Exercise 2.77.

Louis Reasoner tries to evaluate the expression `(magnitude z)` where `z` is the object shown in figure 2.24. To his surprise, instead of the answer 5 he gets an error message from `apply-generic`, saying there is no method for the operation `magnitude` on the types `(complex)`. He shows this interaction to Alyssa P. Hacker, who says "The problem is that the complex-number selectors were never defined for `complex` numbers, just for `polar` and `rectangular` numbers. All you have to do to make this work is add the following to the `complex` package:"

```
(put 'real-part '(complex) real-part)
(put 'imag-part '(complex) imag-part)
(put 'magnitude '(complex) magnitude)
(put 'angle '(complex) angle)
```

Describe in detail why this works. As an example, trace through all the procedures called in evaluating the expression `(magnitude z)` where `z` is the object shown in figure 2.24. In particular, how many times is `apply-generic` invoked? What procedure is dispatched to in each case?

Answer.

Using substitution model, we can obtain a profound perspective on the evolution of process in evaluating `(magnitude z)`:

```
(magnitude z)
(magnitude '(complex rectangular 3 4))
(apply-generic 'magnitude '(complex rectangular 3 4))  ; type-tag: complex
                                                       ; proc: magnitude
(apply magnitude '(rectangular 3 4))
(magnitude '(rectangular 3 4))
(apply-generic 'magnitude '(rectangular 3 4))          ; type-tag: rectangular
                                                       ; proc: magnitude
(apply magnitude '(3 4))
5
```

We have observed that during the process, `apply-generic` is invoked for twice. In the case `apply-generic` was invoked for the first time, it dispatched the `magnitude` procedure in the `complex` package. In another case it was invoked, `apply-generic` dispatched the `magnitude` procedure in the `rectangular` package.