Exercise 4.37.

Ben Bitdiddle claims that the following method for generating Pythagorean triples is more efficient than the one in exercise 4.35. Is he correct? (Hint: Consider the number of possibilities that must be explored.)

```
(define (a-pythagorean-triple-between low high)
  (let ((i (an-integer-between low high))
        (hsq (* high high)))
    (let ((j (an-integer-between i high)))
      (let ((ksq (+ (* i i) (* j j))))
        (require (>= hsq ksq))
        (let ((k (sqrt ksq)))
          (require (integer? k))
          (list i j k))))))
```

Answer.

Ben is correct. To see why it is, we have to compare the number of possibilities each algorithm must traverse. We denote N_1 the number of possibilities explored by the one in exercise 4.35 and N_2 for Ben's proposal. For convinience, we choose to abbreviate the variable *low* and *high* to l and h respectively.

Since all integers between two given bounds will be explored, the number of possibilities that must be explored depends on the depth of "looping" and the width of the bound. "Looping" over three nested sequence, the procedure for generating Pythagorean triples in exercise 4.35 investigate possibilities in a number of:

$$\begin{aligned} N_1 &= \sum_{i=l}^{h} 1 \times \sum_{j=i}^{h} 1 \times \sum_{k=j}^{h} 1 \\ &= (h-l+1) \times \frac{(h-l+1)}{2} \times \frac{h - \frac{(h-l+1)}{2} + 1}{2} \\ &= \frac{(h-l+1)^2}{2} \times \frac{h+l+1}{4} \end{aligned}$$

The method proposed by Ben never loops for the variable k, so the amount of possibilities it investigates is:

$$\begin{aligned} N_2 &= \sum_{i=l}^{h} 1 \times \sum_{j=i}^{h} 1 \\ &= (h-l+1) \times \frac{(h-l+1)}{2} \\ &= \frac{(h-l+1)^2}{2} \end{aligned}$$

Mathematically, $N_1 < N_2$ when both l and h equal to 1; $N_1 = N_2$ for $l = 1$ and $h = 2$. For all other values of l and h, N_1 is always greater than N_2, because the factor exceeds 1 as. In terms of the evaluator, the one of exercise 4.35 is competent only when the arguments has not exceeds 2 but lost for most cases. Ben's claim therefore, is reasonable.

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