Analytical and Computer Cartography

Lecture 8: Algorithms, mosaicing, and conflation

Cartographic Transformations

- Attribute Data (e.g. classification)
- Locational properties (e.g. projection)
- Graphics (e.g. symbolization)
- Information content of maps (e.g. data structure conversion)

Basic Transformation Questions

- Is a transformation quantifiable?
- Can the transformation process be automated? (Alan Turing: Turing Machine and the halting problem, Alonzo Church: Lambda calculus)
- Is a transformation invertible?
- Is a transformation stable?

Types of Transformations

- Map scale
- Dimension
- Symbolic content
- Data structures

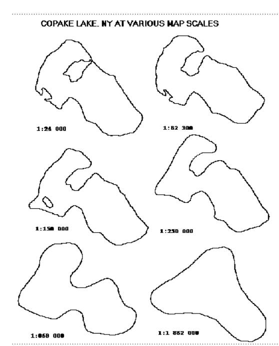
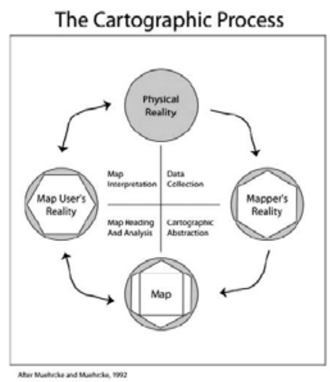


Figure 10.5 Copake Lake (in New York State) at different map scales.

Why do we need to transform?

- We may wish to compare maps collected at different scales.
- We may wish to convert the geometry of the map base.
- We may wish or need to change the map data structure.
- Almost ALL mapping stages involve transformations!

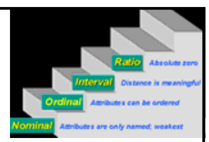
The mapping process



State Changes and Transformations

- Cartographers are interested in the full set of state transformations
- Each map may have an optimal path through the set
- Design cartography primarily concentrates on the last, or symbolization transformation, and now uses human subjects testing and cognitive engineering
- Four types of transformations shape the mapping process:
  - Geocoding (transforming entities to objects: levels, dimension, data structure)
  - Map Scale
  - Locational Attributes or Map Base
  - Symbolization

Levels of Measurement



- Robinson's Classification was based on dimension and level of measurement
- Level of measurement idea is from Stevens (1946)
- Nominal data assume only existence and type. An example is a text label on a map
- Ordinal data assume only ranking. Relations are like "greater than"
- Interval data have an arbitrary numerical value, with relative value Example: Elevation.
- Ratio data have an absolute zero and scale

Robinson & Sale

Content scaling level	Defining relations	FORM OF CARTOGRAPHIC SYMBOL		
		POINT	LINE	AREA
Nominal	Equivalence	Wholesale and retail establishments	Highway connectivity	Land ownership
Ordinal	Equivalence Greater than	Medium Small Large Population centers	Roads by degree of improvement	Low Med High Crop yield
Interval	Equivalence Greater than Ratio of intervals	+ 147 + 210 + 132 + 122 Spot elevations	Graticule	1824 1845 1862 Date of settlement
Ratio	Equivalence Greater than Ratio of intervals Ratio of scale values	Area proportional to population	20 30 2010 Population density isopleths	Value proportional to population density

Figure 10.1 Classification by scaling and dimension. (After Robinson and Sale, *Elements of Cartography, 3d ed.* ,© 1969, by John Wiley & Sons, Inc. Used with permission.)§

Unwin's Classification

DATA TYPES

	Point	Line	Area	Volume
Nominal	City	Road	Name of unit	Precipitation or soil type
Ordinal	Large city	Major road	Rich county	Heavy precipitation Good soil
Interval Ratio	Total population	Traffic flow	Per capita income	Precip. in mm Cation exchange

MAP TYPES

	Point	Line	Area	Volume
Nominal	Dot map	Network map	Colored area map	Freely colored map
Ordinal	Symbol map	Ordered network map	Ordered colored map	Ordered chromatic map
Interval Ratio	Graduated symbol map	Flow map	Choropleth map	Contour map

Figure 10.3 Map data and map types. (After Unwin, 1981.)§

Transformations in Choropleth Mapping

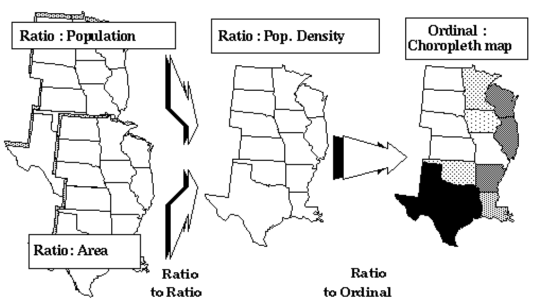
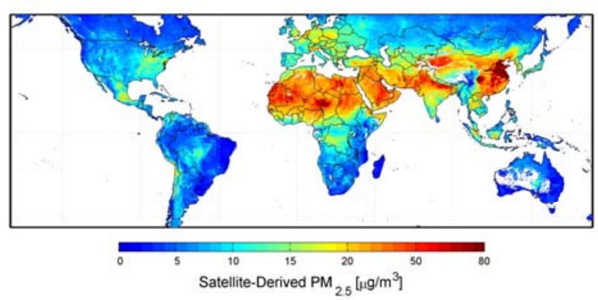


Figure 10.2 Level of measurement transformations for choropleth mapping.§

Air pollution Particulates

Point data (2 instruments and a computer model) -> interpolated to grid > ranked color sequence

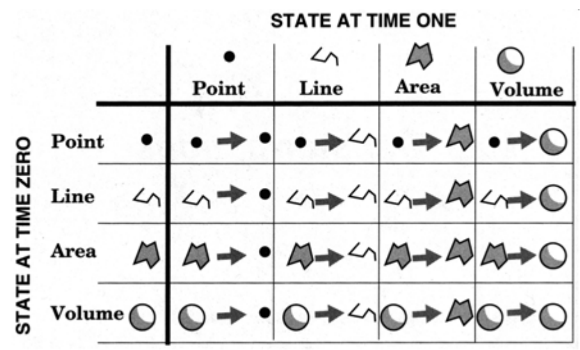
Source: NASA, 2013 and Aaron van Donkelaar and Randall Martin at Dalhousie University



Transformations of Object Dimension

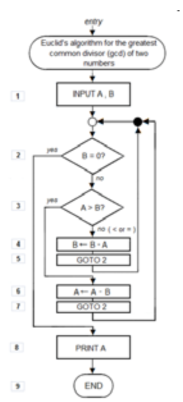
- The four dimension levels of data can be represented at only one level in each state, though a map can contain multiple layer symbols
- Transformations can move data between states
- Full set of state zero to state one transformations is then 16 possible transformations
- Lab exercises fall into several of these.
- Dimensional transformations are only one type
- When dimension collapses to "none" result is a measurement

Dimensional Transformations



Algorithm

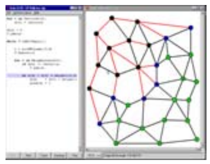
- In mathematics and computer science, an **algorithm** is an effective method expressed as a finite list of well-defined instructions for calculating a function
- Algorithms are used for calculation, data processing, and automated reasoning
- Usually has inputs, result and loops
- Importance of termination
- Divide and conquer



Transformations and Algorithms

- In mathematics, transformations are expressed as equations
- Solutions, inversion and so forth are by algebra, calculus etc.
- In computer science, a set of transformations defining a process is called an algorithm
- Any process that can be reduced to a set of steps can be automated by an algorithm (Church/Turing hypothesis)
- *data structures + transformational algorithms = maps*

Types of Algorithms in mapping

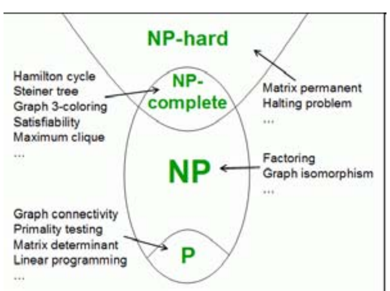


- mathematical
- sorting
- searching
- string processing
- geometrical algorithms (computational geometry)
- graph algorithms
- complex , e.g. decomposition
- In CS, an algorithms is implemented as a function *output = f (inputs)*
- **Inputs can be { data, parameters, objects }**

Graphic algorithms

- Algorithm: method for solving problems, suited for computer implementation (Sedgewick, 1984)
- "Most algorithms of interest involve complicated methods of organizing the data involved in the computation. Objects created this way are called data structures."
- Recursion
- Task decomposition
- Divide and conquer
- Special case vs. Generic solution e.g. vertical lines
- Partitioning: Sequential vs. Parallel (Data and Process)
- Big-O notation and complexity theory
- Solution/Halting problem: Tractability

Complexity Theory



P Polynomial time

N Non-deterministic

T Trivial

Problems needing complex algorithms

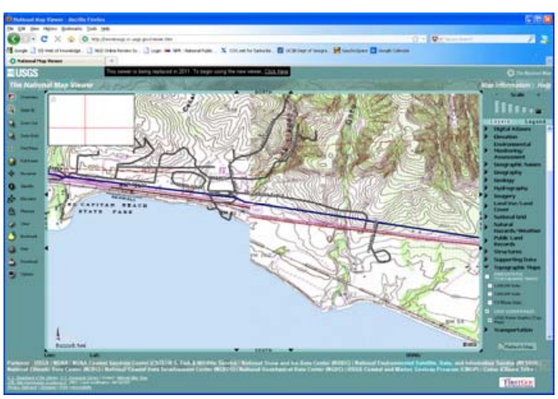
- Mosaicing
- Forward: Given a large map, divide into regular or uneven tiles in an optimal way
- Inverse: Given a tiled map, assemble it back into a single network
  - Detect and eliminate errors
  - Adjust geometry
  - Join divided features
- Conflation
- Given two maps merge their features
  - Geometric error
  - Attribute error
  - Errors of omission and comission

MOSAICING

- Maps collected separately, or map partitioned for storage/search
- Given *n* tiles, with common boundaries, create a new set of common objects that match the geometry in both tiles
- Seam stitching
- Shift
- Mosaicing as conflation

On the 120th meridian

National Map Viewer: 120th meridian



Dissolve

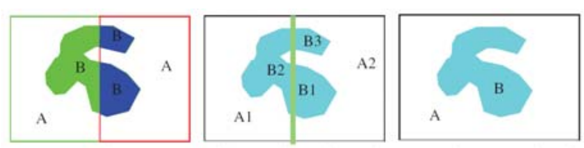
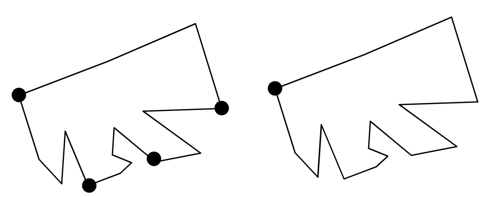
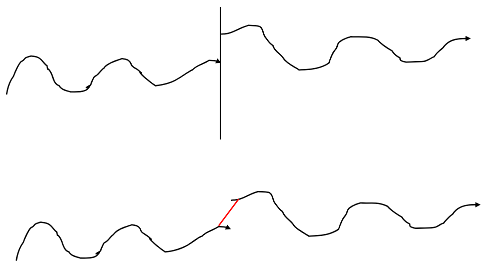


Figure 9.3 Steps in the *dissolve* operation. Left: Two maps show one feature, split across a map edge. Center: Attribute and graphic database have three records for type "B." Right: After dissolve, edge lines are removed and the three type "B" records are amalgamated into a single feature, dissolving the border.



Averaging: Not a solution!



Rubber sheeting

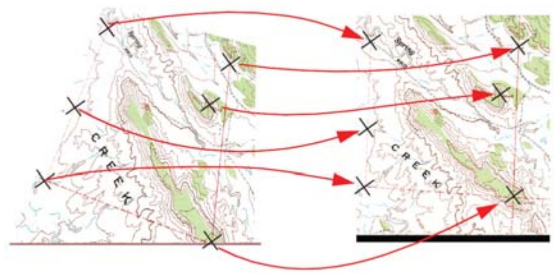
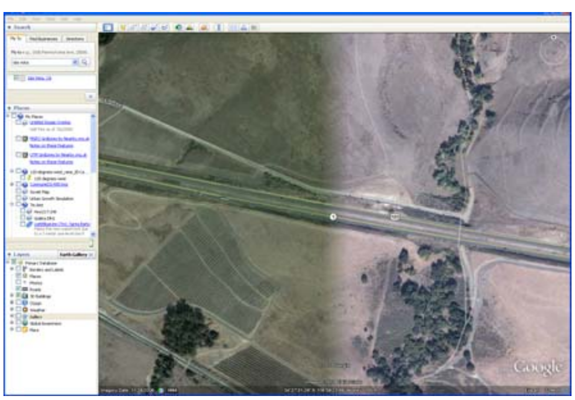
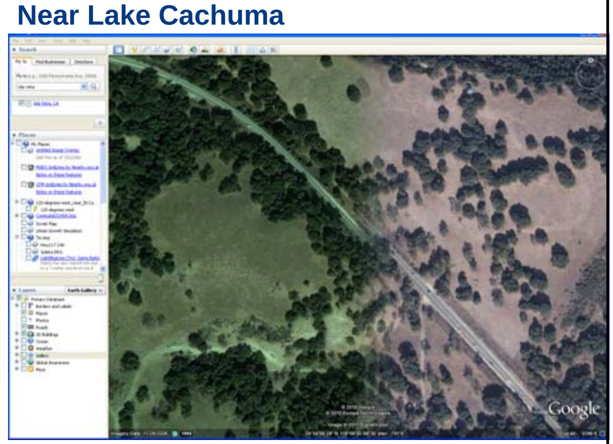


Figure 9.8 The *rubber sheeting method*. A map with unknown geometry (say an air photo taken or scanned map) can be distorted so that its geometry matches that of another map. Pairs of points must be available both on the image and on the map showing the same place or feature location, called control points. Within the GIS, rubber sheeting warps the geometry statistically into that of the map, so that the two geometries match.

Seams



Near Lake Cachuma



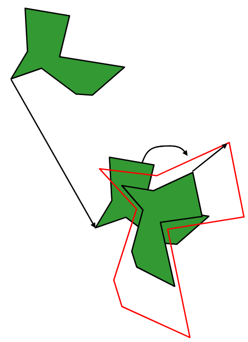
CONFLATION

- **Given two input objects with different (contrary) geometry, generate a single output that conflates the objects**
- **Six-parameter affine** (TRS) **Local affine**

$$A_T(f_{x,y}) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

- **Issues:**
  - **Point selection**
  - **Random vs. systematic error**
  - **"truth"**

Affine transformations



$$[p_x' \ p_y' \ 1] = [p_x \ p_y \ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ c_x & c_y & 1 \end{bmatrix}$$

Translation

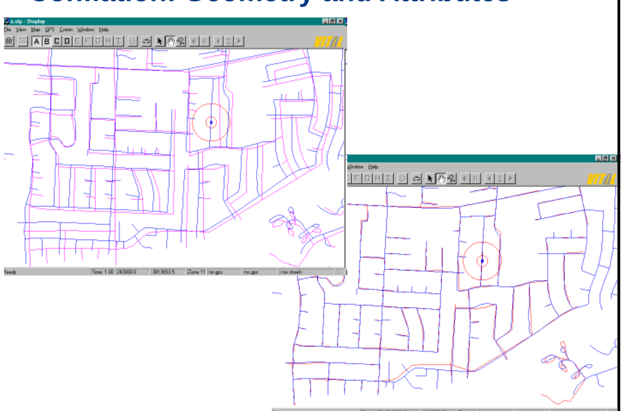
$$[p_x' \ p_y' \ 1] = [p_x \ p_y \ 1] \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation

$$[p_x' \ p_y' \ 1] = [p_x \ p_y \ 1] \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

SCALE

Conflation: Geometry and Attributes



Geometric mismatch

ERROR

Input → **Output**

- **Generic Function Output = f (Input)**
- **O = f(I)**
- **I' = F^{-1}(O)**
- **I I' = Identity?**

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **If not, errors exist.**

Categorical map accuracy

Class Accuracy: Ground Truth

Contingency Table: A pixel by pixel comparison of ground reference class to satellite-based map class.

	Cover Type	Satellite Map Class (pixel counts) Conifer	Hardwood	Grass	Barren	Total	Producers Accuracy, Percent Correct (1-omission error)
ground reference	Conifer	911	20	1	0	932	97.7
	Hardwood	40	343	72	2	457	75.1
	Grass	0	62	176	14	252	69.8
	Barren	0	0	19	27	46	58.7
	Total	951	425	268	43	1687	
	Users Accuracy, Percent Correct (1-commission error)	95.8	80.7	65.7	62.8		86.4

Kappa: An accuracy statistic that permits two or more contingency matrices to be compared.

The statistic adjusts overall accuracy to account for chance agreement.

Use kappa to statistically test for agreement between two contingency matrices.

Summary

- Transformations can impact dimension, data level, scale, symbols
- Transformations are chained, and include map reading and interpretation
- Algorithms can make a transformation computable
- *data structures + transformational algorithms = maps*
- Algorithm computability covered by computational complexity theory
- Examples of hard problems include tiling and conflation
- Methods exist for quantifying and analyzing map error