# Time Series Analysis

## Homework Assignment #4

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### Problems

1. (i) Prove formulas (47) and (48) in Lecture Notes #3.
   (ii) Calculate the KL distance between two binomial distributions: $\mathrm{Bin}(n,p)$ and $\mathrm{Bin}(n,q)$ (express the result in a simple form).

2. Show that the innovations $v_t = Y_t - \mathsf{E}(Y_t|Y_{1:t-1})$ in the linear state space model (equation (15) of Lecture Notes #4) are mutually independent for different values of $t$.

3. Download 5 years worth of most recent daily levels of TSLA and S&P 500 (use adjusted daily closes). Let $r_t$ and $m_t$ denote the daily returns on TSLA and SPX, respectively. Consider the following CAPM model:

$$r_t = \alpha_t + \beta_t m_t + u_t, \tag{1}$$

   where $u_t$ are the residuals. The parameters $\alpha_t$ and $\beta_t$ are estimated daily by means of a linear regression based on a 3 month (63 days) rolling window. You are interested in observing $\alpha_t$ as a trading signal.

   (i) We will formulate this model as the following linear state space model. Set $X_t = \alpha_t$, $Y_t = r_t - \beta_t m_t$; then

$$\begin{aligned} X_{t+1} &= X_t + \varepsilon_{t+1}, \\ Y_t &= X_t + u_t. \end{aligned}$$

Assume that the variances of $\varepsilon_t$ and $u_t$ are constant, and treat the values of $\beta_t$ as known.

(ii) Construct and run the Kalman filter for this model. Plot the graph of the observed vs filtered values of $\alpha_t$. Plot the graph of the smoothed values of $\alpha_t$. Evaluate the predictive power of this model.

**This assignment is due on October 5**