$a_1 = bq + aq + ap$ $\qquad$ $a_2 = b_1 q + a_1 q + a_1 p$

$b_1 = bp + aq$ $\qquad$ $b_2 = b_1 p + a_1 q$

$a_2 = bq' + aq^2 + aq'$

$b_2 = bp' + aq'$

$$a_2 = (bp + aq)\,q + (bq + aq + ap)\,q + (bq + aq + ap)\,p$$

$$= bpq + aq^2 + bq^2 + aq^2 + apq + bqp + aqp + ap^2$$

$$= b(pq + q^2 + qp) + a(q^2 + q^2 + pq + qp + p^2)$$

$$= b\underbrace{(2pq + q^2)}_{q'} + a(2q^2 + 2qp + p^2) \quad \checkmark$$

$$+ a\underbrace{(2qp + q^2)}_{a_1} + a\underbrace{(q^2 + p^2)}_{a_1}$$

$$b_2 = (bq + aq)\,p + (bq + aq + ap)\,q$$

$$= bqp + aqp + bq^2 + aq^2 + apq$$

$$= b\underbrace{(qp^2 + q^2)}_{p'} + a\underbrace{(2qp + q^2)}_{q'}$$

$$\begin{cases} p' = p^2 + q^2 \\ q' = 2pq + q^2 \end{cases}$$