Exercise 2.15.

Eva Lu Ator, another user, has also noticed the different intervals computed by different but algebraically equivalent expressions. She says that a formula to compute with intervals using Alyssa's system will produce tighter error bounds if it can be written in such a form that no variable that represents an uncertain number is repeated. Thus, she says, `par2` is a "better" program for parallel resistances than `par1`. Is she right? Why?

Answer.

Eva is right. To prove the assertion, we'd better find out where the extra error was introduced by tracing the evoluation of the expression of parallel equivalent resistance from $\frac{1}{1/R_1 + 1/R_2}$ to $\frac{R_1 R_2}{R_1 + R_2}$. Using the transformation rule of fraction arithmetic, we have:

$$\begin{aligned}
\frac{1}{1/R_1 + 1/R_2} &= \frac{1}{\frac{R_2}{R_2} \cdot \frac{1}{R_1} + \frac{R_1}{R_1} \cdot \frac{1}{R_2}} \\
&= \frac{1}{\frac{R_1 + R_2}{R_1 R_2}} \\
&= \frac{R_1 R_2}{R_1 + R_2}
\end{aligned}$$

The transformation of the expression lies on a equation that we take for granted:

$$\frac{x}{x} = 1$$

In an ideal condition, where the element x is an accurate number, this equation holds. But it becomes invalid when comes to deal with variable that represents an uncertain number and this is indeed the case in Alyssa's system (we have seen this in exercise 2.14). So the error was introduced for a second time with the appreance of $\frac{R_2}{R_2}$ and $\frac{R_1}{R_1}$ in the process of transformation. Therefore, by comparision, `par2` is a "better" program for parallel resistances than `par1` and this bears Eva's prediction out.