Exercise 1.35. Show that the golden ratio ϕ (section 1.2.2) is a fixed point of the transformation $x \mapsto 1 + {}^1\!/_x$, and use this fact to compute ϕ by means of the `fixed-point` procedure.

Answer. Note that $x = 1 + {}^1\!/_x$ is a simple transformation of the equation $x^2 - x - 1 = 0$; to derive it, multiply x to both sides of the equation

$$x^2 = x + 1$$

then move $x + 1$ to the left of the equation

$$x^2 - x - 1 = 0$$

Obviously, the roots of this quadratic equation are

$$\begin{cases} x_1 & = & \frac{\sqrt{5} - 1}{2} \\ x_2 & = & \frac{\sqrt{5} + 1}{2} \end{cases}$$

that is, $\frac{\sqrt{5} - 1}{2}$ and $\frac{\sqrt{5} + 1}{2}$ are the fixed point of the transformation $x \mapsto 1 + {}^1\!/_x$.

On the other hand, since the value of golden ratio is $\frac{\sqrt{5} - 1}{2}$. Hence, the golden ratio ϕ is a fixed point of the transformation $x \mapsto 1 + {}^1\!/_x$.

Now we can use the `fixed-point` procedure to obtain the value of ϕ:

```
(fixed-point (lambda (x)
               (+ 1.0 (/ 1.0 x)))
             1.0)
;Value: 1.6180327868852458
```

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