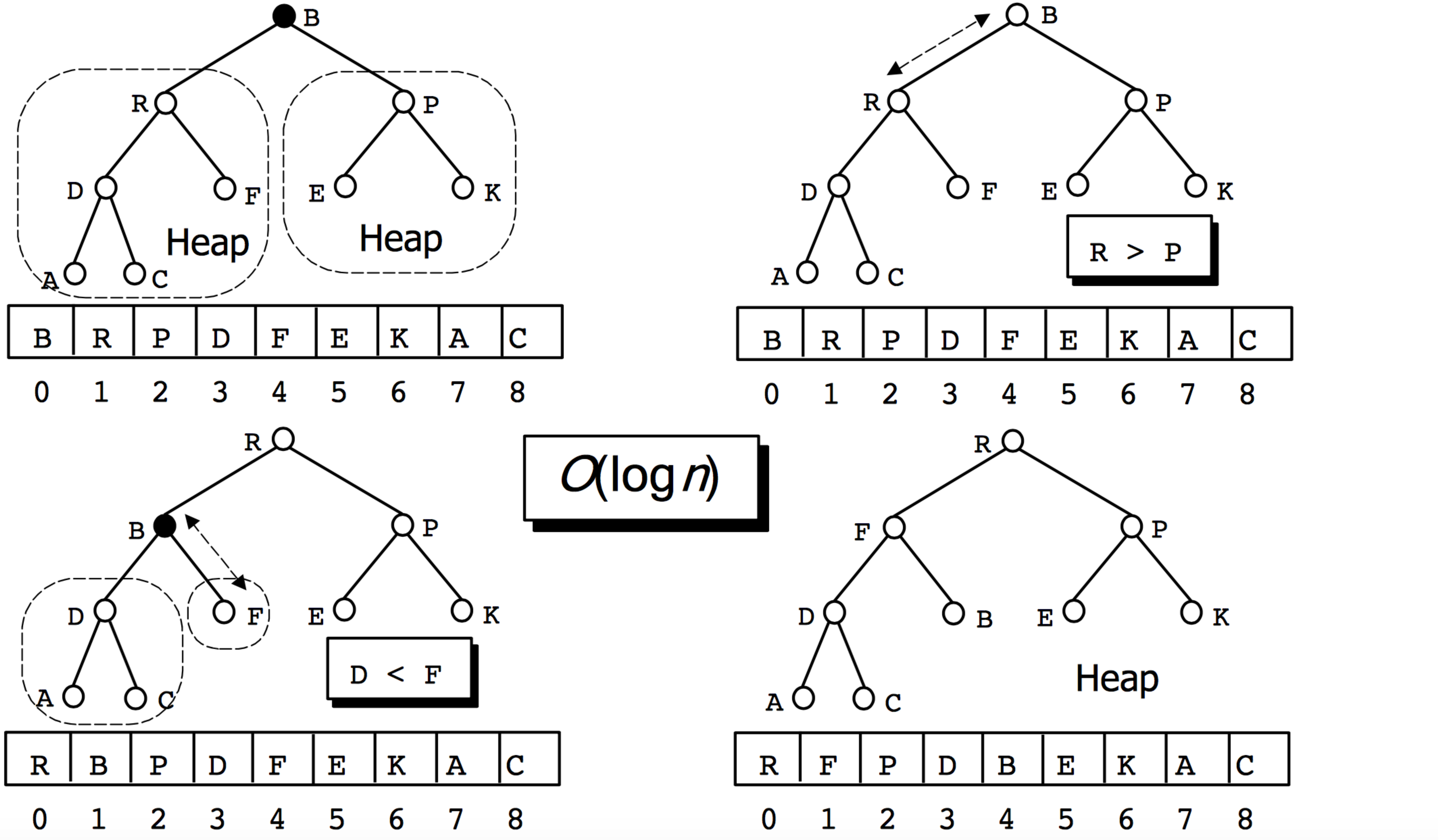


FIGURE 6.11 Bottom-up construction of a heap for the list 2, 9, 7, 6, 5, 8. The double-headed arrows show key comparisons verifying the parental dominance.

Heapify

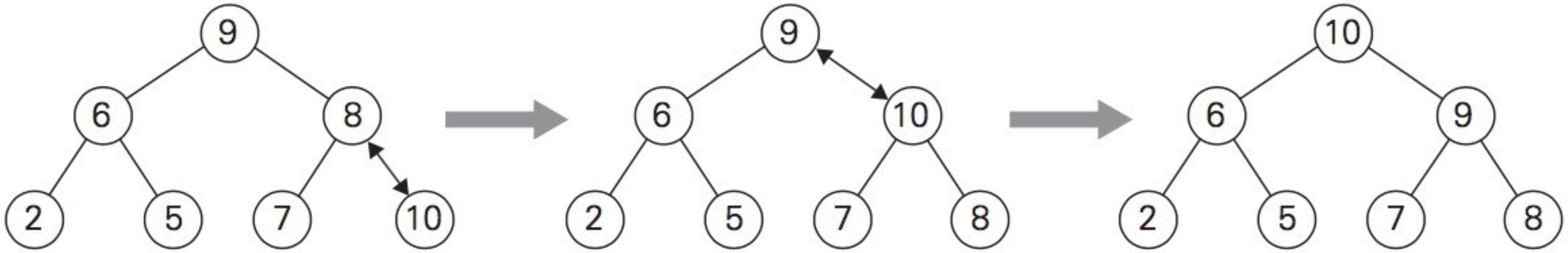

B
R
P
D
F
E
K
A
C
Heap
Heap
B R P D F E K A C
0 1 2 3 4 5 6 7 8
R > P
B R P D F E K A C
0 1 2 3 4 5 6 7 8
D < F
R B P D F E K A C
0 1 2 3 4 5 6 7 8
O(log n)
Heap
R F P D B E K A C
0 1 2 3 4 5 6 7 8

ALGORITHM *HeapBottomUp*$(H[1..n])$

//Constructs a heap from elements of a given array
// by the bottom-up algorithm
//Input: An array $H[1..n]$ of orderable items
//Output: A heap $H[1..n]$
for $i \leftarrow \lfloor n/2 \rfloor$ **downto** 1 **do**
    $k \leftarrow i$; $v \leftarrow H[k]$
    $heap \leftarrow$ **false**
    while not $heap$ **and** $2 * k \leq n$ **do**
        $j \leftarrow 2 * k$
        if $j < n$ //there are two children
            if $H[j] < H[j+1]$ $j \leftarrow j+1$
        if $v \geq H[j]$
            $heap \leftarrow$ **true**
        else $H[k] \leftarrow H[j]$; $k \leftarrow j$
    $H[k] \leftarrow v$

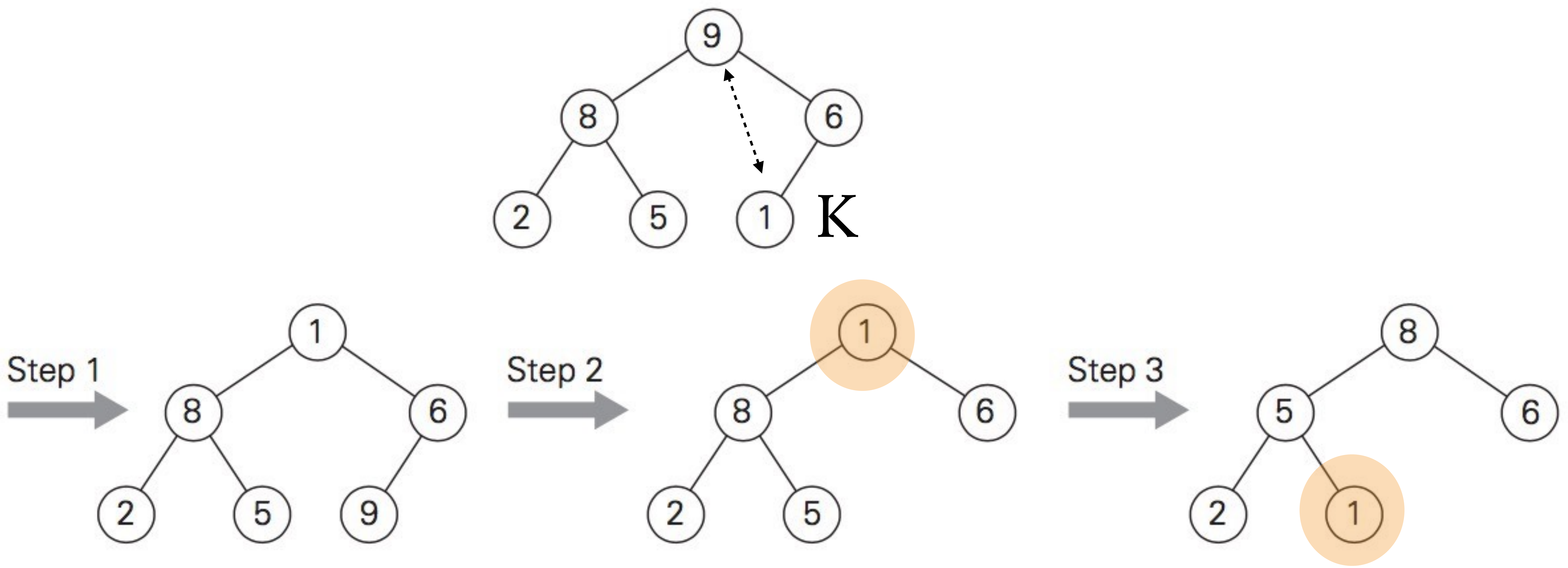
Inserting Key into Heap

- Insert the new key to the last leaf.
- Repeatedly swap the new key with its parent until the parental dominance is satisfied.
- Also be used as *top-down heap construction*



Deleting Key from Heap

- Swap the key to delete with the key K in last leaf.
- Delete the last leaf.
- Heapify the tree by sifting K down the tree

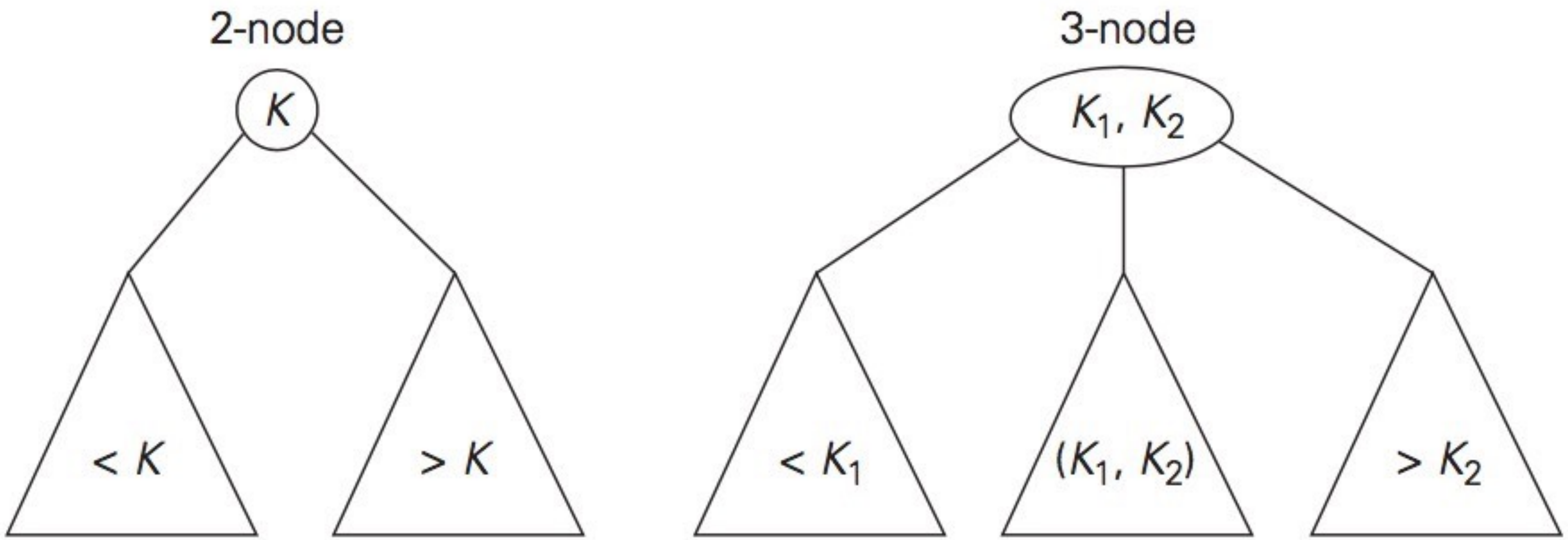


Heapsort

- ★ Stage 1 (heap construction):
  Construct a heap for a given array with HeapBottomUP().
- ★ Stage 2 (maximum deletion):
  Apply the root-deletion operation n−1 times to the remaining heap.
  - ★ The largest value (root) will be deleted first.
  - ★ Array elements are eliminated in decreasing order.
- ★ Since an element being deleted is placed last, the resulting array will be exactly the original array sorted in an increasing order.

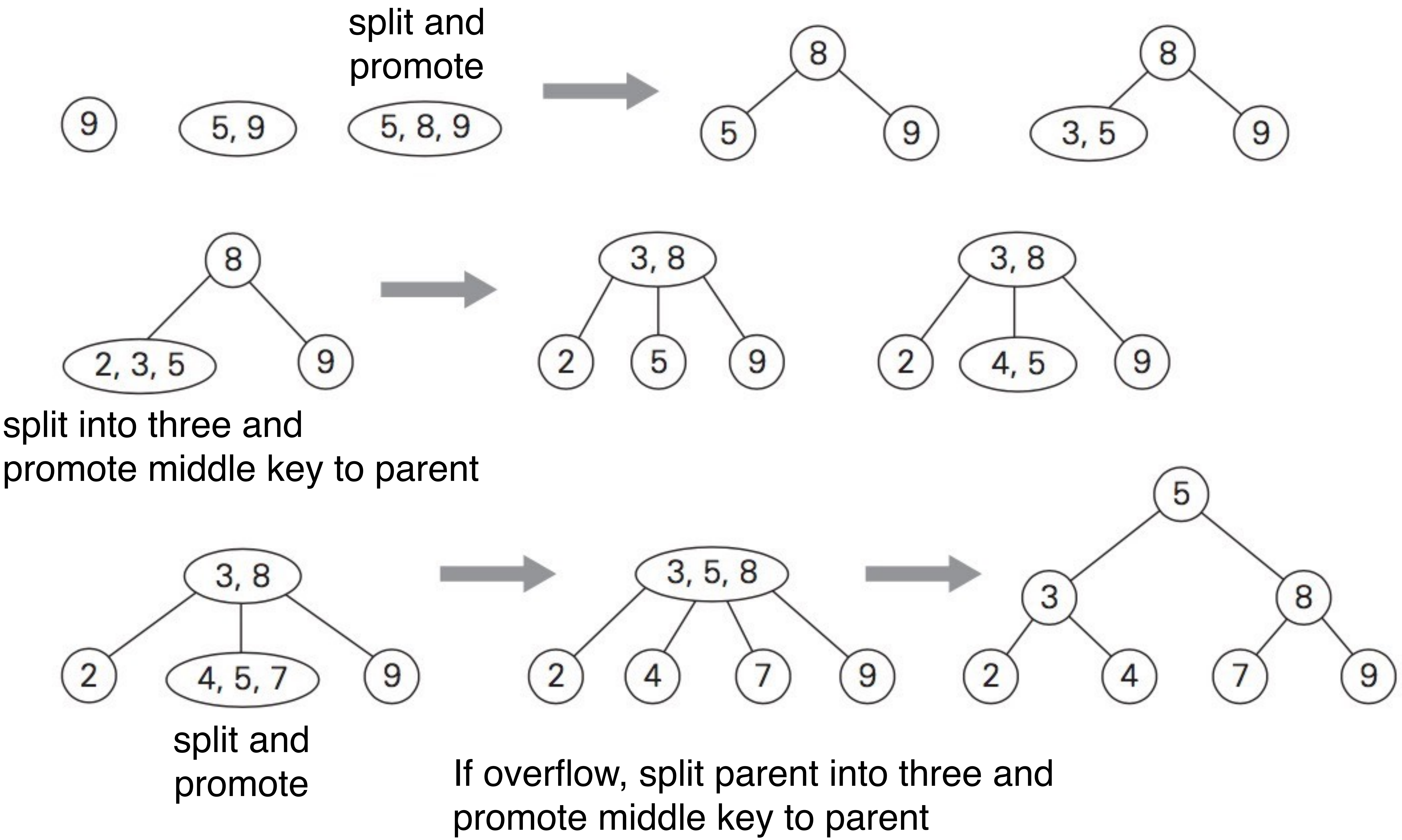
2-3 Tree

- A 2-3 tree is a search tree that
  - may have 2-nodes and 3-nodes
  - height-balanced (all leaves are on the same level)



2-3 Tree

Sequence Inserted: 9, 5, 8, 3, 2, 4, 7



Other Applications of Trees

- Red-Black Tree
- B Tree
- 2-3 Tree

ASCII Code

- ASCII characters use ***fixed-length codes*** (7 bits)
- For a textfile with 1000 characters, what is the file size?
- Suppose the file contains only 7 unique characters 'A' to 'G'
  - Can we use 3 bits instead of 8 bits?
  - What is the file size and ***compression ratio*** now?

Code	Char	Code	Char	Code	Char	Code	Char	Code	Char	Code	Char
32	[space]	48	0	64	@	80	P	96	`	112	p
33	!	49	1	65	A	81	Q	97	a	113	q
34	"	50	2	66	B	82	R	98	b	114	r
35	#	51	3	67	C	83	S	99	c	115	s
36	$	52	4	68	D	84	T	100	d	116	t
37	%	53	5	69	E	85	U	101	e	117	u
38	&	54	6	70	F	86	V	102	f	118	v
39	'	55	7	71	G	87	W	103	g	119	w
40	(	56	8	72	H	88	X	104	h	120	x
41	)	57	9	73	I	89	Y	105	i	121	y
42	*	58	:	74	J	90	Z	106	j	122	z
43	+	59	;	75	K	91	[	107	k	123	{
44	,	60	<	76	L	92	\	108	l	124	|
45	-	61	=	77	M	93	]	109	m	125	}
46	.	62	>	78	N	94	^	110	n	126	~
47	/	63	?	79	O	95	_	111	o	127	[backspace]

Huffman Code

- Can do better by using ***Prefix-free codes*** like ***Huffman Code***

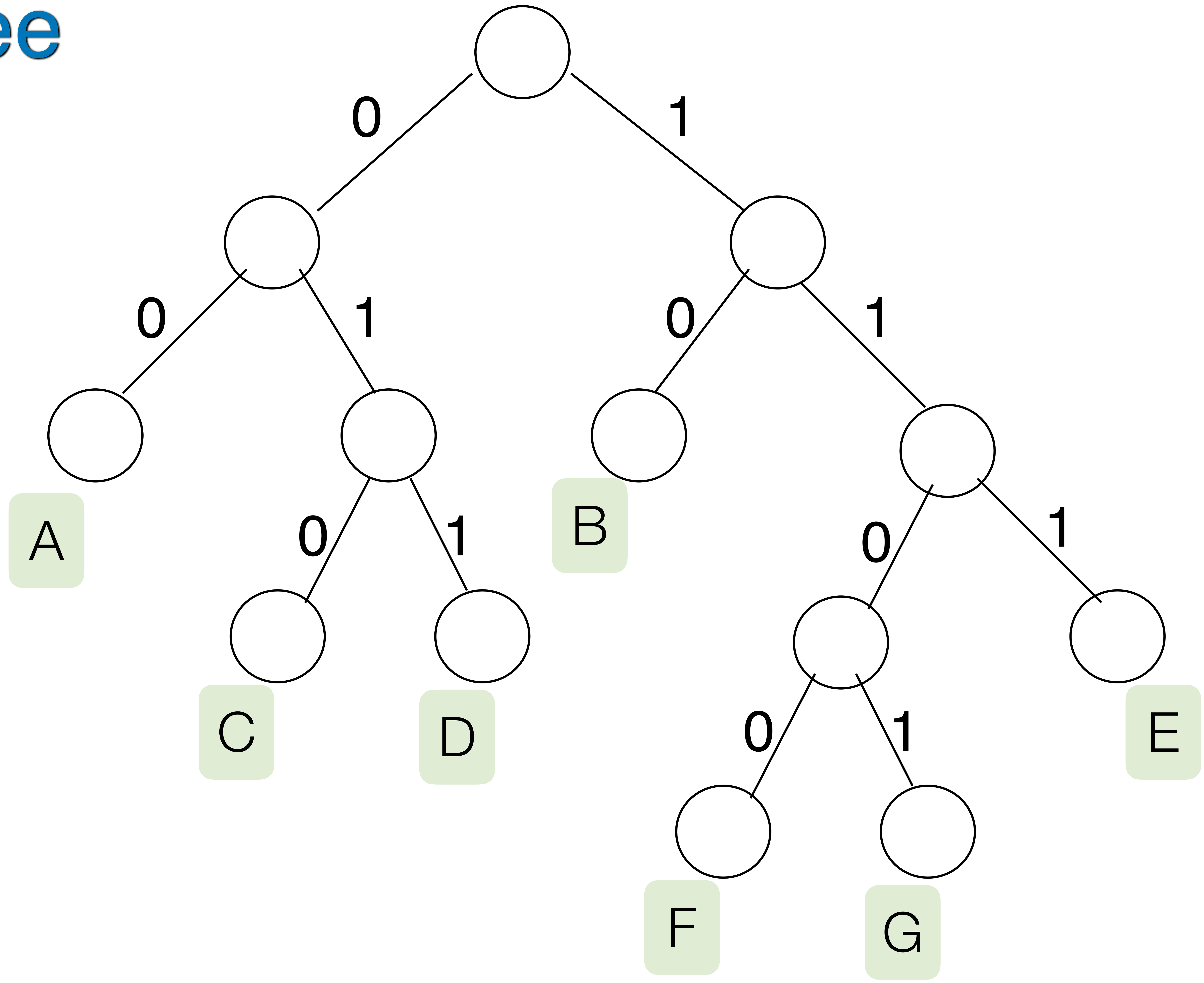
Symbol	Codewords
A	01
B	11
C	000
D	001
E	101
F	1000
G	1001

No codeword is a prefix of another codeword.

Average bits per symbol $\sum_{i=1}^{n} l_i p_i$

Huffman Tree

David Huffman (1952)

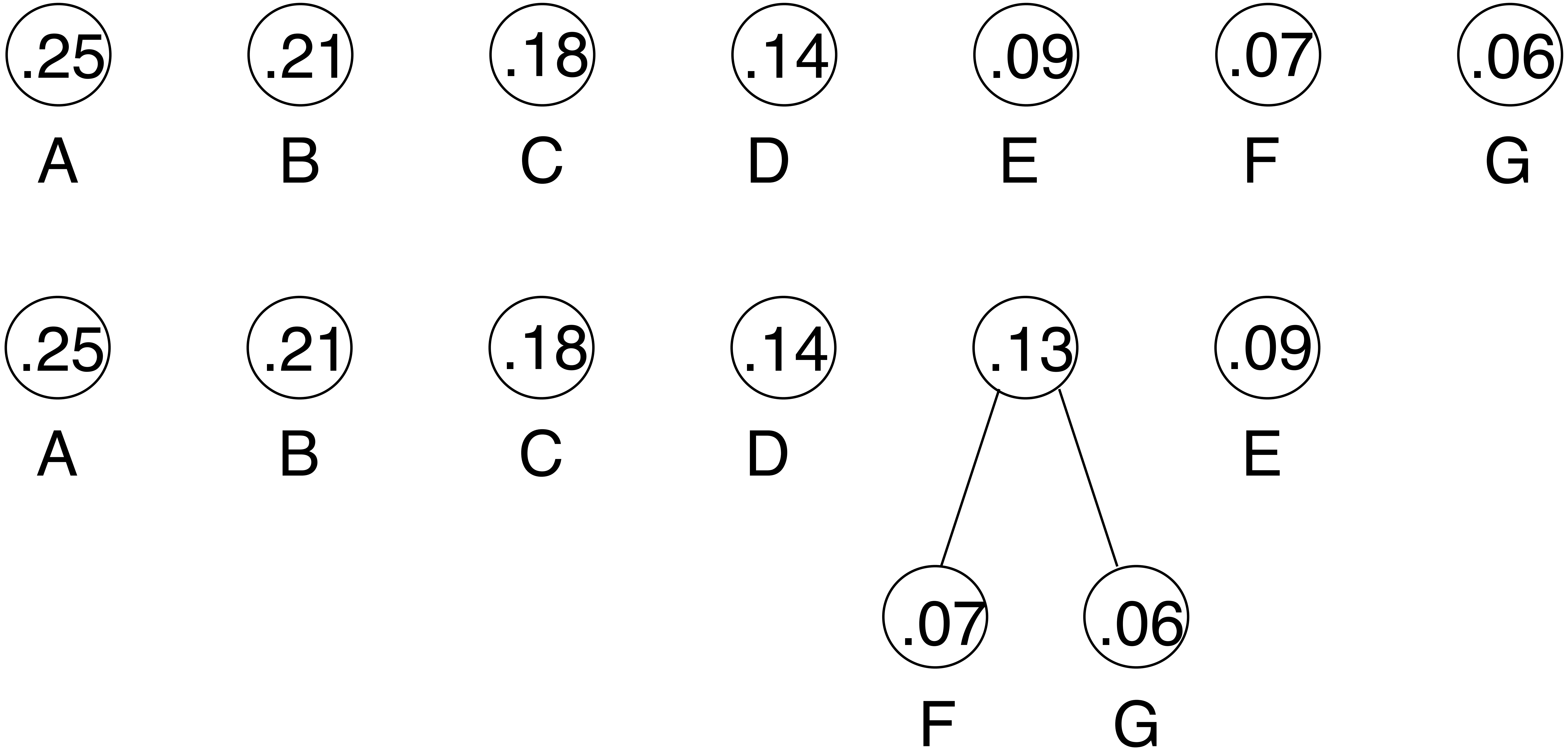


Huffman Algorithm

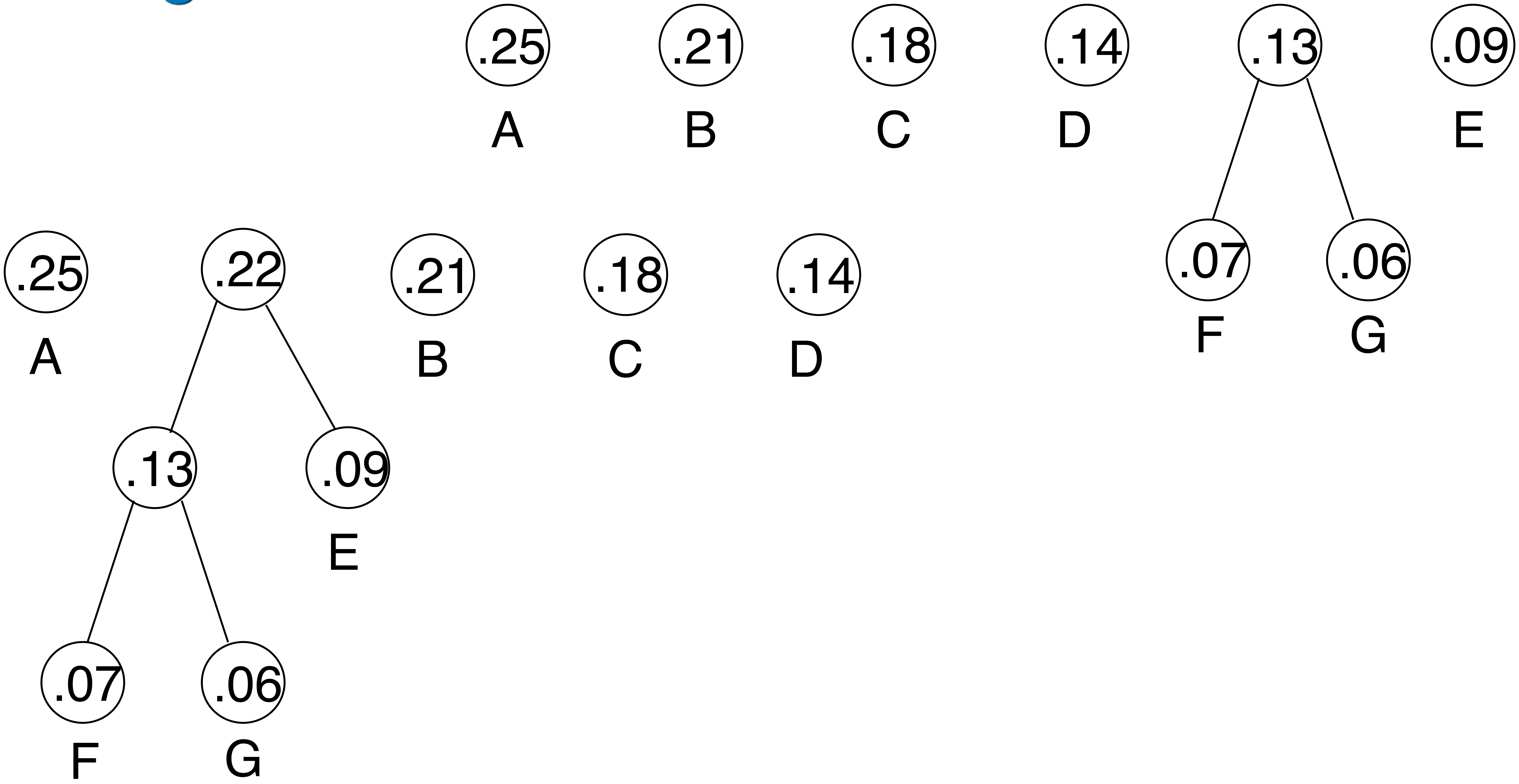
- Greedy algorithm to construct a Huffman tree of n symbols
  - Start from n single-node trees and combine them to a single tree
  - Generate short/long codewords to high/low frequency symbols

Symbol	Frequency
A	0.25
B	0.21
C	0.18
D	0.14
E	0.09
F	0.07
G	0.06

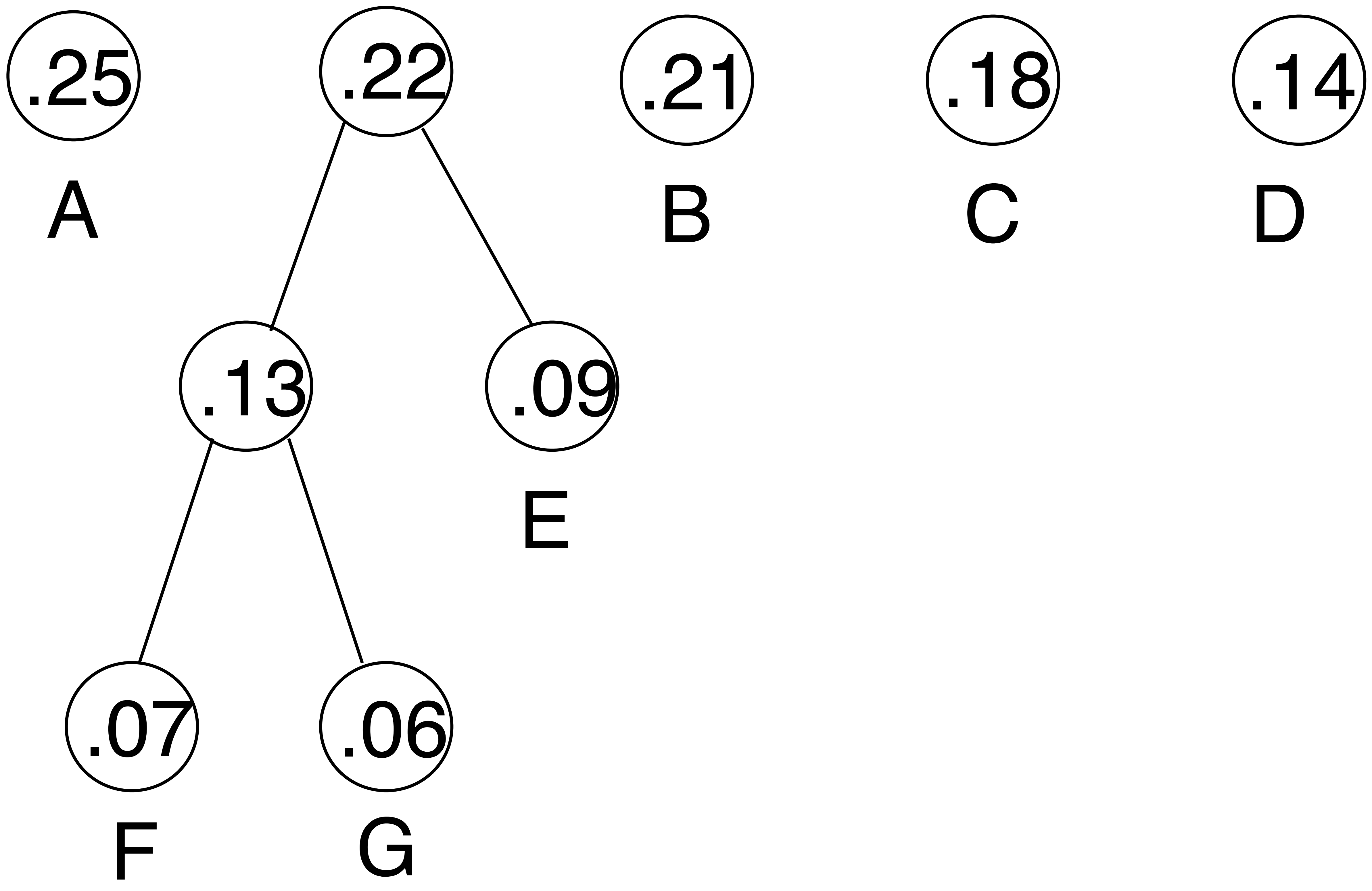
Huffman Algorithm

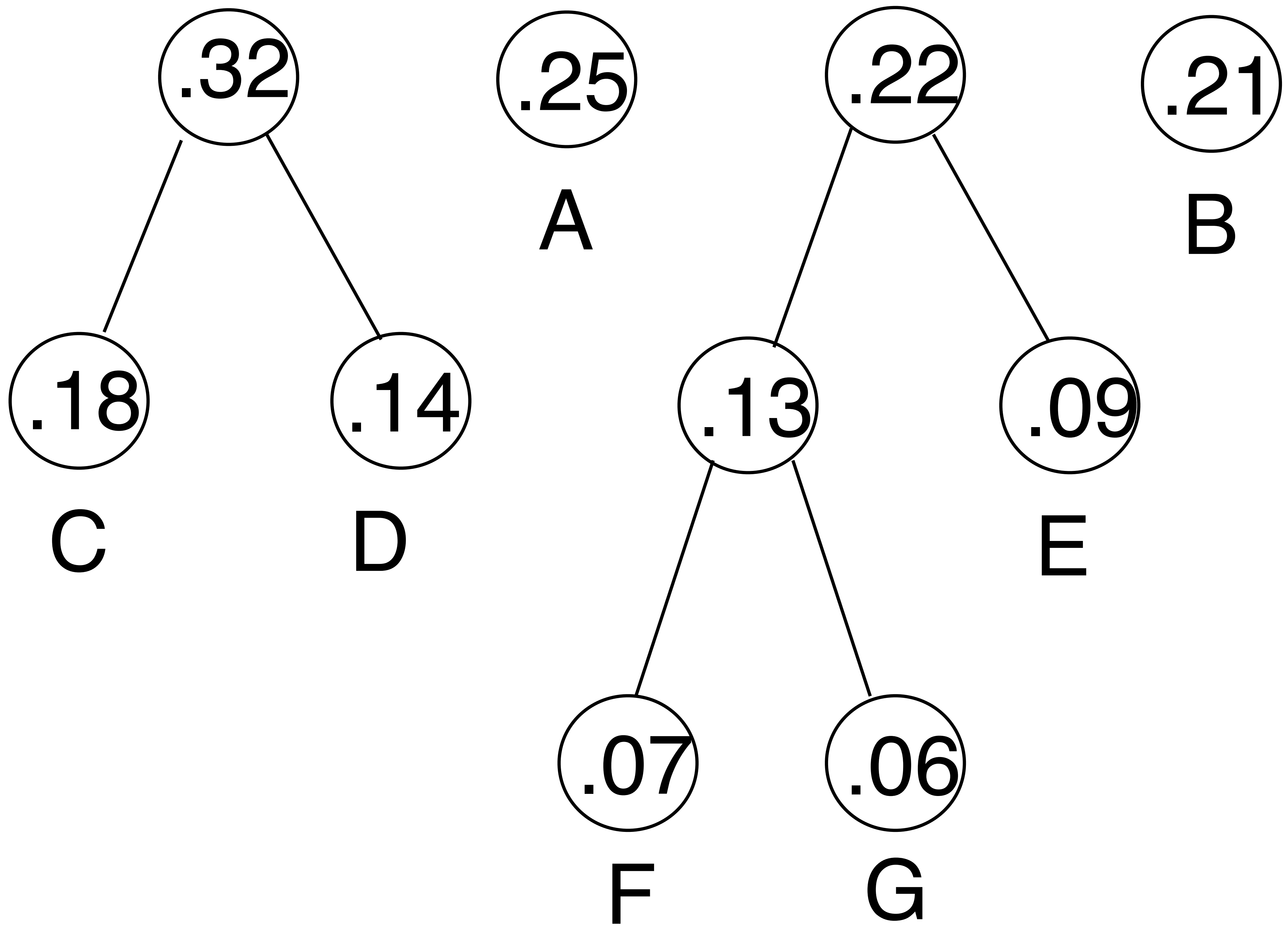


Huffman Algorithm



Huffman Algorithm





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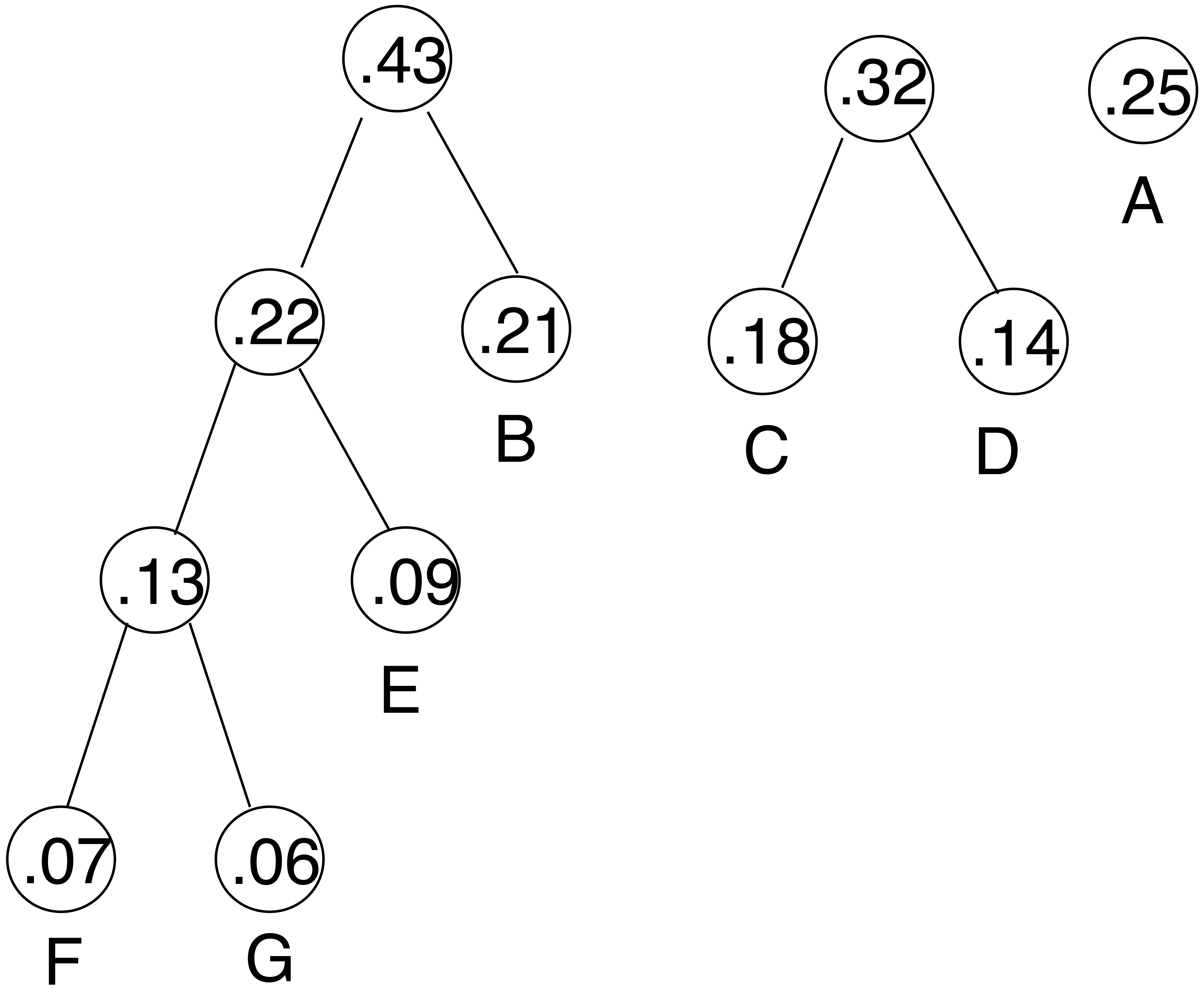

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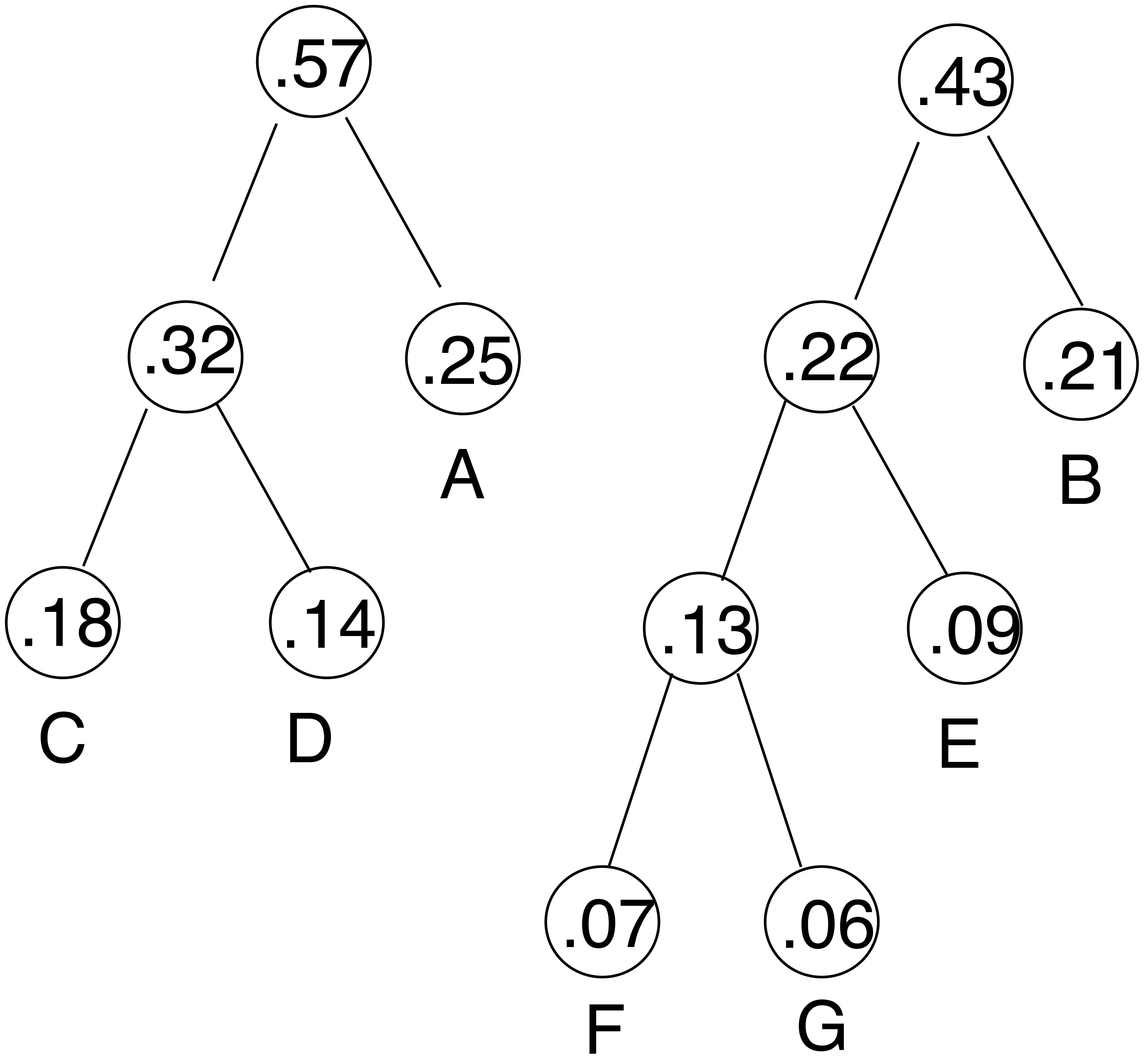
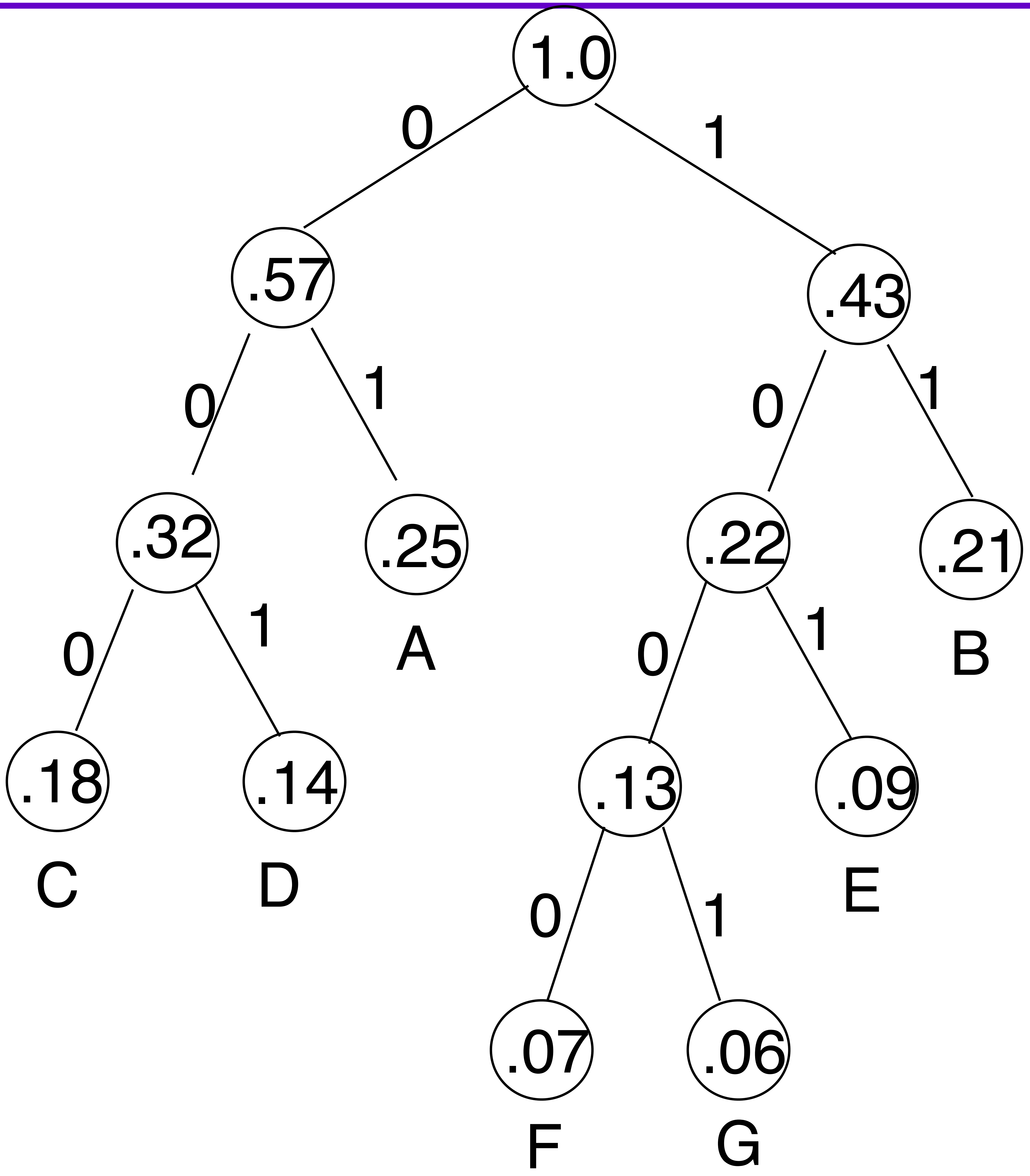


Result



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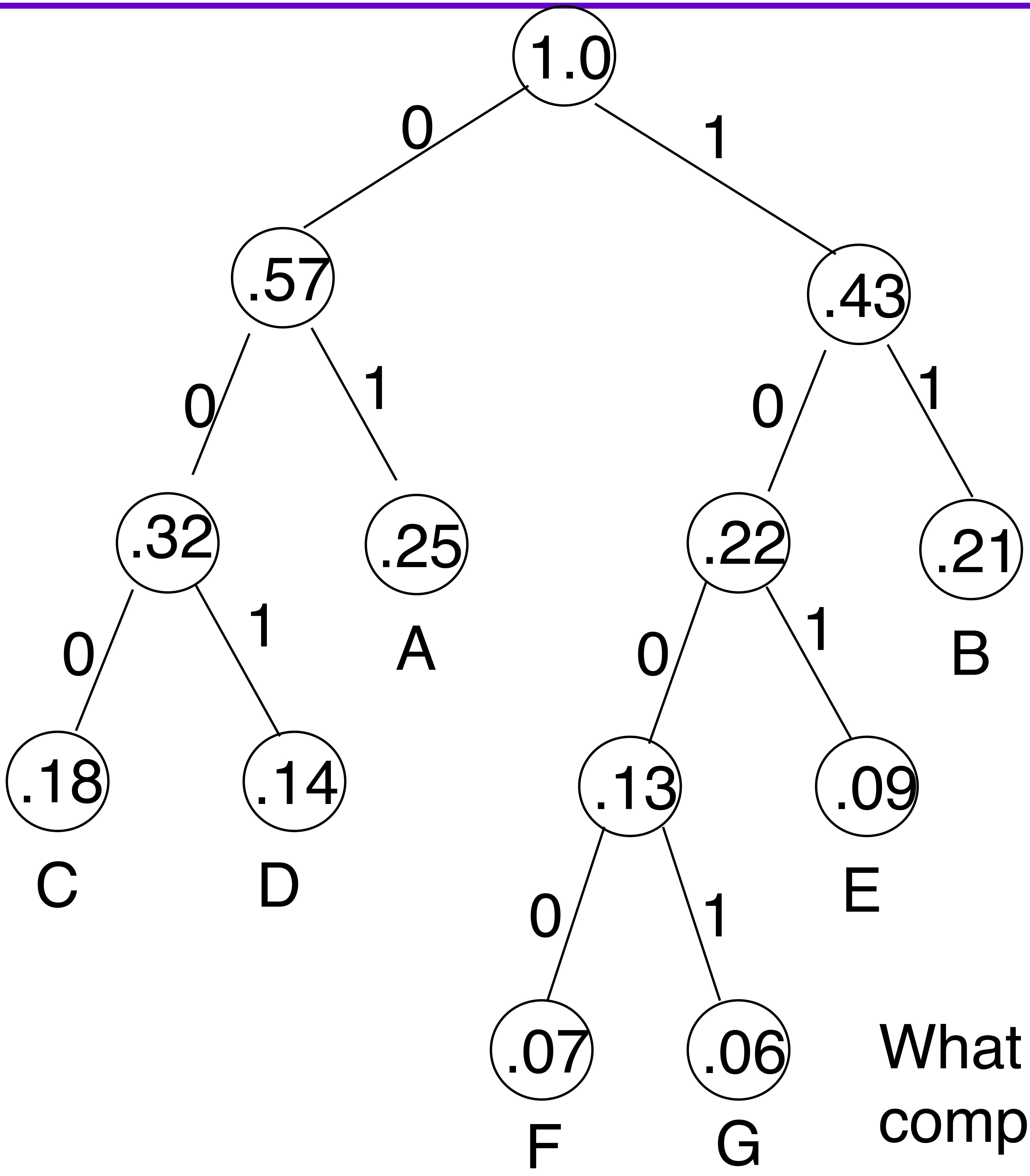


What is the best-case compression ratio for Huffman code?

Note on Huffman Algorithm

- ★ If you sort the initial trees from smallest to largest weight, a different huffman tree will be generated.
- ★ The same huffman tree must be used for both encoding and decoding Huffman tree

Huffman's Algorithm

Step 1 Initialize n one-node trees and label them with the symbols of the alphabet given. Record the frequency of each symbol in its tree's root to indicate the tree's ***weight***. (More generally, the weight of a tree will be equal to the sum of the frequencies in the tree's leaves.)

Step 2 Repeat the following operation until a single tree is obtained. Find two trees with the smallest weight (ties can be broken arbitrarily, but see Problem 2 in this section's exercises). Make them the left and right subtree of a new tree and record the sum of their weights in the root of the new tree as its weight.

Huffman's Algorithm: In-Class exercise

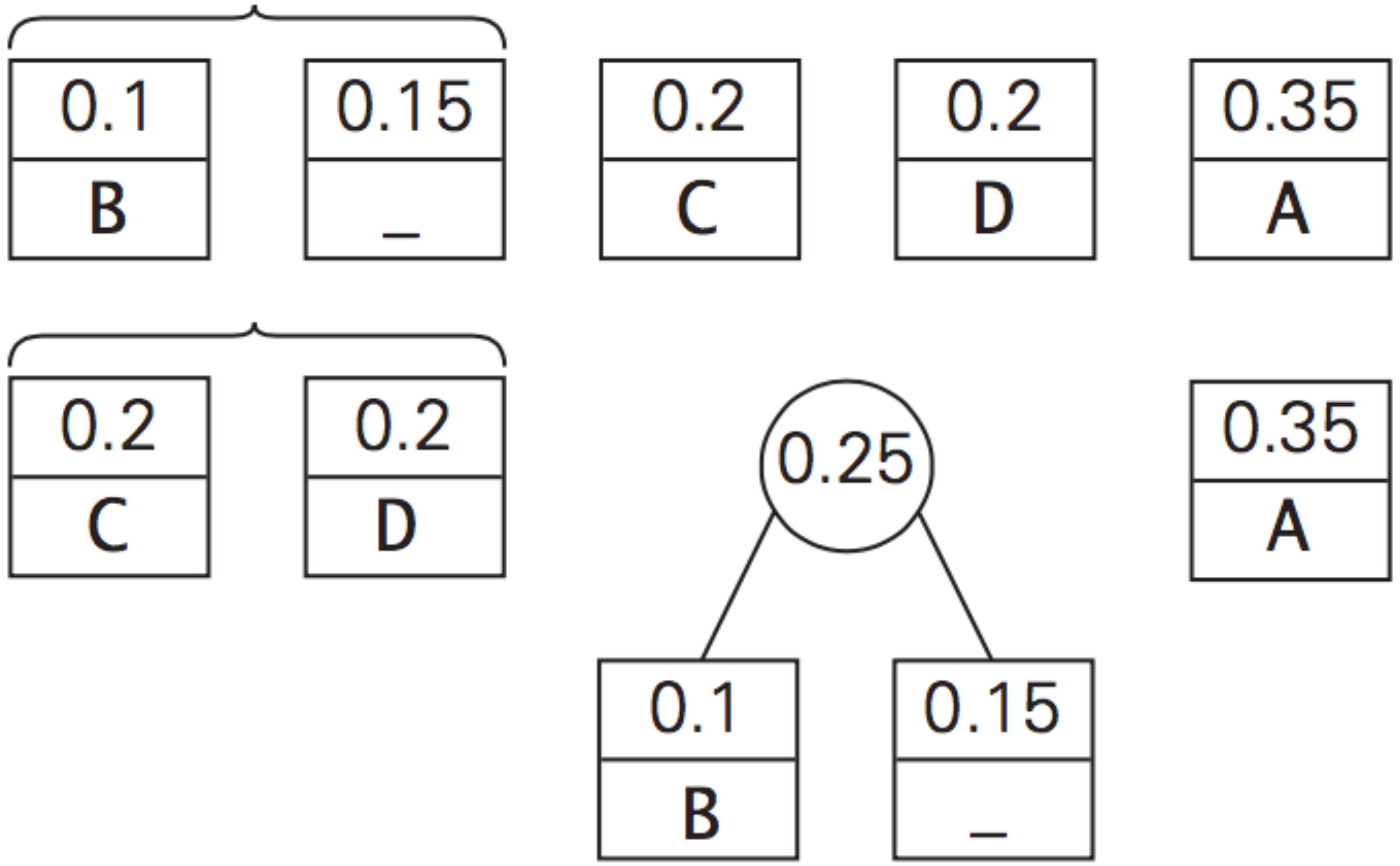
- Create a Huffman tree and the resulting codewords from the following occurrence frequencies

symbol	A	B	C	D	_
frequency	0.35	0.1	0.2	0.2	0.15

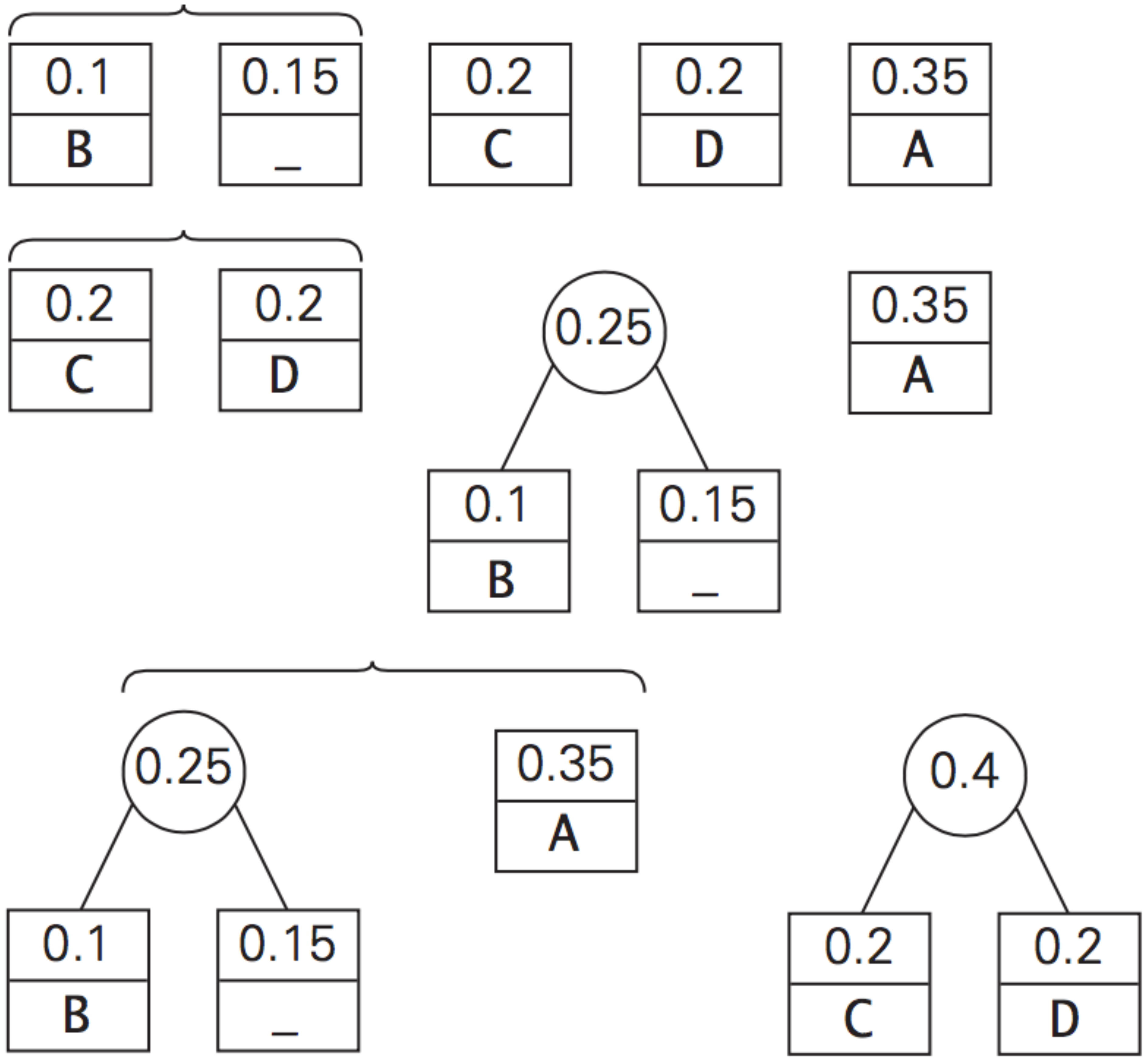
  - What are the huffman codes for this input?
  - What is the average number of bits per symbol?

Huffman’s Algorithm: In-Class exercise

Greedy Technique



Huffman's Algorithm: In-Class exercise



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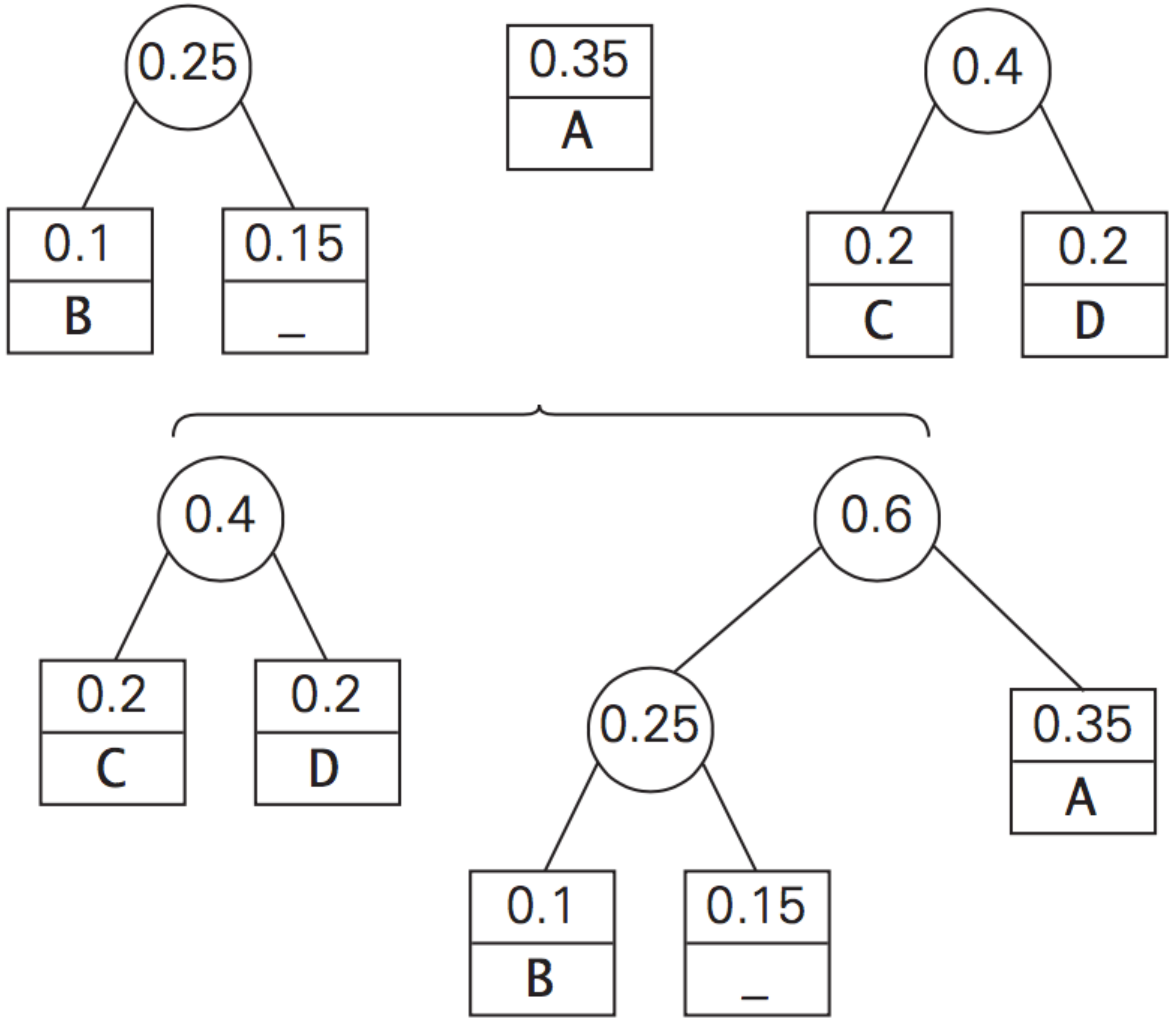

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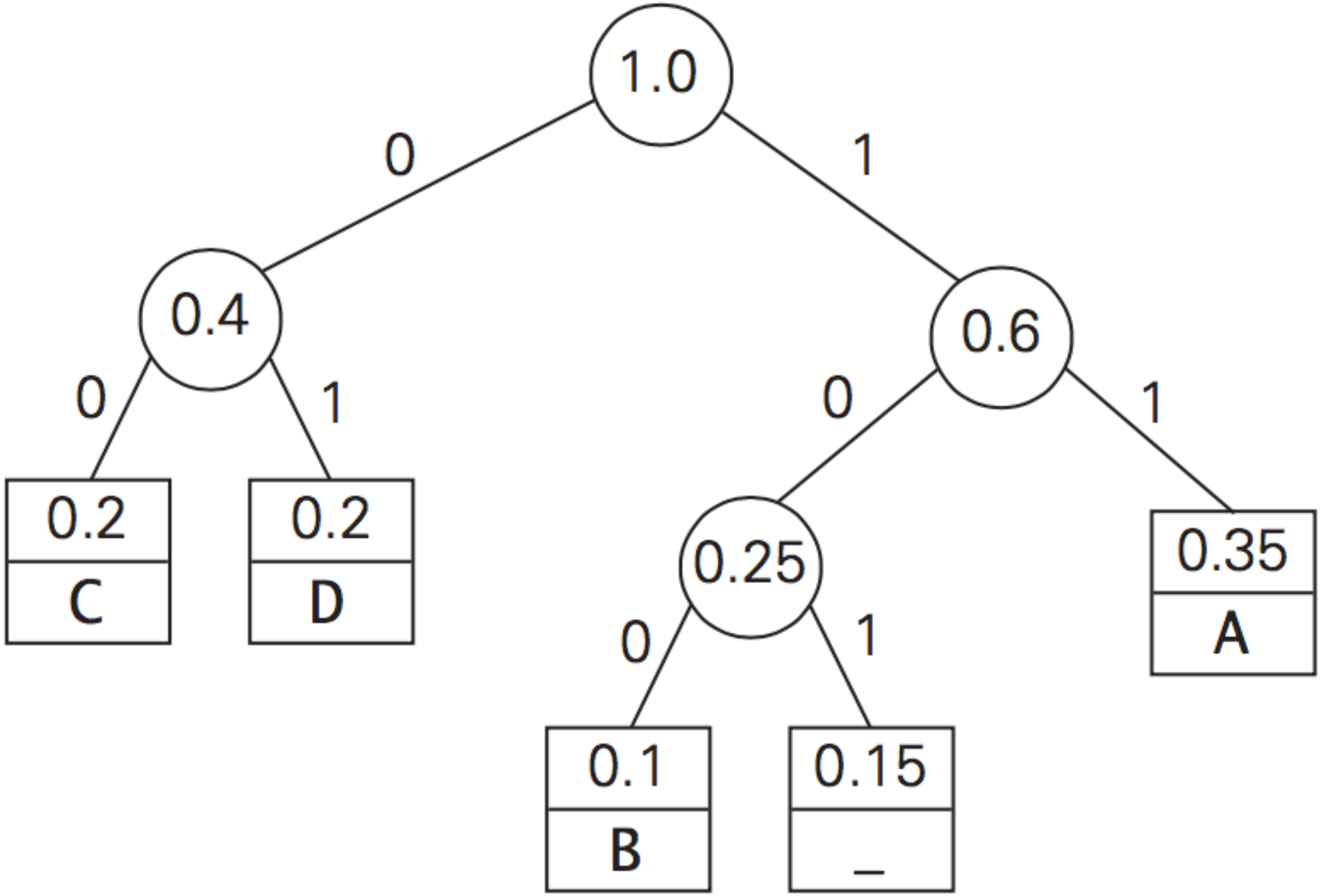

Huffman's Algorithm: In-Class exercise

The resulting codewords are as follows:

symbol	A	B	C	D	_
frequency	0.35	0.1	0.2	0.2	0.15
codeword	11	100	00	01	101

Hence, DAD is encoded as 011101, and 10011011011101 is decoded as BAD_AD.

With the occurrence frequencies given and the codeword lengths obtained, the average number of bits per symbol in this code is

$$2 \cdot 0.35 + 3 \cdot 0.1 + 2 \cdot 0.2 + 2 \cdot 0.2 + 3 \cdot 0.15 = 2.25.$$

Questions and Answers