

Time Series Analysis

Homework Assignment #3

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Problems

1. **Programming project.** This is an **open ended project** whose purpose is to analyze causal relationships between the VIX levels and realized volatility. Your solution to this project should not be submitted to the course TAs. Instead, your homework team will give a 15 min in class presentation on October 5.

Consider the last 20 years worth of *daily returns* of the S&P 500 index and *daily levels* of the volatility VIX index. As usual, the daily return of S&P 500 on day t is calculated as $L(t)/L(t-1) - 1$, where $L(t)$ is the close of S&P 500 on day t . For the sake of concreteness, realized volatilities of returns are defined as ordinary standard deviations of the 21 most recent daily returns.

The data is available on Yahoo Finance and WRDS (if you have access to Bloomberg, the symbols for these indices are SPX Index, and VIX Index, respectively).

- (i) The VIX index is supposed to forecast future 1-calendar-month (= 21 business days) realized volatility of the S&P 500 index. Your goal is to carry out a study validating or disproving this expectation.

- (ii) The first natural question that arises is whether there is a causal relation between the (downward) move of the index and the (upward) move of VIX. One way to approach this question is to fit the data to the model (42) of Lecture Note #3, and carry out a test for Granger causality between the two components of this time series. Can you decide whether VIX Granger causes S&P 500 or the other way around?
 - (iii) Another question is how well does VIX predict future 1 month realized volatility. Set up a testing framework and evaluate its out of sample performance. As usual, beware of look ahead data snooping.
 - (iv) You can also pose and answer other related questions.
 - (v) For this project, you can use whatever methodologies you deem appropriate. By way of suggestion I am including two recent review papers on time series causality, but you are not limited to the methodologies discussed in these papers.
2. Prove formula (60) for the kurtosis of the $GARCH(1, 1)$ model stated in Lecture Notes #2.
 3. Derive formulas (8) and (9) in Lecture Notes #3.
 4. Use the historical data from Problem 1 to test the predictive power of the $GARCH(1, 1)$ model. Using a rolling window of training data (select its length!), estimate the parameters of the model through the range of available data and use formulas (64) - (65) for calculating predicted values of the volatility. Evaluate the quality of the model forecasts.

This assignment (Problems 2 - 4) is due on September 28