Name: ____________________

Student ID number: ____________________

PRACTICE FINAL EXAM - FALL 2019

STAT 153 - Introduction to Time Series

December 16, 2019

General comments:

1. On the actual final, the formula sheet will be attached to the back of the exam, which we will all rip off before the exam begins. For this practice exam, the formula sheet will be posted on bCourses in the "Final Exam Preparation" folder by end of the workday on Friday 12/13/19.
2. The final exam will be timed at just under 3 hours (3 hours minus how long it takes us to pass out exams). However, the final exam was written with the intent to take less than 2 hours, so don't feel odd if you finish early!
3. This practice exam does not cover every single topic, so just because it's not on this practice exam doesn't mean it's not on the final. The intent of this practice exam is to give you practice at taking an exam in the style of the final.

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1. (a) Define (weak) stationarity.

   (b) Do all white noise processes follow weak stationarity? If yes, explain why. If no, provide a counter example.

   (c) Are all stationary processes considered white noise? If yes, explain why. If no, provide a counter example.

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2. Consider the time series $X_t = a + bt + ct^2 + cos(2\pi ft) + Z_t$, where $a, b, c \in R$, $f = 1/3$, and $\{Z_t\}$ is white noise.

   (a) $\{X_t\}$ is not stationary. Give an example of positive integers j, k that make $\{Y_t\}$ stationary, where $Y_t = \nabla_j \nabla_k X_t$.

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(b) What is the best predictor of X_n given $X_1, ..., X_{n-1}$?

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3. Consider the equation $X_t - X_{t-4} + 0.5X_{t-8} = Z_t$ where $\{Z_t\}$ is white noise.

   (a) Let $Y_t = X_{4t}$ (such that $Y_0 = X_0$, $Y_1 = X_4$, $Y_2 = X_8$, ...). Show that $\{Y_t\}$ is an AR(2) process.

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(b) What are the roots of the AR polynomial of Y_t? Is Y_t causal? Does it have a unique stationary solution? (If you were not able to complete part a, assume $\phi_1 = 1$ and $\phi_2 = -0.5$.)

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4. Show how the Yule Walker equations can estimate the parameters of an AR(1) model (ϕ, σ^2) with the sample ACVF values $\hat{\gamma}(0) = 3, \hat{\gamma}(1) = 2$.

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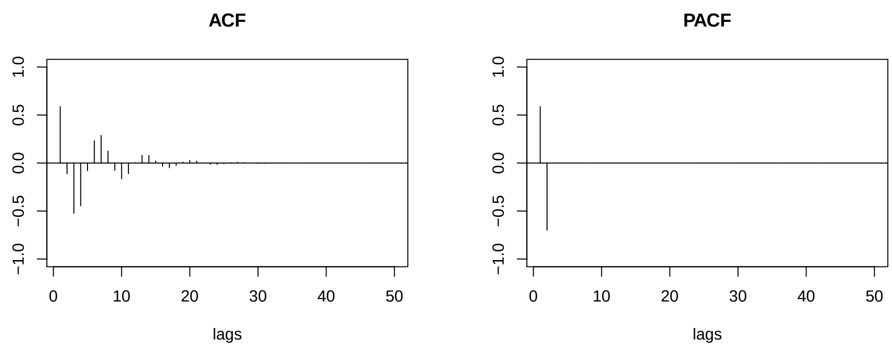
5. Below are the theoretical ACF plots and PACF plots for three different models. Note that there are no bars for lag 0 in either the ACF or PACF plots. Also note that these are simple MSARMA models

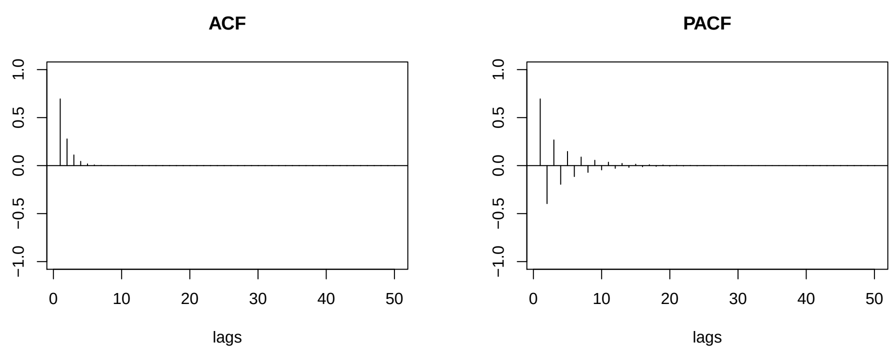
$$ARMA(p,q)x(P,Q)_S$$

such that $p + q + P + Q \leq \mathbf{2}$, so possibilities include AR(1), AR(2), ARMA(1,1), ARMA(1,0)x(0,1)$_{12}$, ARMA(0,0)x(1,1)$_{20}$, etc. Above each pair of plots in the blanks provided, write in the appropriate values of p, q, P, Q, and S, where S is the period length for the seasonal ARMA component. Let $S = 0$ if no seasonal component is present.

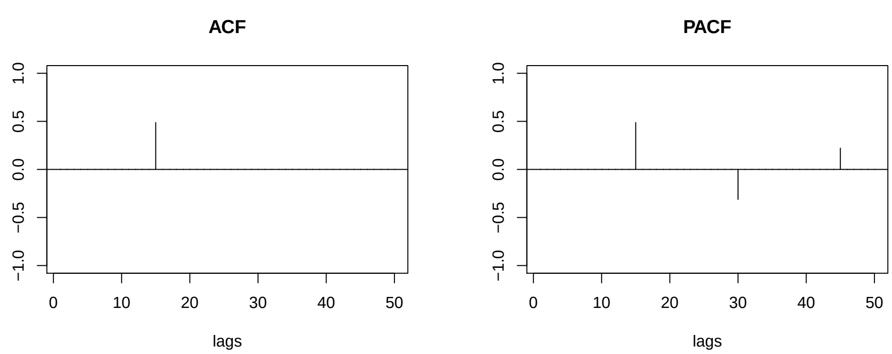
(a) ARMA(_____ , _____)x(_____ , _____)_____



(b) ARMA(_____ , _____)x(_____ , _____)_____



(c) ARMA(_____ , _____)x(_____ , _____)_____



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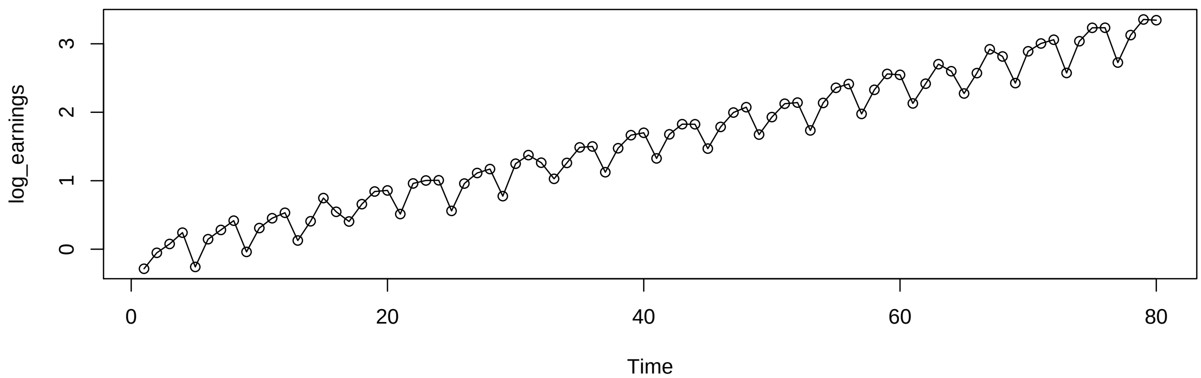
6. These are the quarterly earnings for Mom and Pop's Diner. We have already taken the log of the original data to stabilize the variance.



(a) How could you model the trend and seasonality of this time series, without using differencing? Write the suggested equation.

(b) How could you remove the trend and seasonality of this time series using differencing? Briefly explain why this removes the trend and seasonality.

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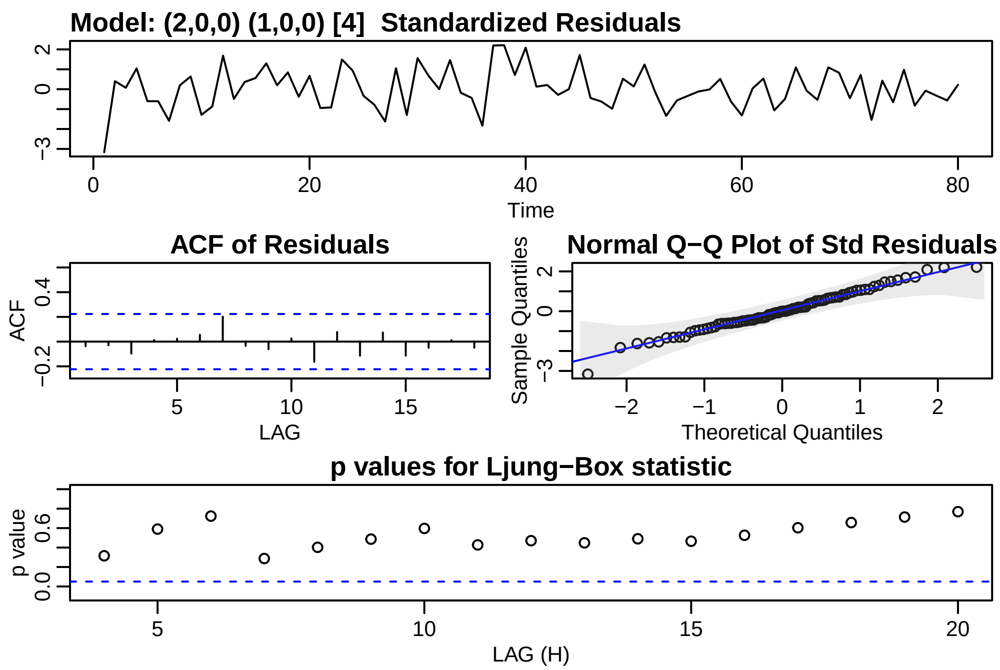
(c) So, instead, the data were de-trended by someone else, and they modeled the noise with auto.arima(): ARMA$(2,0)x(1,0)_4$. You have been taught by Professor Fisher that this is a good tool, but perhaps not the best final model, so you are a tad suspicious. Below are the sarima() diagnostic plots. Comment briefly on each plot. Then, conclude whether or not this model seems to fit the noise well.



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(d) Below are some model fit diagnostics of the ARMA$(2,0)x(1,0)_4$ compared to the client's preference for no noise model (i.e. ARMA(0,0)). Defend your preference for the more complicated model to your client using the information below, and be sure to distinguish in-sample versus out-of-sample. Note that RMSE is the square root of the mean squared error.

Noise Model	AIC	AICc	BIC	Cross-validated RMSE
ARMA$(2,0)x(1,0)_4$	-1.94	-1.93	-1.79	0.017
ARMA$(0,0)$	-1.83	-1.83	-1.77	0.048

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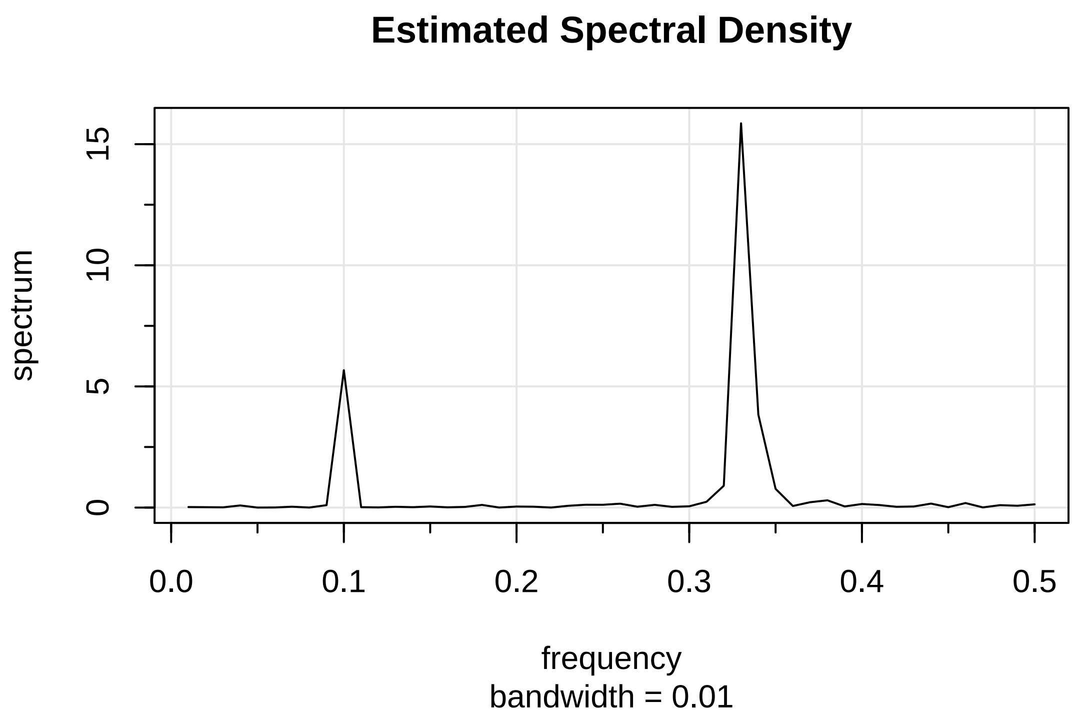
7. On this problem you'll find frequencies and periods.

   (a) The plot below is the spectral density of a process estimated from data. Does this process seem like it could be modeled with sinusoids? If so, what are the PERIODS the sinusoids should have?



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(b) Below are the Fourier coefficients for a time series of length 13. This time series was generated from a single sinusoid plus white noise. What approximately was the FREQUENCY of the sinusoid?

$0.29 + 0.00i$

$-0.50 + 0.52i$

$0.36 - 0.15i$

$-1.31 + 0.01i$

$0.00 - 0.01i$

$3.85 + 5.17i$

$0.22 - 0.35i$

$0.22 + 0.35i$

$3.85 - 5.17i$

$0.00 + 0.01i$

$-1.31 - 0.01i$

$0.36 + 0.15i$

$-0.50 - 0.52i$

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8. Calculate the spectral density of AR(1) and simplify such that no imaginary component remains.

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FORMULA SHEET

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