Analytical and Computer Cartography

Lecture 1: Review of Geodesy and Scale

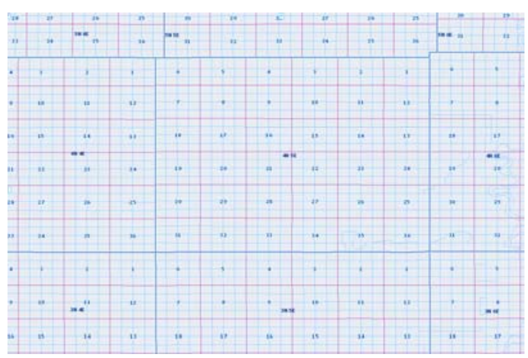
Transformations in Mapping

- T1: True earth to globe
  - Lengths scaled by the Representative Fraction e.g. 1:1M
  - Real world objects become symbols (representations) e.g. road to a red line
  - Need to choose earth model (Datum)
- T2: 3D earth to flat map
  - Map projection transformation
  - Inherent distortion!

Earth models

- Plane, Sphere, Ellipsoid, Geoid
- Much simple survey assumes flat, then corrects (e.g. Township and Range system)
- Sphere assumed since Pythagoras. Aristotle introduced logic proofs
- Ellipsoid since discrepancies in lengths of degrees of latitude discovered
- Geoid involves detailed spherical harmonic model based on gravimetry

USPLSS

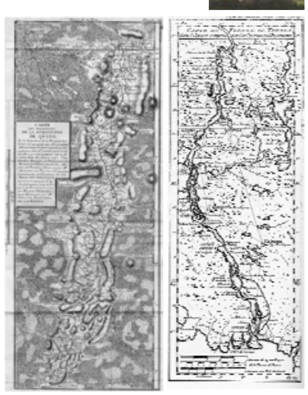


Aristotle (384–322 BC) Arguments for a spherical earth

- Every portion of the Earth tends toward the center until by compression and convergence they form a sphere. (*De caelo*, 297a9–21)
- Travelers going south see southern constellations rise higher above the horizon; and
- The shadow of Earth on the Moon during a lunar eclipse is round. (*De caelo*, 297b31–298a10)

By observation

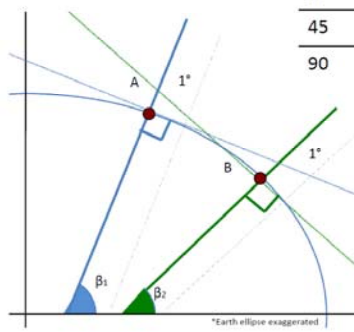
Oblate vs. Prolate ellipsoid



French Academy of Sciences sent expeditions to Peru (now Ecuador) in1735 and Sweden (1736-37) Proved along with Cassini's meridian at Paris that the length of a degree became longer at higher latitudes

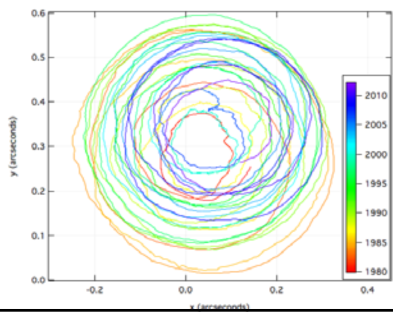
Geodetic latitude

Latitude	Length of 1° of Latitude
0	110574.27m
45	111131.75m
90	111693.92m



More consequences…

- Chandler’s wobble
- 9 m with a period of 433 days (plus)

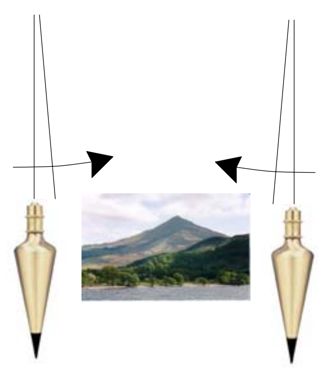


The Attraction of Mountains

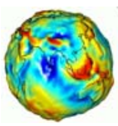
Schiehallion, Scotland

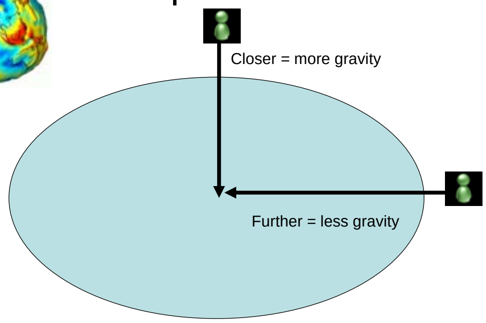
- Experiment to estimate the mass of the Earth in 1774
- Isolated position and regular shape led it to be selected by Charles Mason (of Mason-Dixon fame)
- The deflection of a pendulum used to compute Newton's Gravitational constant *G*
- Measurements made by Astronomer Royal, Nevil Maskelyne
- Assisted in the task by mathematician Charles Hutton, who devised a graphical system to represent heights, later known as contour lines

Deflection of the vertical

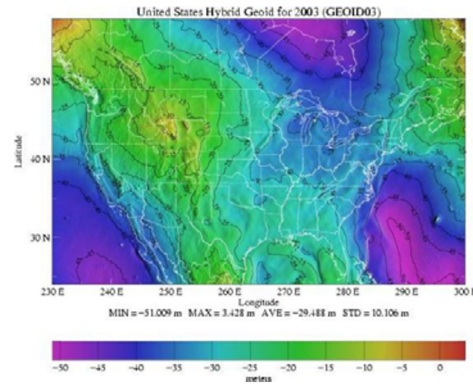


Implications





Gravity as a variable: Height

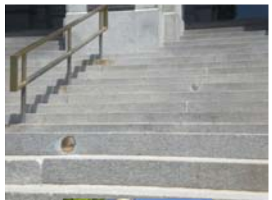


The Datum

- An ellipsoid gives the base elevation for mapping, called a datum
- US mapping standardized in the 1920s at Meades Ranch, Kansas
- First US national mapping efforts used NAD27, changed over to NAD83

The “mile high” city

Datums matter

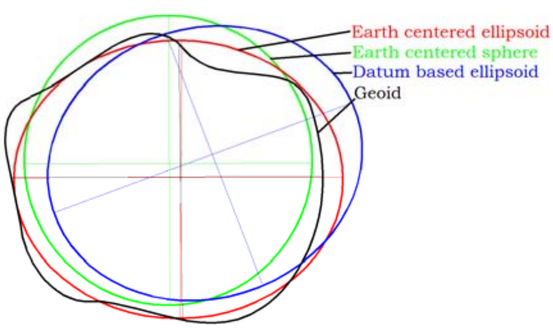


- Three markers on the steps of the Colorado State Capitol that identify this point.
- The first marker was installed in 1909 on the fifteenth step, but was stolen several times before a more permanent "ONE MILE ABOVE SEA LEVEL" marker was engraved in 1947.
- The second marker was placed in 1969 by Colorado State University engineering students on the eighteenth step, after resurveying the elevation.
- Third marker was placed in 2003, after surveyors using the new national vertical datum, determined that the thirteenth step was exactly at one-mile above sea level.

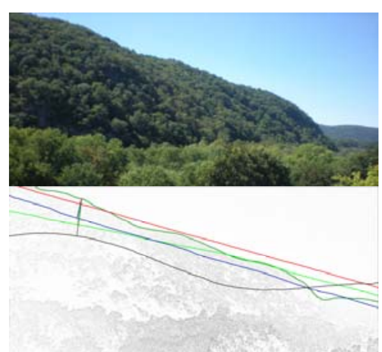
Datum change also “moves” location

104m East of the Observatory

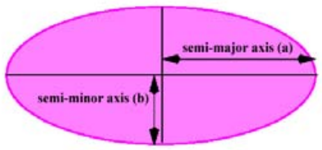
Earth Models and Datums



Which height?



WGS84



$$f = 1 - \frac{b}{a}$$

For the WGS84
$a = 6{,}378{,}137$
$b = 6{,}356{,}752.3$, so $f = 1/298.257$

More exact data

a = 6,378,137 meters
f = 1/298.25722 3563

Figure 2.3 The ellipsoid. The long axis is the major axis, the short the minor axis. Half of each of these lengths is used to calculate the flattening of the ellipsoid.

In summary, for the earth

- Close to a sphere
- Slightly (1/298) an oblate ellipsoid
- Gravity and therefore "down" varies, so we need an earth centered model
- Many ellipsoids measured, some standardized for mapping e.g. WGS84
- Changing the datum, also changes height and position!

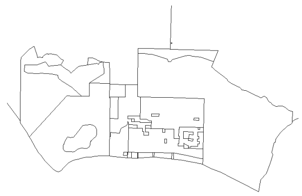
Map Scale

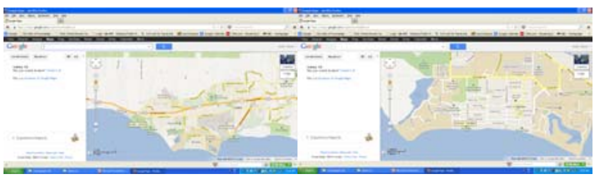
- *Map scale* is based on the representative fraction, the ratio of a distance on the map to the same distance on the ground
- Most maps fall between 1:1 million and 1:1000
- Digital and web maps are *scale-less* because maps can be enlarged and reduced and plotted at many scales other than that of the original data
- But in fact, all maps when displayed have a scale

The Scale Problem

- Zooming (Trick is to use Level of Detail)
- Single map data base at most detailed scale or multiple representations at key scales?
- All map data have the scale of the source "stamped" on them
- Maps at coarse scale need few features and labels, maps at too fine a scale are unreadable

Amount of detail





Ideally, for scale

- The map is at a scale with a memorable RF e.g. 1:10 000
- Scale is the same in all directions and at every point on the map
- Scale is given to the user in some way
- Units are logical
- Reader can interpret, e.g. measure off distances

Scale of a baseball earth

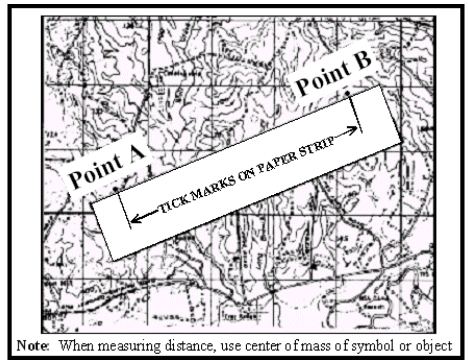
- Baseball circumference = 226 mm
- Earth circumference approx 40 million meters
- RF is : 1:177 million

Calculating ground distance from a map

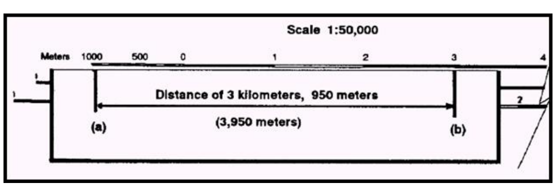
Known scale, calculate distances

- Map distance = 300mm
- Map scale = 1:50 000
- RF = MD / GD so GD = MD / RF
- 1/RF is the denominator
- GD = 300 mm x 50 000 = 15 000m

The paper strip-to-graphic scale trick



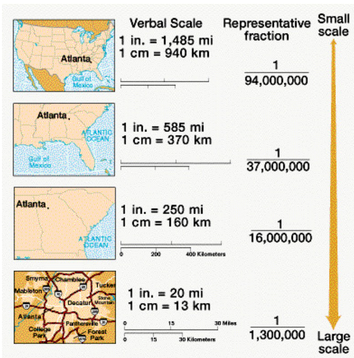
Move the paper to the graphic scale
Read off distance



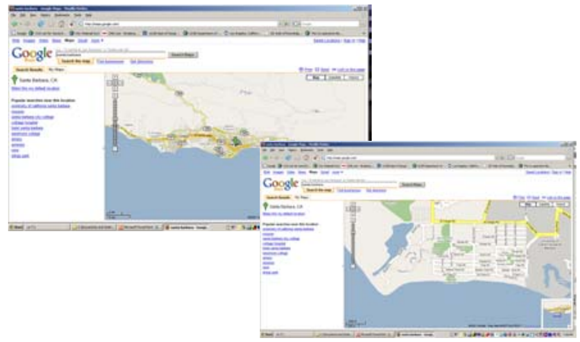
Graphic scale: Units and Divisions

1 Toise was exactly 6 *pieds* (feet) (about 1.949 metres) in France until 1812.
1 Toise was exactly 2 metres in France between 1812 and 1 January 1840
1 Toise = 1.8 metres in Switzerland.
Lieue commune=4452m

Bigger range



Zoom: New features at new scale



In summary

- Earth model the first important base for mapping
- Once a datum and ellipsoid is chosen, the scale transformation can take place
- Converting from datum to datum involves going back to earth centered then retransforming
- Scale impacts cartographic content and map merging/tiling