Practice Test for Final Exam of Math 2318: Linear Algebra
Spring 2016

This exam consists of two parts with a total of 100 points: Part I, worth 44 points, are true and false, and short answer questions. Part II, worth 56 points, are 5 problems to be solved. Supporting work is required.

Part I (44 pts): Ture/False and Short Answer Questions

1. (**26 pts.**) Answer each of the following True (if the statement is correct) or False (if the statement is false). Each problem is worth 2 points. Supporting work is not required here.

______ 1. The columns of any 4×5 matrix is linearly dependent.

______ 2. If a system $A\vec{x} = \vec{b}$ $(\vec{b} \neq \vec{0})$ has more than one solution, then so does the homogenous system $A\vec{x} = \vec{0}$.

______ 3. If none of the vectors in the set $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ in R^3 is a multiple of one of the other vectors, then S is linearly independent.

______ 4. The null space of an $m \times n$ matrix is a subspace of R^n.

______ 5. Let H be the set of all vectors $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where $x_2 = x_1 + 1$. Then H is a subspace of R^2.

______ 6. For a $m \times n$ matrix A, Then the dim Col A + dim Nul $A = m$.

______ 7. For matrices A and B, if $AB = 0$, then either $A = 0$ or $B = 0$.

______ 8. If A and B are square and invertible, then AB is invertible and $(AB)^{-1} = A^{-1}B^{-1}$.

______ 9. If the columns of a square matrix A are linearly dependent, then det $A = 0$.

______ 10. If B is produced by interchanging two rows of A, then det $B =$ det A.

______ 11. Similar matrices always have exactly the same eigenvectors.

______ 12. If A is diagonalizable, then A^{-1} is also diagonalizable.

______ 13. If A is similar to a diagonalizable matrix B, then A is also diagonalizable.

______ 14. Not every orthogonal set in R^n is linearly independent.

2. (**18 pts.**) Short answer. Each one is 3 points. Show the work/reason briefly.

(a) Carefully state the definition of an *eigenvalue and eigenvector* of a matrix.

(b) Complete the following definition: The set $\{\vec{v_1}, \cdots, \vec{v_p}\}$ is called to be linearly independent if

(c) Complete the following definition: a subset H of vectors is called a subspace of R^n if it satisfies the following three conditions:

(d) A set of vectors $\{\vec{v}_1, \vec{v}_p, \cdots, \vec{v}_p\}$ is called a basis of subspace H if

(e) Is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$ linearly independent or dependent?

(f) If A is a 4×7 matrix with 2 pivot columns, what are the dimensions of the null space and the column space of A?

Part II. (54 pts) There are **six** problems to be solved out. Supporting work is required.

3. (**10 pts**) Determine if the following three vectors forms a basis in R^3 or not.

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} \right\}$$

4. (**8 pts.**) Let T be a linear transformation: $R^2 \to R^2$ first rotates points around the origin in counterclockwise direction by $\pi/2$ radians, then translate the points from the origin to the point $(1,1)$. Find a matrix representation of T in homogeneous coordinates.

5. (**8 pts.**) Determine which of the following matrices are invertible and which are not with as little work as possible. In each case, give a brief reason (one sentence at most) why the matrix is/is not invertible.

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 6 & 0 \\ -1 & -2 & 1 \\ 2 & 4 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 5 \end{bmatrix}$

6. (**14 pts**) Consider the following matrix A and its reduced echelon form:

$$A = \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 1 & -1 & 6 & 5 & -3 \\ -2 & 0 & -6 & 1 & -2 \\ 4 & 1 & 9 & 1 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the null space of A.
(b) Find a basis for the column space of A.

7. (**16pts.**) It is known that one of the eigenvalue of the following matrix A is 5.
(a) Find all the eigenvalues of A
(b) Find the bases corresponding to each of the eigenvalue.
(c) What is the form of matrix P if $A = PDP^{-1}$. Here D is a diagonal matrix.

$$A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$$