Geography 128 Winter Quarter 2011

Lecture 4: A Transformational View of Cartography

Tobler , W. R. (1979). “A Transformational View of Cartography.” *American Cartographer,* vol. 6, no. 2, pp. 101–106.

A Transformational View of Cartography

W. R. Tobler

ABSTRACT. Cartographic transformations are applied to locative geographic data and to substantive geographic data. Conversion between locative aliases are between points, lines, and areas. Substantive transformations occur in map interpolation, filtering, and generalization, and in map reading. The theoretical importance of the inverses is in the study of error propagation effects.

Leonard Bernstein, in a recent television lecture, made an exciting, and largely successful, attempt to describe musical concepts in terms of Noam Chomsky's ideas concerning transformational grammars, as originally devised for linguistics.[2,4] A similar, though less ambitious, attempt is made here to look at a range of cartographic activities from a transformational point of view. The treatment is not particularly Chompskian, although some work on picture languages is available.[7,11,21,23] generalizing lines on maps. As can be seen in his paper the method embodies a notion of spatial resolution.[20] Since that time considerable further work has been done on line generalization, and it is now known that the problem admits of more solutions than were recognized in Perkal's early paper.[9,25,26,41]

At the time Perkal wrote, computer cartography had not reached the large scale implementation which it now enjoys. Automatic plotters were not commonplace and he did not have avail-

A simplified view

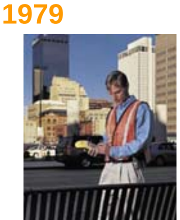
Map input

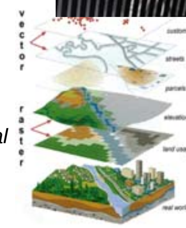
Transformation

Map output

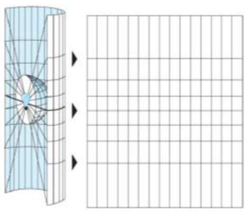
Waldo Tobler’s Classic Paper, 1979

- *“…the entire process of making, and using a map can be viewed as a sequence of transformations*”.
- Types of Cartographic Transformation
  - Geometrical Transformations: to “*manipulate the locative aspects of the geographical data*”
    - Map Projections
  - Substantive Transformations: to “*modify the substantive geographical data*”
    - Map Generalization

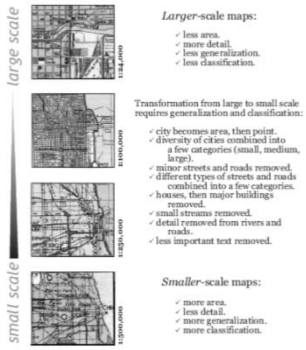




Examples of Cartographic Transformations



ESRI, Understanding Map Projections

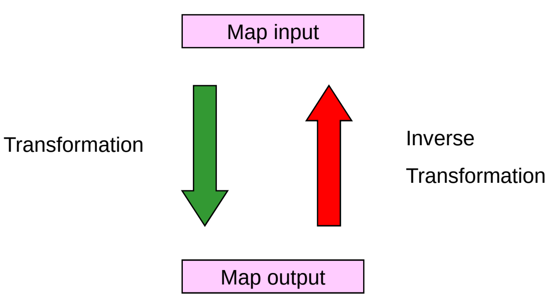


John Krygier and Denis Wood, *Making Maps: a visual guide to map design for GIS*

Cartographic Transformations

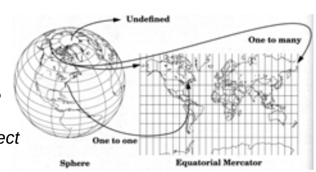
- The base of Computer Cartography
- The core of Analytical Cartography
- Forms of Cartographic Transformation
  - Geometry
  - Attribute
  - Symbolization
  - Scale
  - Data Structure and Data Model
  - Map Type
  - …

A simplified view



Invertibility

- "*…whether or not a Cartographic Transformation can be undone or reversed to produce the initial starting conditions*" –Clarke, 1995
- Stable Transformation
  - invertible
  - "*controllable and therefore are effectively programmed and modeled, especially with respect to the error introduced*"



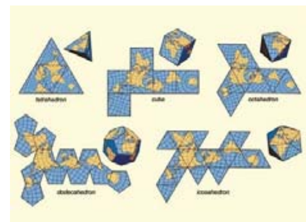
- Unstable Transformation
  - NOT invertible
  - "*the inverse transformation produces chaos*"

For example: Map generalization

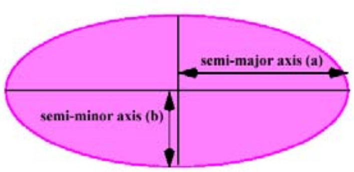
- Detailed map to coarse: throw away points and detail
- Coarse map to detailed: not possible without new data

Transformations to a projection

- Earth to model
- Model to scaled data (representation)
- Data to projection
- Projection to rendering



T1: Earth to Ellipsoid



$$f = 1 - \frac{b}{a}$$

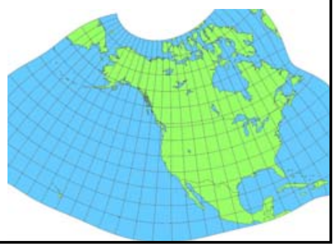
For the WGS84
a = 6,378,137
b = 6,356,752.3, so f = 1/298.257

Figure 2.3 The ellipsoid. The long axis is the major axis, the short the minor axis. Half of each of these lengths is used to calculate the flattening of the ellipsoid.

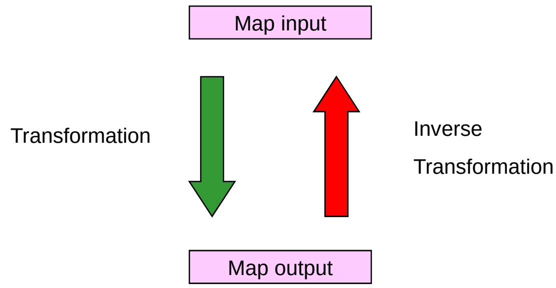
T2: Ellipsoid to Globe

T3: Map Projections

- A transformation of the spherical or ellipsoidal earth onto a flat map is called a map projection.
- The map projection can be onto a flat surface or a surface that can be made flat by cutting, such as a cylinder or a cone.



Our simplified view



Forward and Inverse Projection Techniques

X,Y = f (φ, λ)	*Forward equation*
φ, λ = f (X,Y)	*Inverse equation*

- *Mercator projection (spherical assumption):*

Forward equations:

$X = R * (\lambda - \lambda_0)$

$Y = R * \ln(\tan(\pi/4 + \varphi/2))$

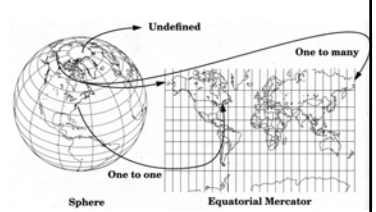
Inverse equations:

$\varphi = \pi/2 - 2 * \arctan(e^{-y/R})$

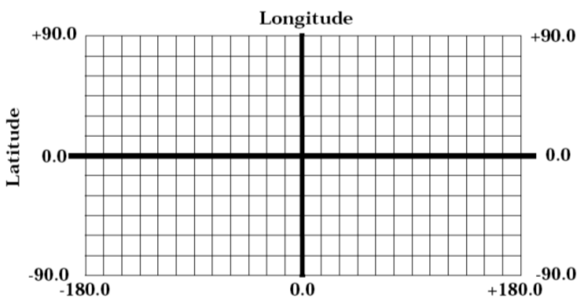
$\lambda = x/R + \lambda_0$

Map Projection Transformations

- Map projections represent many different types of transformation
- Perfectly invertible (one-to-one)
- One-to-many
- Many-to-one
- Undefined (non-invertible)
- Imperfectly invertible, e.g. on ellipsoid and geoid, computational error, rounding etc.
- Some transformations use iterative methods i.e. algorithms, not formulas



Geographic Coordinate Transformation



Matrices: Review

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1\times1+2\times3 & 1\times2+2\times4 \\ 3\times1+4\times3 & 3\times2+4\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$\lambda\mathbf{A} = \lambda\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{pmatrix} = \begin{pmatrix} \lambda A_{11} & \lambda A_{12} & \cdots & \lambda A_{1m} \\ \lambda A_{21} & \lambda A_{22} & \cdots & \lambda A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda A_{n1} & \lambda A_{n2} & \cdots & \lambda A_{nm} \end{pmatrix}.$$

Has n rows

Has m columns

Has values

Has a leading diagonal

Identity matrix

Matrix multiplication

Matrix inversion (T^{-1})

Powers, squares, etc

Maps as matrices

- $\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$ can have an *n* x 2 matrix with as many coordinates as rows as we want
- Can also have *x, y*, and *z* as an *n* x 3 matrix
- Transformation matrix only needs to be 2 x 2
- Or 3 x 3 if (*x, y, z*)

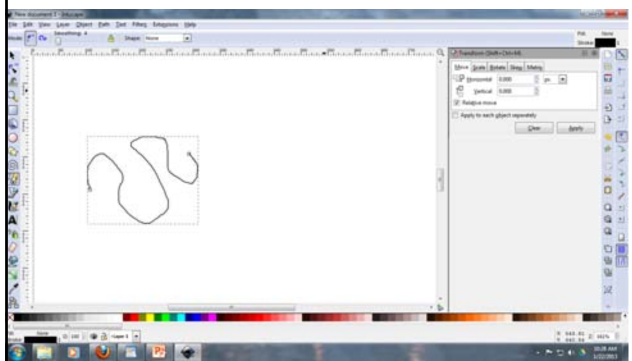
World Outline
(or any set of points, lines or areas)

- Consists of many sets of coordinate pairs (x, y)
- Can be represented as a list or set X, where X is a 2 x n matrix of points
- Matrix multiplication: Possible when one dimension is equal in sequence
- E.g. 2 x 2 times a 2 x 10
- The transformation can be represented also as a matrix multiplication, with transformation T
- Works across sequences of transformations

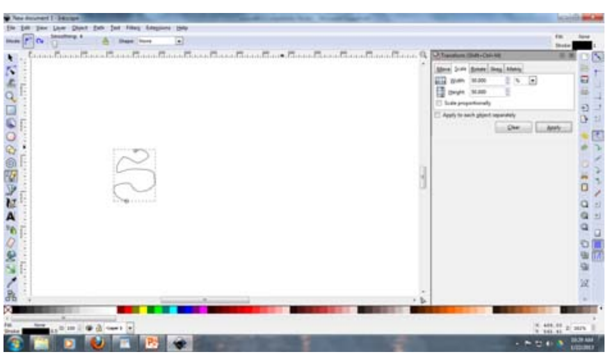
Sequences

- Data matrix X (n x 2)
- Transformation matrix T1 (2 x 2)
- Transformation matrix T2 (2 x 2)
- Multiply T1 by T2 gives a new 2 x 2 matrix that performs BOTH transformations!
- Can repeat as often as we want
- Examples: Scaling, rotations, changes of origin, projections, etc.
- Original matrix X, transformation T, result X'

For example, in Inkscape



Rotate 60 deg CCW, then scale to 50%



Map Transformation Algebra

- Matrices have inverses, which reverse effect of multiplication to yield the identity matrix
- Error creeps in when inversion does not result in identity matrix

$$X = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_n & y_n \end{bmatrix}$$

$XT = X'$ Normal transformation.

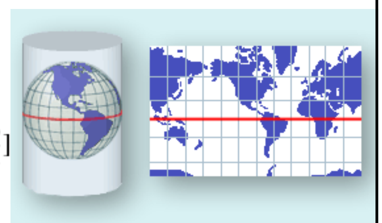
$X'T^{-1} = X + E$ Inverse transformation.

$TT^{-1} = E \quad TT^{-1} = I$ Transformation both with and without error.

Equatorial Mercator Transformation

$$x' = Rs(\lambda - \lambda_0)$$

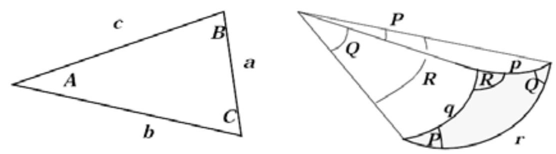
$$y' = Rs\log(\tan[\frac{\pi}{4} + \frac{\phi}{2}])$$



$$\begin{bmatrix} x \\ y \end{bmatrix} = T\begin{bmatrix} \lambda \\ \phi \end{bmatrix} \qquad \begin{bmatrix} \lambda \\ \phi \end{bmatrix} = T^{-1}\begin{bmatrix} x \\ y \end{bmatrix}$$

Planar Geometry vs. Spherical Geometry

- Rule of Sines – Distance between points



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{\sin p}{\sin P} = \frac{\sin q}{\sin Q} = \frac{\sin r}{\sin R}$$

$$\text{radians} = \frac{\pi}{180} \times \left(\text{degrees} + \frac{\text{minutes}}{60} + \frac{\text{seconds}}{60 \times 60}\right)$$

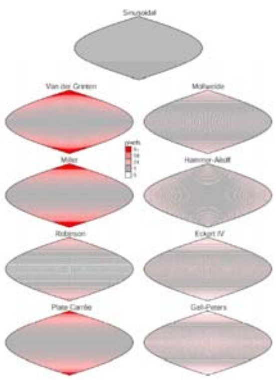
Making projections

- Writing code modules
- Needs point data input
- Rasters create error, often autocorrelated
- Output must be stable
- Numerical handling
- Problem of repetition
- Store data in latitude and longitude, project on the fly?
- Tools for projections: Open source, code libraries, toolkits, web sites

Map projection software

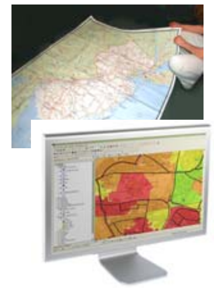
- General Cartographic Transformation Package
- C-language (K&R style) version of GCTP
- PROJ.4 (Most popular)
- National Geodetic Survey (NOAA/NGS)
- MicroCAM
- GMT-3 The Generic Mapping Tools
- GeoConverter
- JMPL (Java Map Projections Library)
- PyProj (Python)
- GDAL
- Commercial
  - GeoCart
  - The Geographic Calculator

Some issues and research problems



- **Display of pixel loss and replication in reprojecting raster data from the sinusoidal projection**
  Denis White

Real Maps and Virtual Maps



- Joel Morrison (1974) called for an expanded definition of map in the computing era
- Harold Moellering (1977, 1980, and 1984) defined real and virtual maps
- Two crucial characteristics
  - Whether a map is directly viewable as a cartographic image
  - Whether it has a permanent tangible reality

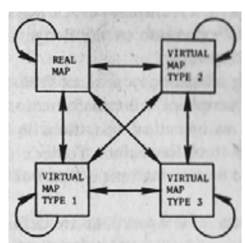
Classes of Real & Virtual Maps

Directly viewable as a cartographic image

Permanent tangible reality	Yes	No
Yes	*Real Map* Conventional Sheet Map Globe Orthophoto Map Machine Drawn Map Computer Output Microfilm Block Diagram Plastic Relief Model	*Virtual Map-Type 2* Traditional Field Data Gazetteer Anaglyph Film Animation Hologram (stored) Fourier Transform (stored) Laser Disk Data
No	*Virtual Map-Type 1* CRT Map Image a) refresh b) storage tube c) plasma panel Cognitive Map (two-dimensional image)	*Virtual Map-Type 3* Digital Memory (data) Magnetic Disk or Tape (data) Video Animation Digital Terrain Model Cognitive Map (relational geographic information)

Moellering, 1984, Real Maps, Virtual Maps and Interactive Cartography

Transformations between Real & Virtual Maps



Moellering, 1984, Real Maps, Virtual Maps and Interactive Cartography

- Real => Real – conventional cartographic processing;
- Real => Virtual 3 – digitizing spatial data and storing them in a digital database;
- Virtual 1 => Real – making hard copy image of a CRT screen image;
- Virtual 3 => Real – digital cartographic plotting/drawing from a spatial database;
- Virtual 3 => Virtual 1 – CRT display of digital spatial data from hard disk to CRT;
- Virtual 1 => Virtual 3 – CRT screen editing spatial data stored on hard disk;
- Virtual 2 => Virtual 3 – reading digital data from CD-ROM and storing them on hard disk;
- Virtual 3 => Virtual 3 – mathematical transformation of digital spatial data resident on computer magnetic media.These transformations are sometimes called Tobler's transformations.

Moellering, 2000, The Scope and Conceptual Content of Analytical Cartography

Summary