CS61A Lab 8: Recursion

Week 8, 2011

We have provided the skeleton file lab8.py for you to write your solutions in (it includes rlist functions for you to use). You can also copy it to your account by using the following in your terminal:

```
cp ~cs61a/lib/lab/lab8.py .
```

Iteration and Recursion

For the following 2 questions, write both an iterative solution (uses a `while` or `for` loop) and a recursive solution (no `while` or `for` loops).

0.) Write a function `contains` that takes a number `n` as the first argument and a rlist of numbers `l` as the second argument, and returns whether `n` is in `l`.

1.) The greatest common divisor of two positive integers `a` and `b` is the largest integer which evenly divides both numbers (with no remainder). Euclid, a Greek mathematician in 300 BC, realized that the greatest common divisor of `a` and `b` is the smaller value if it evenly divides the larger value or the same as the greatest common divisor of the smaller value and the remainder of the larger value divided by the smaller value. So if `a` is greater than `b` and `a` is not divisible by `b` then:

```
gcd(a, b) == gcd(b, a % b)
```

Write the `gcd` function using Euclid's algorithm.

Recursion

2.) It is a common problem to find whether an element `p` exists in a sorted list.

There is an efficient algorithm known as binary search that cuts the number of elements you are currently looking through by 50% at each iteration. Each iteration of `binary_search` works like this.

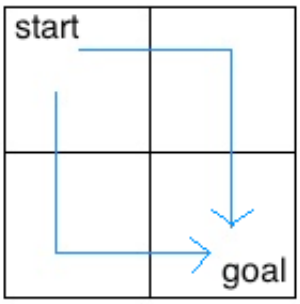
1. If the list is empty, return `False`.
2. Compare `p` to the middle element of the list.
    1. Terminate and return `True` if the middle element is equal to `p`.
3. If `p` is bigger than the middle element, call `binary_search` with the second half of the list (ie. ignore the smaller half).
4. If `p` is smaller than the middle element, call `binary_search` with the first half of the list (ie. ignore the bigger half).

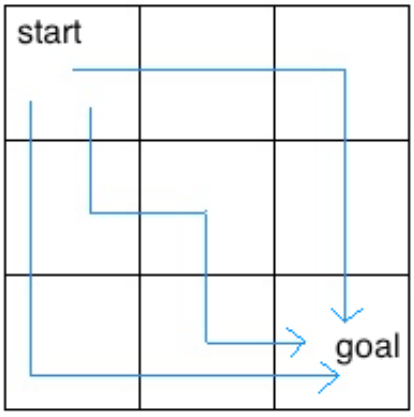
Write a recursive solution for `binary_search`. Given an element and a list, it should return `True` if the element in the list, and `False` otherwise. When there is no clear "middle" element of the list (ie. when the list is even) take the smaller of the two middle elements. Also, never include the middle element when slicing the list for the next iteration (otherwise the function might never stop).

3.) Write the function `deep_map`, which works almost the same as `map` except that if one of the items is a list, we also apply the function to each of its items. Here are some examples of how `deep_map` should work:

```
>>> deep_map(lambda x: x * x, [1, 2, [3, [4], 5], 6])
[1, 4, [9, [16], 25], 36]
>>> deep_map(lambda x: x + 2, [4, 22, [23, 35], 9])
[6, 24, [25, 37], 11]
```

4.) Consider an insect in a MxN grid. The insect starts at the top left corner, (0,0), and wants to end up at the bottom right corner, (M-1,N-1). The insect is only capable of moving right or down. Write a function `count_paths` that takes a grid length and width and returns the number of different paths the insect can take from the start to the goal. [There is an analytic solution to this problem, but try to answer it procedurally using recursion].





For example, the 2x2 grid has a total of two ways for the insect to move from the start to the goal. For the 3x3 grid, the insect has 6 different paths (only 3 are shown above).