1 Encryption

Consider the following PRP:

	000	001	010	011	100	101	110	111	m
000	011	001	111	010	000	101	110	100	
001	101	110	010	000	111	100	001	011	
010	001	110	100	111	011	010	000	101	
011	101	011	100	111	110	000	010	001	
100	001	101	110	010	100	011	000	111	
101	000	001	010	011	100	110	101	111	
110	100	000	011	101	111	001	010	110	
111	110	101	001	011	111	000	010	100	
k									

1.1

Check that this is a PRP (in particular, check that $F(k, x)$ is a permutation for any fixed key k).

1.2

Use the to encrypt the following messages:

- `100` using CTR with IV 000 and key 010
  Proof. [SOLUTION: `011`] □
- `100` using CBC with IV 000 and key 010
  Proof. [SOLUTION: `110`] □
- `100` using OFB with IV 000 and key 010
  Proof. [SOLUTION: `011`] □
- `011 010 101 100` using CTR with IV 100 and key 010
  Proof. [SOLUTION: `101 000 110 101`] □
- `011 010 101 100` using CBC with IV 100 and key 010
  Proof. [SOLUTION: `001 100 010 011`] □

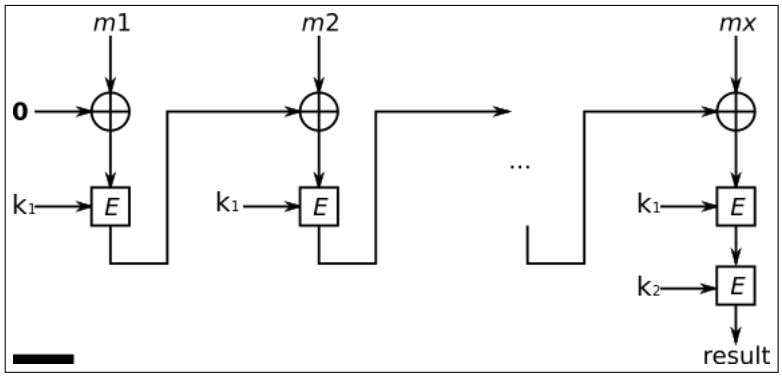
- `011 010 101 100` using OFB with IV 100 and key 010
  Proof. [SOLUTION: `101 001 001 010`] □
- `100 010 101 100` using CTR with IV 100 and key 010
  Proof. [SOLUTION: `010 000 110 101`] □
- `100 010 101 100` using CBC with IV 100 and key 010
  Proof. [SOLUTION: `111 010 001 010`] □
- `100 010 101 100` using OFB with IV 100 and key 010
  Proof. [SOLUTION: `010 001 001 010`] □
- `011 010 101 100` using CTR with IV 010 and key 010
  Proof. [SOLUTION: `111 110 011 110`] □
- `011 010 101 100` using CBC with IV 010 and key 010
  Proof. [SOLUTION: `010 111 100 111`] □
- `011 010 101 100` using OFB with IV 010 and key 010
  Proof. [SOLUTION: `111 100 110 000`] □
- `011 010 101 100` using CTR with IV 100 and key 100
  Proof. [SOLUTION: `111 110 010 011`] □
- `011 010 101 100` using CBC with IV 100 and key 100
  Proof. [SOLUTION: `111 100 111 110`] □
- `011 010 101 100` using OFB with IV 100 and key 100
  Proof. [SOLUTION: `111 110 001 000`] □

2

Pick a partner (someone new) and:

- Agree on a 3-bit key and padding scheme.
- Ask them to send you a message that has between 7 and 14 bits, using CBC with a random IV.

Recall that CBC-MAC is like CBC encryption, except with a constant IV and one extra PRP application at the end.



3

Calculate the MAC of `011 010 111 000` using CBC-MAC with keys ($k_1 = 010, k_2 = 011$).

Proof. [SOLUTION: 111 using the old homework.] □

4 CBC-MAC IV

Suppose the IV for CBC-MAC were random instead of 0. Would this help with security? Why/why not?

Proof. [SOLUTION: In theory, it would allow you to send the same message multiple times without giving away what the message is.

This is nice, but:

- It makes the message signature longer if you need to include the IV.
- MACs are usually used to encrypted semantically secure cipher texts, which should be unique.

] □

5

The diagram above uses k_2 to encrypt the last block, instead of reusing k_1. What would happen if we allowed the same key to be used?

Proof. [SOLUTION: Let $\overline{0}$ be the block of all zeros.
Ask for a signature

$$s = Sign\Big(k, \overline{0}\Big)$$

Output the following 3-block message

$$\overline{0} \,\Big\|\, \overline{0} \,\Big\|\, s$$

with the signature s.
Since this is a signature on a message we haven't seen, this is a forgery.
The "trick" is that $E(k, E(km))$
] □

6 MAC Security Game

The idea behind the MAC security game is that:

- The adversary can ask for signatures on as many messages as (s)he wants, one at a time
- The challenger replies with the signature on that message (using the secret key k).
- The adversary wins if they output a message and a signature (m, s) so that $Verify(k, m, s)$. (This is *different* from all the other security games we've seen so far).

Draw the diagram for this security game, and write the definition of advantage.

7

Do the same as problem 2, but also agree on keys k_1, k_2 for CBC-MAC and include the MAC with the message.

8 MAC Forgery

Suppose we didn't do the last encryption at the end, and allowed messages of any length to be encrypted. Show that we would lose security. In particular, show that someone with two valid message-signature pairs (using the same key[s]):

- (m, s)
- (m', s')

can create a new message and a signature to go with it (m'', s'') that verifies as valid.

(Hint 1: First, try this with the assumption that m and m' are one block each.)

(Hint 2: Try to place m and m' together, and use the output of the MAC to produce the same IVs during the CBC-MAC of the m' portion as in the original m' signature.)

Proof. [SOLUTION:
The signature $s = Sign(m, s)$ is the last block of the encryption of m.
Change the first block of m' to $m'[0] \oplus s$ to produce m^*

$$m^* = (m[0] \oplus s) \Big\| m[1] \Big\| m[2] \Big\| ...$$

Consider the encryption of the following:

$$m' = m \Big\| m^*$$

It will have the signature s', so this allows us to produce a forgery.
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