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Solution for Problem 1 The difference between data structures and algorithms is the difference between form and function. Data structures gives your data form, it determines how you store your data. Algorithms determine how you work with your data. They are related concepts in that all algorithms operate on some data structure even it is as simple as a list.

Solution for Problem 2

Proof. Show that $3n^2 + n\sqrt{n} = O(n^2)$

$$\lim_{n \rightarrow \infty} \frac{3n^2 + n\sqrt{n}}{n^2} = 3$$

so

$$3n^2 = O(n^2)$$

□

Solution for Problem 3

Proof. Show that $2(n + 100\sqrt{n})(\log(n))^2 = o(\frac{n\sqrt{n}}{\log(n)})$

$$\lim_{n \rightarrow \infty} \frac{2(n + 100\sqrt{n})(\log(n))^2}{\frac{n\sqrt{n}}{\log(n)}} = 0$$

so

$$2(n + 100\sqrt{n})(\log(n))^2 = o(\frac{n\sqrt{n}}{\log(n)})$$

□

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Solution for Problem 4

Proof. it will always be true that $f(n) \leq g(n)$ or that $g(n) \leq f(n)$ so we can say that

$$2(\max\{f(n), g(n)\}) \geq f(n) + g(n)$$

givin that defenition of θ implies that

$$0 < \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} < \infty$$

if

$$\lim_{n \rightarrow \infty} \frac{2 \max\{f(n), g(n)\}}{\max\{f(n), g(n)\}} = 2$$

then

$$\lim_{n \rightarrow \infty} \frac{f(n) + g(n)}{\max\{f(n), g(n)\}} \leq 2$$

because

$$\frac{2 \max\{f(n), g(n)\}}{\max\{f(n), g(n)\}} \geq \frac{f(n) + g(n)}{\max\{f(n), g(n)\}}$$

a

$$\lim_{n \rightarrow \infty} \frac{f(n) + g(n)}{\max\{f(n), g(n)\}} > 0$$

because

$$\lim_{n \rightarrow \infty} \frac{f(n) + g(n)}{f(n)} > 0 \text{ and } \lim_{n \rightarrow \infty} \frac{f(n) + g(n)}{g(n)} > 0$$

so by definition $f(n) + g(n) = \theta(\max\{f(n), g(n)\})$

□

Solution for Problem 5

Proof. is $f(n) = \omega(f(\sqrt{n}))$ always true?

$f(n) = \omega(f(\sqrt{n}))$ is shown by

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n}} = \infty$$

so as long as $n_0 > 0$ it is true $f(n) = \omega(f(\sqrt{n}))$

□

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Solution for Problem 6

1. Pseudocode:

max(*a*, *lower*, *upper*)

Input: A Section of an array *a* with bounds at *lower* and *upper*

Result: The largest number in *a* will be returned

```
1 begin
2   if upper ≠ lower then
3     mid ← ((upper − lower)/2) + lower;
4     return max{max(a, lower, mid), max(a, mid + 1, upper)};
5   else
6     return lower;
```

2. θ analysis using tree The recurrence for that describes this algorithm is

$$T(N) = 2T(N/2) + 1$$

There will be $\log(n)$ layers to the tree

The sum of each layer will be 2^d where d is the depth of the layer// So the bottom in terms of n will be

$$\begin{aligned} 2^{\log_2 n} &= \\ n^{(\log_2)} &= \\ &= n \end{aligned}$$

the summation is

$$\begin{aligned} 2^0 + 2^1 + 2^2 + \dots + n \\ \sum_{i=0}^{\log_2(n)} 2^i = 2n - 1 \end{aligned}$$

so O is

$$O(n)$$

3. Asymptotic Comparison

Asymptotically the algorithms are the same

4. Reality Comparison

In practice the d-c algorithm will be half as fast as the simple method because it is doing twice as many comparisons

Solution for Problem 7

1. **Towers of Hanoi** The problem is divided smaller steps that consist of moving the n th ring to the middle peg then to the far right peg, n is increased from the smallest ring to the largest until all of the rings have been moved
2. **Multiplication of numbers** For a given x and y , x and y is broken into two parts x_h and x_l where the high and low parts are each half of the number represented as an array then the algorithm recurses with the parameters a x array and a y (four times total) when the arrays reach a size of one they are multiplied together then combined with bit shifts.
3. **Finding n th greatest number** This algorithm splits the array into three parts, a greater, less than, and equal to then some number (in the array) then counts the number of items in each part and then if the count of items in the less than array is less than n (the place of the number in a sorted version of the array) then it recurses and runs the algorithm again on the less than array, if it is greater than the less than count it adds the count of equal items to the count if n is now less than the count then it recurses on to the equals array with $n = n - \text{count of items in the less than array}$. If none of these are true then it recurses onto the greater than array with $n = n - \text{the count of all items not in the greater than array}$.