Exercise 1.38.

In 1737, the Swiss mathematician Leonhard Euler published a memoir *De Fractionibus Continuis*, which included a continued fraction expansion for $e - 2$, where e is the base of the natural logarithms. In this fraction, the N_i are all 1, and the D_i are successively 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, ... Write a program that uses your `cont-frac` procedure from exercise 1.37 to approximate e, based on Euler's expansion.

Answer.

Obviously, in order to the `cont-frac` procedure to approximate e, we first have to find out the pattern submerged in the series of D_i

$$1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, \ldots$$

As is shown in Table 1, the value of i diverses from 1 whenever $i \bmod 3 = 2$. Further more, the value of D_i can

i	1	2	3	4	5	6	7	8	9	10	11	...
D_i	1	2	1	1	4	1	1	6	1	1	8	...

Table 1. Reflection between i and D_i

be obtained through

$$D_i = \frac{i+1}{3} \times 2$$

Notice that what we obtained from the program uses `cont-frac` based on Euler's expansion is $e - 2$, not e. So, we have to add the value returned by `cont-frac` by 2 to approximate e.

```
(define (e-appr i)
  (+ (cont-frac (lambda (i) 1.0)
                (if (= (remainder i 3) 2)
                    (lambda (i)
                      (* 2
                         (/ (+ i 1) 3)))
                    (lambda (i) 1.0))
                i)
     2))
```