Exercise 1.15. The sine of an angle (specified in radians) can be computed by making use of the approximation $\sin x \approx x$ if x is sufficiently small, and the trigonometric identity

$$\sin x = 3 \sin \frac{x}{3} - 4 \sin^3 \frac{x}{3}$$

to reduce the size of the argument of sin. (For purposes of this exercise an angle is considered "sufficiently small" if its magnitude is not greater than 0.1 radians.) These ideas are incorporated in the following procedures:

```
(define (cube x) (* x x x))
(define (p x) (- (* 3 x) (* 4 (cube x))))
(define (sine angle)
   (if (not (> (abs angle) 0.1))
       angle
       (p (sine (/ angle 3.0)))))
```

a) How many times is the procedure `p` applied when `(sine 12.15)` is evaluated?

b) What is the order of growth in space and number of steps (as a function of a) used by the process generated by the `sine` procedure when `(sine a)` is evaluated?

Answer. In order to understand how the computation process evolves when `(sine 12.15)` is evaluated, let's first describethe computational process generated here in terms of *substitution model*:

```
;; Expanding
(sine 12.15)
(p (sine 4.05))
(p (p (sine 1.35)))
(p (p (p (sine 0.45))))
(p (p (p (p (sine 0.15)))))
(p (p (p (p (p (sine 0.05))))))
;; Reducing
(p (p (p (p (p 0.05)))))
(p (p (p (p 0.14950000000000002))))
(p (p (p 0.43513455050000005)))
(p (p 0.9758465331678772))
(p -0.7895631144708228)
-0.39980345741334
```

a) Given the process generated above, one might intuitively figure out that it takes 5 times to get `p` applied when `(sine 12.15)` is evaluated.

b) We may easily draw out a conclusion that what the process generated above while evaluating `(sine 12.15)` is a linear recursive process and the number of steps grows proportionally to the input a. Thus, the steps required for this process grows as $\Theta(a)$. We also saw that the space require grows as $\Theta(a)$.