Hw3

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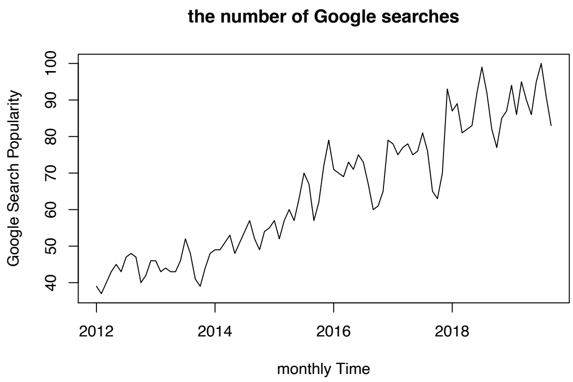
10/5/2021

Computer Exercises

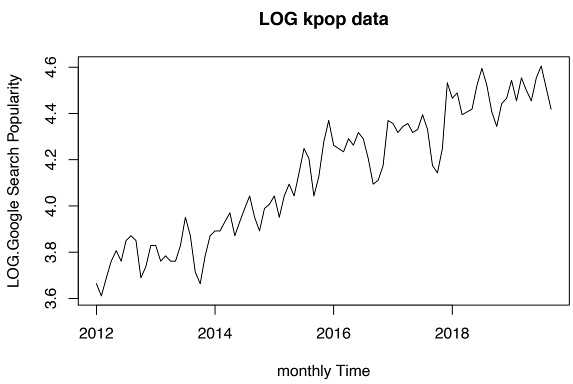
1a variance stabilizing transform

Assume the Variance is in quadratic form of mean, $Var(Xt) = C\mu_t^2$, choose $F() = log()$.

```
dataset = read.csv("/Users/li/Desktop/STAT153/HW3/kpop.csv")
t = dataset$timestamp
kpop = dataset$kpop
plot(t,kpop,type = "l", ylab = "Google Search Popularity", xlab = "monthly Time", main = "the number of
```

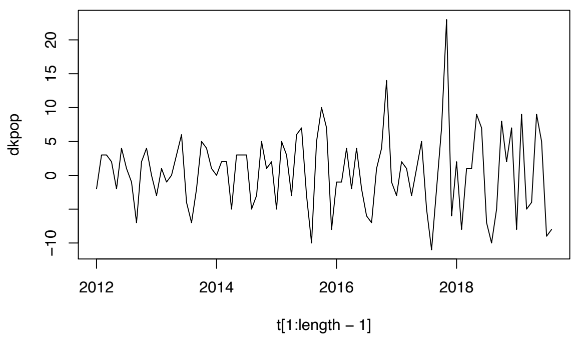



```
kpop.log = log(kpop) # assume Var(Xt)=C*ut^2, choose F()=log().
plot(t,kpop.log,type = "l", ylab = "LOG.Google Search Popularity", xlab = "monthly Time", main = "LOG k
```

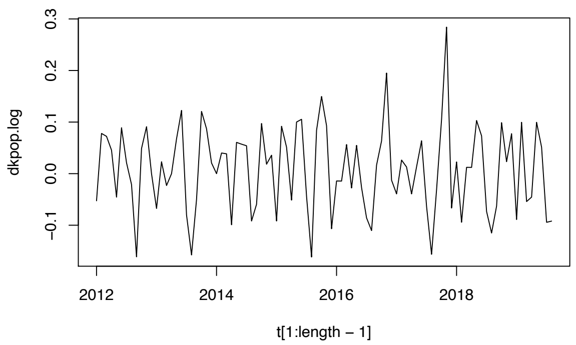


1b difference

```r
length = length(kpop)
dkpop = kpop[2:length]-kpop[1:length-1]
dkpop.log =kpop.log[2:length]-kpop.log[1:length-1]
plot(t[1:length-1],dkpop,type = "l" )
```




```r
plot(t[1:length-1],dkpop.log,type = "l")
```



The difference of log(kpop) is more plausibly stationary.

1c Forecast

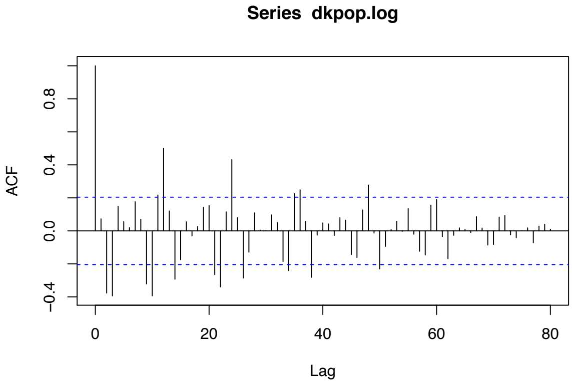
```r
#choose log(kpop) as basic data.
log.predict = kpop.log[length-1] + mean(dkpop.log)
predict = exp(log.predict+var(dkpop.log)/2)  #E(exp(x))=exp(EX)*exp(Var(X)/2)
print("The predict value Yt+1 is")
```

```
## [1] "The predict value Yt+1 is"
```

```r
print(predict)
```

```
## [1] 92.06665
```

```r
acf(dkpop.log,lag.max = 80)
```



1c Forecast It doesn't look like white noise because ACF shows it probably has periodic terms. So the result in 1c may be biased due to the lack of periodic term.

Theoretical exercises

2

2. (a) for $X_t = W_t - \delta W_{t-1}$

$\gamma'(h) = Cov(X_t, X_{t+h}) = Cov(W_t - \delta W_{t-1}, W_{t+h} - \delta W_{t+h-1}) =$ ~~for h>1~~ ~~h=1~~

$= \gamma(h) - \delta\gamma(h+1) - \delta\gamma(h-1) + \delta^2\gamma(h)$

$= (\delta^2+1)\gamma(h) - \delta(\gamma(h+1) + \gamma(h-1))$

$\delta = 0.4$

$\gamma_X(h) = 1.16\,\gamma(h) - 0.4[\gamma(h+1) + \gamma(h-1)]$

$\delta = 2.5$

$\gamma_Y(h) = 7.25\,\gamma(h) - 2.5(\gamma(h+1) + \gamma(h-1))$

(b) $\rho_X(h) = \dfrac{\gamma_X(h)}{\gamma_X(0)} = \dfrac{1.16\gamma(h) - 0.4(\gamma(h+1) + \gamma(h-1))}{1.16\gamma(0) - 0.8\gamma(1)}$

$= \dfrac{2.9\gamma(h) - \gamma(h+1) - \gamma(h-1)}{2.9\gamma(0) - 2\gamma(1)}$

$\rho_Y(h) = \dfrac{\gamma_Y(h)}{\gamma_Y(0)} = \dfrac{7.25\gamma(h) - 2.5(\gamma(h+1) + \gamma(h-1))}{7.25\gamma(0) - \quad 5\gamma(1)}$

$= \dfrac{2.9\gamma(h) - \gamma(h+1) - \gamma(h-1)}{2.9\gamma(0) - 2\gamma(1)}$

(c) $X_t = W_t - 0.4W_{t-1}$

$= \theta_1(B)W_t$. $\quad \theta_1(B) = I - 0.4B$.

$Y_t = W_t - 2.5W_{t-1}$

$= \theta_2(B)W_t$. $\quad \theta_2(B) = I - 2.5B$

(d) For X_t: $\theta_1(x) = 0$. $1 - 0.4x = 0$. $x = 2.5 > 1$ X_t is Invertible.

For Y_t: $\theta_2(x) = 0$. $1 - 2.5x = 0$. $x = 0.4 < 1$ Y_t is not invertible

(e) $\dfrac{1}{\theta_1(B)} X_t = W_t$,

$(1 + 0.4B + (0.4B)^2 + \ldots) X_t = W_t$.

$\sum_{j=0}^{\infty} 0.4^j X_{t-j} = W_t$.

3. (a) $\phi(z) = 1 + \frac{1}{12}z^2 - \frac{7}{12}z = 0.$ $\quad z_1 = 3 \quad z_2 = 4$ casual & have unique stationary solution.

$\theta(z) = 1.$ invertible

(b) $(1-B)X_t = \frac{1}{2}(2-B)(1-B)W_t.$

$X_t = \frac{1}{2}(2-B)W_t.$ unique stationary solution & casual

$\theta(z) = 1 - \frac{1}{2}z.$ $\quad z = 2.$ invertible

(c) $\phi(z) = (1-\frac{1}{3})z = 0.$ $\quad z = 3.$ have unique stationary solution, & casual

$\theta(z) = 1 - z = 0,$ $\quad z = 1.$ not invertible,

4. choose (c). $(1-\frac{1}{3}B)X_t = (1-B)W_t$

(a)

$$X_t = \frac{1-B}{1-\frac{1}{3}B}W_t = (1-B)\left(\sum_{j=0}^{\infty}(\tfrac{1}{3}B)^j\right)W_t$$

$$= \left[\sum_{j=0}^{\infty}(\tfrac{1}{3}B)^j - \sum_{j=0}^{\infty}(\tfrac{1}{3})^j B^{j+1}\right]W_t$$

$$= \left[1 + \sum_{j=1}^{\infty}(\tfrac{1}{3}B)^j - 3\sum_{k=1}^{\infty}(\tfrac{1}{3})^k B^k\right]W_t$$

$$= \left[1 - 2\sum_{j=1}^{\infty}(\tfrac{1}{3}B)^j\right]W_t$$

(b) $\gamma(h) = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\theta_j\theta_h \operatorname{Cov}(W_{t-j}, W_{t+h-k})$ $\quad j = k-h$

$$= \sum_{j=0}^{\infty}\theta_j\theta_{j+h}\cdot\sigma_w^2$$

$$= \left[\theta_0\theta_h + \sum_{j=1}^{\infty}2(\tfrac{1}{3})^j\cdot 2(\tfrac{1}{3})^{j+h}\right]\sigma_w^2$$

$$= \left[-2(\tfrac{1}{3})^h + 4\sum_{j=1}^{\infty}(\tfrac{1}{3})^{2j+h}\right]\sigma_w^2$$

$$= \left[-2(\tfrac{1}{3})^h + 4\times(\tfrac{1}{3})^h\times\frac{1}{8}\right]\sigma_w^2 = -\frac{1}{2}\times(\tfrac{1}{3})^{h-1}\sigma_w^2$$

(c) $\rho(h) = \frac{\gamma(h)}{\gamma(0)} = (\tfrac{1}{3})^h$