Statistics 153 Homework 6

Due by 11:59 pm, Monday, Dec 6 , 2021

1. **Periodogram vs. Spectral Density**

    Consider the Lynx data from the *astsa* library.

    a. Plot the Lynx time series. Does this look like something we could model with sinusoids?   *(5 point)*

    b. Plot the periodogram. The three highest spikes on the periodogram are for which values of j? Do you think these represent three separate sinusoids?   *(5 point)*

    c. The *mvspec()* function plots an estimate of the spectral density. Recall that this is effectively smoothing over the periodogram, except now the x-axis is not j but $\lambda = j/n$. Plot the empirical estimate of the spectral density. Now using *abline()*, add to this plot 3 red vertical lines at the λ values that match your j values from part b.   *(5 point)*

    d. Based on the figure you created in part c, how many sinusoids created the three significant periodogram spikes in part b? What does this say about "leakage"? It may help your argument to find the j values or λ values for the top of the spikes in part c's figure.   *(5 point)*

2. **Spectral Density of AR Processes**

    Let W_t be a white noise process with variance 1. Consider the AR(2) process:

    $$(1 - .9B^2)X_t = W_t$$

    a. Compute the transfer and power transfer functions associated with the AR polynomial $(1 - .9B^2)$. Also, compute the spectral density $f_X(\lambda)$.   *(20 points)*

    b. Plot the spectral density $f_X(\lambda)$. Do you think X_t will oscillate? If so, what period?   *(10 point)*

    c. Simulate X_t for 50 time steps. Is the simulation consistent with your answer to (b)?   *(10 point)*

    d. Consider the linear filter with weights $a_{-1} = a_0 = a_1 = \frac{1}{3}$; $a_j = 0$ otherwise. Let Y_t be the time series obtained by applying this filter to X_t. Compute the transfer function, power transfer function, and spectral density $f_Y(\lambda)$.   *(20 points)*

    e. Plot the spectral density $f_Y(\lambda)$. Do you think Y_t will oscillate? If so, what period?   *(10 point)*

    f. Simulate Y_t by applying the filter from (d) to your simulated X_t from (c). Is the simulation consistent with your answer to (d)?   *(10 point)*