Statistics 153 - Homework 3

Due on Wednesday, October 6, 2021 by 3 pm

Computer exercises:

1. *(Oppa Gangnam Style!)* If anyone remembers, 2012 was the year "Gangnam Style" - the first video to hit over a billion views - came out, and it was around this time that the Hallyu wave really began picking up. To this end, we'll be analyzing the number of Google searches for the term "kpop" over time, starting from 2012 to the present. To begin, download the file `"kpop.csv"`.

   (a) Plot the (raw) data. Choose and apply an appropriate variance stabilizing transform and plot the transformed data. For the transform you used, what dependency of the variance on the mean does it correct for (linear or quadratic)?

   (5 Point)

   (b) Difference both the original and the transformed data, and plot each differenced series. Which one looks more plausibly stationary?

   (5 Point)

   (c) Using whichever dataset from (b) looks more like white noise, provide a forecast for the number of "kpop" queries for next month **using the differencing method**.

   (5 Point)

   (d) Now, plot the acf correlogram of the differenced dataset you used in (c) for 80 lags. Does it look like white noise? Based on this, how confident are you in the your estimate in (c)? (Hint: even excellent signal models may have ACF plots that don't look like white noise... that's where ARMA comes in)

   (5 Point)

Theoretical exercises:

2. *(Uniqueness of ACF?)* Let $\{W_t\}_{t=-\infty}^{\infty}$ be a mean zero sequence of random variables that is stationary with ACVF $\gamma(h) = Cov(W_t, W_{t+h})$. Define $X_t = W_t - .4W_{t-1}$ and $Y_t = W_t - 2.5W_{t-1}$.

    a. Express the autocovariance functions of $\{X_t\}$ and $\{Y_t\}$ in terms of the autocovariance function $\gamma(h)$ of $\{W_t\}$. (Hint: solve once for $W_t - \delta W_{t-1}$, then replace δ with the appropriate coefficient.) (5 Point)

    b. Show that $\{X_t\}$ and $\{Y_t\}$ have the same autocorrelation function. (5 Point)

    c. From now on, assume W_t is white noise. Give the MA operators for $\{X_t\}$ and $\{Y_t\}$. (5 Point)

    d. Which of $\{X_t\}$ and $\{Y_t\}$ is invertible? (5 Point)

    e. For the respective invertible process give the precise form of the AR(∞) representation. (5 Point)

3. *(Check invertibility/causality)* Assume $\{W_t\}$ is a white noise process with variance σ_W^2. For each of the following ARMA processes, is there a unique stationary solution? Is the process invertible and/or causal?

    (a) $X_t + \frac{1}{12}X_{t-2} = \frac{7}{12}X_{t-1} + W_t$ (10 Point)

    (b) $X_t - X_{t-1} = W_t - \frac{3}{2}W_{t-1} + \frac{1}{2}W_{t-2}$ (10 Point)

    (c) $\left(1 - \frac{1}{3}B\right)\left(1 - \frac{3}{2}B\right)\left(1 - \frac{5}{2}B\right)X_t = (1 - B)\left(1 - \frac{3}{2}B\right)\left(1 - \frac{5}{2}B\right)W_t.$ (10 Point)

4. Choose the process in problem 3 (hint: some will be easier than others to work with).

    (a) What is its unique stationary solution? (10 Point)
    Recall that we look for solutions in $X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j}$ form.

    (b) What is this process's autocovariance function? (10 Point)

    (c) What is this process's autocorrelation function? (5 Point)