

Surfaces and Adsorption

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1 About this book

This book intends to provide a freely available resource on concepts in surface science. You are free to copy this work, redistribute, even print and sell the work, provided you adhere to the terms in the [license](#).

The book is a work in progress and is being used to teach a course titled “Surfaces and adsorption”.

We have endeavored to avoid any violations of copyright law in the creation of this book, and we believe that all material used in this book falls under “fair use” as allowed by copyright law. If you are a copyright holder of material used in this book and you believe we have infringed upon your rights, please contact John Kitchin at jkitchin@andrew.cmu.edu to discuss the issue. It is not our intention to violate the law.

1.1 Python

The book uses Python wherever numerical analysis is required. Python is similar in nature to Matlab, but is freely available. We have attempted to introduce the language by examples throughout the book. If you are a new user, you should start at the beginning of the book. If you are experienced with Matlab, the syntax should be easy to read. We recommend the following Python distribution for use with this book.

- Enthought Python Distribution (<http://www.enthought.com/products/epd.php>)

This package is free for academic use, and available for Windows, Macs and Linux. The package includes all the typical python libraries needed for numerical, scientific and graphics computing.

- An alternative python environment that may be suitable is Python(x,y) (<http://code.google.com/p/pythonxy/>). This distribution is Windows focused, and there are not Mac or Linux installers available.
- For editing/writing python code, I like the Spyder editor (<http://code.google.com/p/spyderlib/>). It is also available for Windows, Macs and Linux. You may find the IDLE or SciTE editor that comes with Enthought suitable though.

2 Introduction

2.1 The Importance of Solid Surfaces

- We have never seen anything but the surface of an object.
- Catalysis. Over 90% of all commodity chemical are produced or processed through the use of heterogeneous catalysts. These catalysts are: dispersed metal particles, high surface area zeolites, finely divided oxide powders.
- Corrosion. Destructive oxidation of surfaces or etching for control of surface finishes.
- Brittle fracture. Fracture of solids is often due to segregation of foreign materials to grain boundaries.
- Thermionic emission. Electron emission from heated filaments in TVs (old), electronic tubes etc. Rate of emission depends on surface properties.
- Crystal growth. Growth from solution or from the vapor phase depends on reactions on surfaces and on diffusion on surfaces.
- Semiconductor properties and processing. As the size of devices decreases the surface-to-volume ratio increases and surfaces begin to have an important influence on physical properties.

- Nanophase materials. Solid materials with grains of nanometer dimensions have extremely high grain boundary densities and extraordinary properties.

2.2 Historical Development

- Solids were found to cause reactions. Priestley (1775) $\text{CH}_3\text{CH}_2\text{OH}$ decomposition on Cu. Davy (1817) CO and H_2 oxidation on Pt. Miner's lamp.
- Reactivity increased with porosity. One idea was that the surfaces compress gasses in pores and cause reaction. This was debunked by the fact that porous metal surfaces differ in reactivity.
- Van't Hoff and Sabatier show that surfaces affect the rate but not the equilibrium constant of a reaction. This is a major milestone in the development of chemical thermodynamics. Demonstrates that the equilibrium constant is path independent.
- Several catalytic processes are developed for commerce.
 - Messel (1875) $\text{SO}_2 + \frac{1}{2} \text{O}_2 \rightarrow \text{SO}_3$
 - Mond (1888) $\text{CH}_4 + \frac{1}{2} \text{O}_2 \rightarrow \text{CO} + 2 \text{H}_2$
 - Sabatier (1902) $\text{C}_2\text{H}_4 + \text{H}_2 \rightarrow \text{C}_2\text{H}_6$
 - Haber (1905) $\text{N}_2 + 3 \text{H}_2 \rightarrow 2 \text{NH}_3$
- Langmuir (1915) works on the development of long-life light bulbs for GE and studies the adsorption of gases on hot filaments.
- Davisson and Germer (1927) observe the diffraction of electron from the surface of a Ni crystal and demonstrate that this is due to the wave nature of the electron. Quantum mechanics is proved!
- Modern surface science is born in the 1960's as an outgrowth of space science and the development of instrumentation for achieving ultra-high vacuum (10^{-10} Torr) environments.

2.3 Modern Surface Science

- Atomistic level study of surface imposes extremely stringent demands on experimental methods.
- The total amount of material at the surface of a solid is extremely small. 10^{15} atoms per cm^2 or 10^{-9} moles.
- The surface must be analyzed in the presence of a bulk solid whose contribution to any measurement could swamp that of the surface.

2.4 Surface Sensitivity

- Surface sensitivity must be achieved in order to avoid studying the bulk of a solid rather than the surface of interest.
- Electrons and ions interact very strongly with matter and so they cannot penetrate or escape from the bulk of a solid. In scattering or emission experiments they only sample the surface.

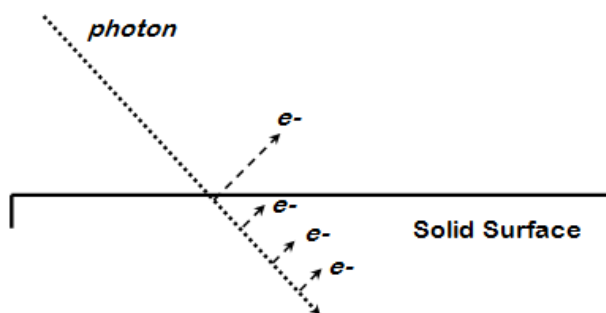


Figure 1: The XPS experiment with electrons coming from the surface only. X-rays penetrate the surface but electrons photomitted from the bulk cannot escape.

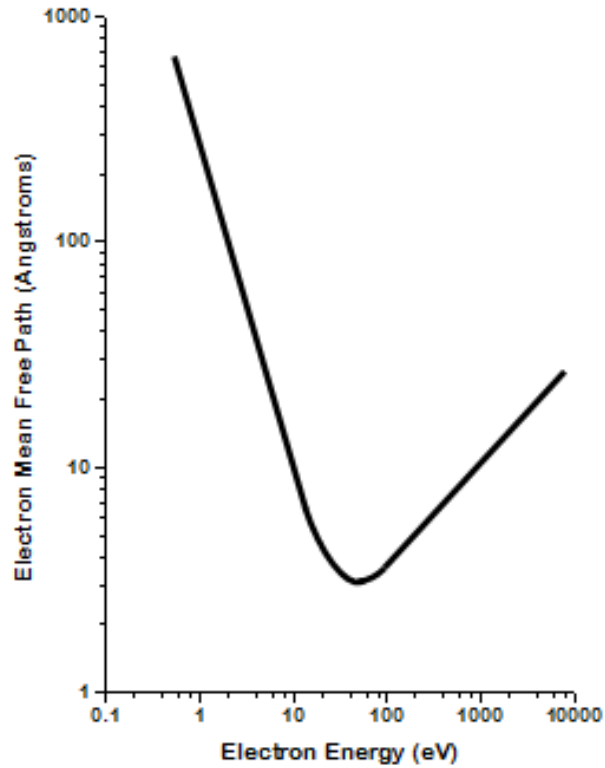


Figure 2: The universal curve of electron mean free paths in solids. The mean free path is the mean distance traveled before an electron is scattered by an atom. This curve has been obtained from measurements made with many materials.

- Ions are even more surface sensitive than electrons. Low energy ions (less than 100 eV) do not penetrate the bulk at all.
- Surface sensitive spectroscopies can almost always be classified into one of four types.
 - ion (or e^-) in \rightarrow ion (or e^-) out
 - ion (or e^-) in \rightarrow photon out
 - photon in \rightarrow ion (or e^-) out
 - photon in \rightarrow photon out

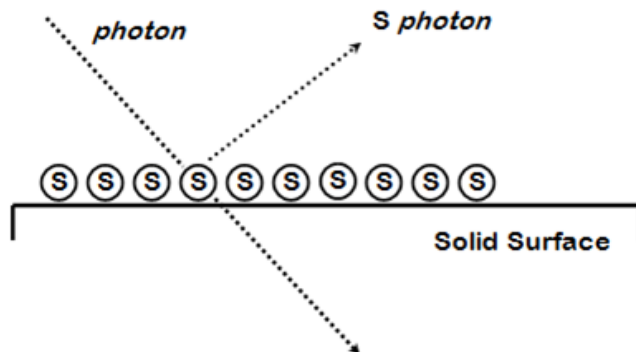


Figure 3: Photon in \rightarrow photon out only detecting sulfur atoms on a surface. If there were high concentrations of sulfur in the bulk then the bulk signal would swamp the signal from the surface atoms.

2.5 Surface Cleanliness

- During the course of an experiment (which may be many minutes to many hours) the state of the surface must remain stable (clean or otherwise). This means that if one studies a clean surface it must not become contaminated by collisions with gas phase molecules.
- Consider the flux of molecules colliding with a surface.

$$F = \frac{1}{4} N_g \langle \nu \rangle$$

Where F is the flux (molecules/m²/s), N_g is the gas molecular density, (molecules/m³), and $\langle \nu \rangle$ is the mean molecular speed of the gas (m/s).

The mean molecular speed is given by kinetic theory:

$$\langle \nu \rangle = \sqrt{\frac{8kT}{\pi m}}$$

Where k is the Boltzman constant (1.38×10^{-23} J/K), T is the absolute temperature, and m is the molar mass (kg).

The ideal gas law gives the density.

$$N_g = \frac{n}{V} = \frac{P}{kT}$$

where P is the pressure, n is moles of gas in the volume.

Putting this together we finally have the flux as

$$F = \frac{P}{\sqrt{2\pi m k T}}$$

Let us consider an example at $P = 1$ bar, $T = 300$ K, and $m = 30$ amu. We will show how to do this in Python.¹

¹We have to import the numpy library. Here we import it with the name np, and then access the functions using a dot notation. For example, the sqrt function is np.sqrt.

```

1  1: import numpy as np
2
3  3: P = 1.0          # bar
4  4: T = 300.0        # K
5  5: m = 30.0         # amu
6  6: kb = 1.3807e-23  # J/K
7
8  8: # conversion factors
9  9: amu2kg = 1.660538921e-27
10 10: bar2Pa = 100000.0
11 11: m2cm = 100.0
12 12:
13 13: F = (P * bar2Pa) / (np.sqrt(2 * np.pi * m * amu2kg * kb * T))
14 14: print 'The flux = {0:1.2e} mlc/cm^2/s'.format(F / m2cm**2)

```

The flux = 2.78e+23 mlc/cm²/s

In line 14 we use the syntax {0:1.2e} to format the answer in scientific notation with two decimal places.

Note the order of magnitude, about 0.5 moles of gas hit a square centimeter, every second. A typical density of surface atoms is $\rho_s = 2.7 \times 10^{14}$ atoms/cm². The collision frequency is then given by:

$$Z_c = \frac{F}{\rho_s} \approx 10^9$$

That is, each surface atom is bombarded about 1 billion times a second! Let us assume that we could start with a clean surface, and that every molecule that hits the surface sticks. We can estimate adsorption rates from the flux then.

Pressure (bar)	adsorption rate (molecules/sec)
1	10 ⁻⁹ /sec
10 ⁻⁹	1/sec
10 ⁻¹²	1/hr

You can see that we must have pressures less than 1×10^{-13} bar to keep surfaces clean for hours at a time to do experiments. Of course not every molecule that hits sticks, so this is only an approximate analysis.

3 Structure of Solid Surfaces

In this section we review the structure of solid surfaces. Recommended readings include chapter 2 in [Masel, 1996](#) and chapter 2 in [Somorjai and Li \[2010\]](#).

- The crystalline structure of solids has been observed since antiquity.
- The existence of crystals and the fact they could be subdivided into identical crystals lead to the original ideas that matter was formed of some elementary components with characteristic shape.
- Structure is defined by the positions of all atoms of all types in a solid.

$$\{(x_i^m, y_i^m, z_i^m)\} i \in I, m - type$$

- When a crystal is cleaved or sliced it exposes a surface with atoms in some ordered array.
- Electron diffraction first proved the existence of atomic order at crystal surfaces (Davisson and Germer, 1927).
- The first direct observation of atoms was on surfaces and was made using Field Ion Microscopy (Muller and Bahadur, Oct. 11, 1955).

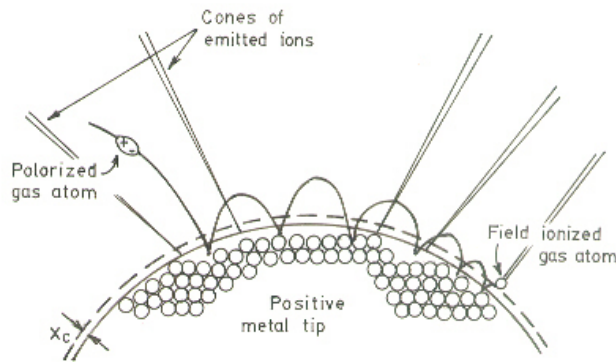


Figure 4: Field ion emission microscope. The metal tips has a high applied potential (≈ 25 kV). He atoms are ionized at the points of high field gradient (atoms at step edges and then accelerated away from the tip along the field lines. They are imaged on a phosphorus screen.

²Read this equation as the (x,y,z) coordinates of the i^{th} atom of type m for every atom i in the structure I .

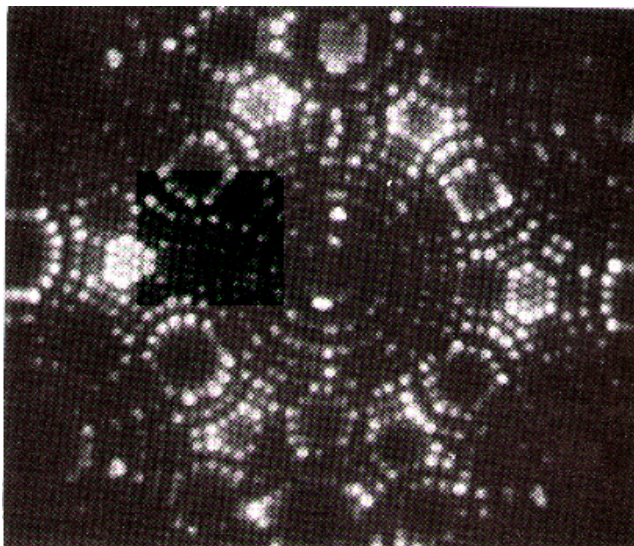
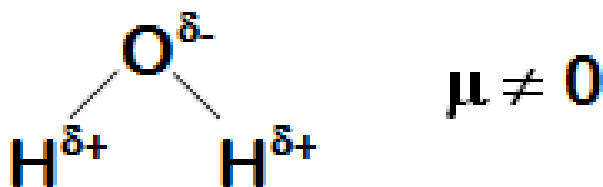


Figure 5: Field ion micrograph of a tungsten tip. The atomic structures of various planes are easily observable.

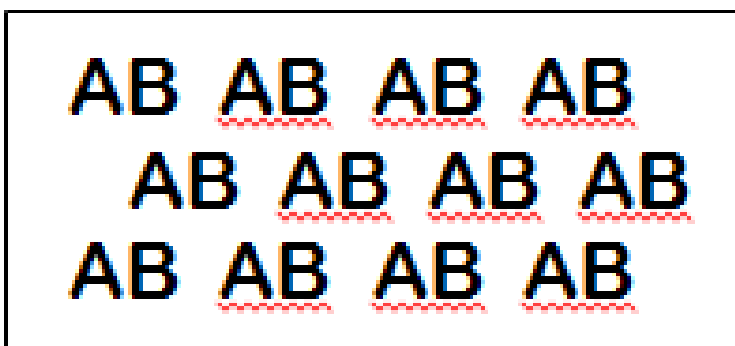
- In general we consider three types of solids and the surfaces that they expose:
 - Amorphous – no long range structure or relationship between the positions of atoms.
 - Periodic – the structure is based on a unit cell that is repeated in space through translation.
 - Quasi-periodic – the structure has long range order in the sense that there are rules that define the positions of all atoms based on the positions of a few, but there is no ‘unit cell’ and no translational repetition of the structure.
- All the surfaces that we will discuss are based on periodic structures and thus some discussion of symmetry is needed.

3.1 Symmetry

- Symmetry is one of the most powerful tools available for understanding and describing the structure of molecules and solids.
- Symmetry often dictates some of the important properties of molecules, solids, and surfaces.
- The existence of dipole moments of molecules can be determined immediately from symmetry arguments: CO_2 , H_2O . Here is an example.



- Second harmonic generation of light occurs only in crystals with no inversion symmetry.



We will see later that symmetry determines which vibrational modes are visible in a vibrational spectroscopy experiment.

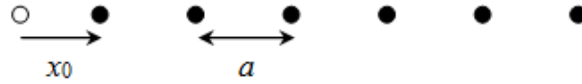
3.1.1 Point Symmetry

- Point group symmetry is used to describe isolated molecules.
- The symmetry elements that can exist in molecules are :
 - mirror planes,
 - rotational axes,
 - inversion symmetry, and
 - rotation-reflection axes.

3.1.2 Symmetry in 1D

- In extended dimensional spaces symmetry is defined by translational periodicity combined with symmetry elements.
- The constraint of translational periodicity limits the types of symmetry elements that can be considered.
- There are two elements that are needed to define the structure of a periodic solid: a lattice and a basis.
- The lattice is a set of abstract points related to one another by translational periodicity which divide space into unit cells.

1D Lattice



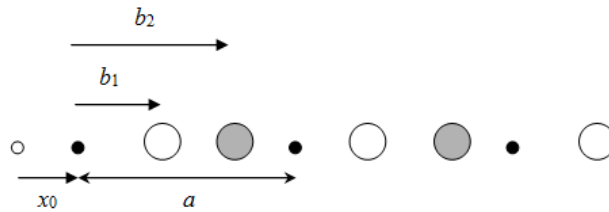
The positions of each of the points are the set $\{x_n\}$ related by

$$x_n = x_0 + n \cdot a$$

$$n \in I$$

- The basis is the set of atoms which are positioned at identical positions $\{b_i\}$ within each of the unit cells.
- The positions of atoms in the unit cell are usually given in terms of the lattice vector lengths, $b = 0 \rightarrow 1$.

Atomic basis



The position of any atom in the solid can then be given as

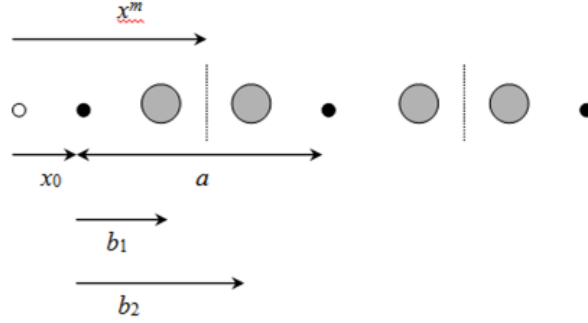
$$x_{i,n} = x_0 + (n + b_i) \cdot a$$

where i is the atom #, and n is the unit cell number.

- A symmetry element relates the position of two or more atoms (of the same type) within the unit cell to one another.

- Specifying the position of one atom and the existence of a symmetry element immediately implies the existence and dictates the position of one or more additional atoms.

Mirror plane



$$x_n^m = x_0 + a \cdot n + \frac{1}{2}(b_1 + b_2) \cdot a$$

- The mirror plane supplies a rule for relating the positions of identical points in the lattice or objects of the basis.
- A mirror plane at position x^m maps a point or atom at a position x into an identical point or atom at position $x' = x^m + (x^m - x) = 2x^m - x$.

Example : Consider the fact that we have atoms at positions $x_{1,n}$

$$x_{1,n} = x + 0 + b^m \cdot a$$

and a mirror plane at

$$x^m = x_0 + b^m \cdot a.$$

This implies the presence of a set of identical atoms at positions $x_{2,n}$.

$$x_{1,n} \implies 2x^m - x_{1,n} \quad (1)$$

$$x_{2,n} = (2x_0 + 2b^m \cdot a) - (x_0 + (n + b_1) \cdot a) \quad (2)$$

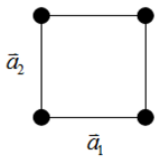
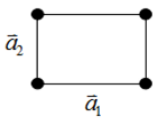
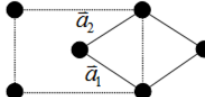
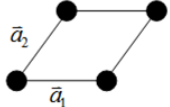
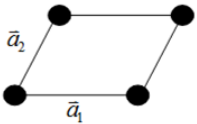
$$= x_0 + (2b^m - n - b_1) \cdot a \quad (3)$$

If the position of the mirror plane happened to be chosen such that $b^m = \frac{1}{2}$ then this would generate all the other atoms in the unit cell shown above.

In other words this generates the positions of the entire set of atoms at positions b_2 within each of the unit cells.

3.1.3 Symmetry in 2D

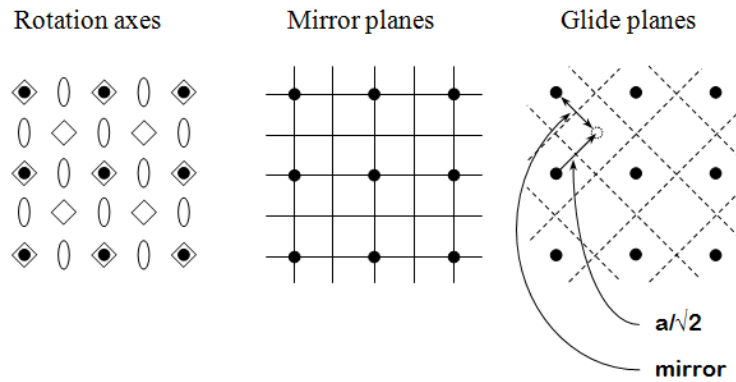
- When one slices a plane through a 3D lattice it generates a surface with the periodicity of a 2D lattice.
- Note that the surface of a real solid is not really 2D. It has some depth since it consists of atoms that are not necessarily all constrained to lie in the plane. The periodicity that we think of is actually due to the projection of the 3D semi-infinite solid onto the hypothetical plane that defines the surface.
- Also, realize that the act of cutting a 3D translationally periodic solid destroys the periodicity in the direction normal to the cut. It leaves only translational periodicity in the two directions parallel to the cutting surface.
- 2D Lattices
 - Symmetry in 2D is defined by a lattice of points combined with several symmetry elements.
 - The lattice in 2D is defined by two vectors which define the sides of the unit cells .
 - The need that the lattice span all space with translational periodicity constrains the lattice vectors to fall into one of five types known as Bravais lattices.

1.	$ \vec{a}_1 = \vec{a}_2 $ $\gamma = 90^\circ$	Square	
2.	$ \vec{a}_1 \neq \vec{a}_2 $ $\gamma = 90^\circ$	Rectangular	
3.	$ \vec{a}_1 = \vec{a}_2 $ $\gamma \neq 90^\circ, 120^\circ$	Centered Rectangular	
4.	$ \vec{a}_1 = \vec{a}_2 $ $\gamma = 120^\circ$	Hexagonal	
5.	$ \vec{a}_1 \neq \vec{a}_2 $ $\gamma \neq 90^\circ, 120^\circ$	Oblique	

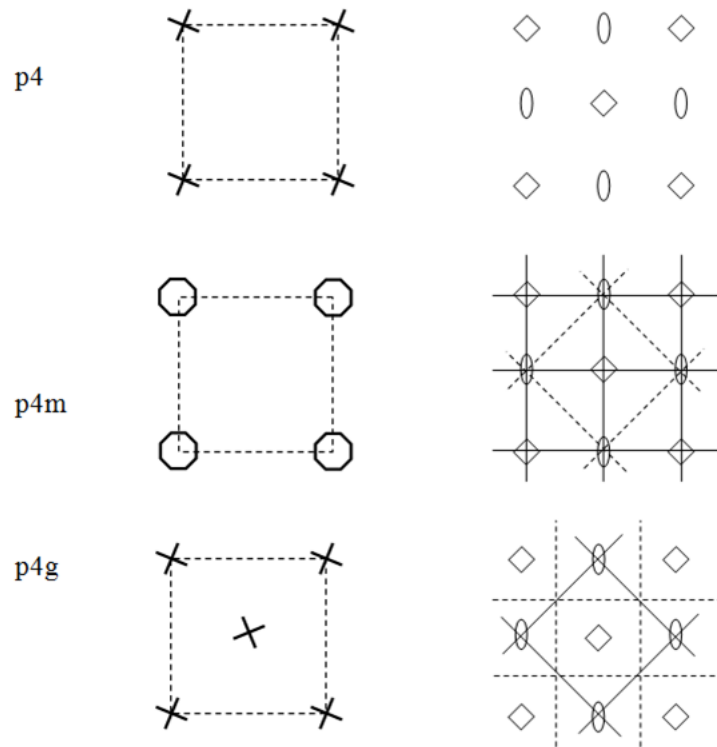
- One thing to note is that the unit cell for the lattice that is called centered rectangular doesn't look rectangular. However, the unit cell can be redrawn in such a way that it is rectangular but has a lattice point at the center and an angle of $\gamma = 90^\circ$.
- This unit cell is no longer “primitive” because it has more than one lattice point per unit cell. It is important to distinguish between primitive and non-primitive lattices. In a non-primitive lattice the points are truly equivalent and therefore there will be atoms that are equivalent and thus indistinguishable.

- Symmetry Elements

- In 2D there are three types of symmetry elements that are consistent with the constraint of translational periodicity.
 1. Rotation Axes : 2-, 3-, 4-, 6-fold
 2. Mirror planes
 3. Glide planes (screw axes according to Masel)

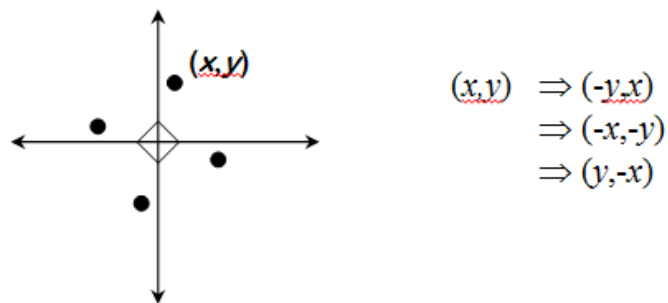


- Note that not all of these symmetry elements must be specified explicitly.
- Specifying two may imply the existence of others.
- The symmetry of the surface will be equal to or lower than that of the corresponding Bravais lattice.
- Combining the five Bravais lattices with the possible symmetry elements results in seventeen 2D plane groups. All 2D periodic structures fall into one of these groups.
- Consider just those that have the square Bravais lattice.



- Note that every one of the symmetry operations can be used to generate one point in the lattice from any other.

Example: Consider a four-fold axis at the origin (0,0) and a point at (x,y).



These transformations can be written down as matrices. The notation for the application of a rotation about fourfold axis by 90° is $\hat{C}_4^{(1)}$.

$$\hat{C}_4^{(1)} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \hat{C}_4^{(1)} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$$

$$\hat{C}_4^{(2)} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \hat{C}_4^{(2)} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

$$\hat{C}_4^{(3)} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Note also that there is a rigorous mathematical meaning to these matrix operations. Two rotations by 90° about a fourfold axis would be represented by the following.

$$\hat{C}_4^{(1)} \cdot \hat{C}_4^{(1)} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \hat{C}_4^{(2)} = \hat{C}_2^{(1)}$$

This is the basis for a very powerful set of ideas that form the basis of “group theory”. [Carter \[1998\]](#), [Bishop \[1973\]](#)

Before continuing, we briefly show how to verify the results above. In python we have three options to describe a matrix, and it is important to know the differences.

```

1:  import numpy as np
2:
3:  C4l_list = [[0, -1],
4:             [1,  0]]   #(list)
5:
6:  C4l_array = np.array([[0, -1],
7:                       [1,  0]])   #(array)
8:
9:  C4l_matrix = np.matrix([[0, -1],
10:                         [1,  0]])   #(matrix)
11:
12:  C4l_matrix = np.matrix([[0, -1], [1,  0]])   #(matrix-alt)
```

In line 4 above, we created a list of lists; that is, there are two lists inside a list. This is not a matrix, and does not act like a matrix.

In line 7 we created a numpy.array. This is closer to a matrix, but most mathematical operations act element-wise.

In line 9 we create a numpy.matrix object. This acts like a matrix for mathematical operations.

In each of the three examples discussed above, the second row of the data was aligned with spaces so you could “see” the intended arrangement of numbers. Since the second row is “inside” a pair of brackets or parentheses, this indentation is meaningless. It is perfectly fine to define the matrix as in line 12, it is just a little harder to read.

Now we consider verifying that two four-fold rotations are equivalent to a two fold rotation.

```

1: import numpy as np
2:
3: C21 = np.matrix([[-1, 0], [0, -1]])
4:
5: C41 = np.matrix([[0, -1],      #(matrix)
6:                  [1,  0]])
7:
8: print(C41 * C41)                #(dot)
9:
10: print C41 * C41 == C21          #(equal)
11: print np.all(C41 * C41 == C21) #(all)

```

```

[[-1  0]
 [ 0 -1]]
[[ True  True]
 [ True  True]]
True

```

In line 7 we define a matrix. This is similar to a matrix in Matlab, except that you have to use a matrix function to create it. The rows are aligned for visual clarity. Because the rows are inside the function parentheses, the indentation of the second row is unimportant. In line 17 we multiply the matrix with itself. This works on a matrix, but not on a list or array.

You can visually see that the product of the two 4-fold matrices is equal to a 2-fold rotation matrix. In line 10 we compare the two quantities. Note the comparison is element-wise, and we get a new matrix of Boolean values. That is inconvenient to examine, we want a single value saying whether the matrices are equal or not. In line 11 we use the `numpy.all` command to tell us whether all of the elements in the matrix are True or not.

```

1: import numpy as np
2:
3: C21 = np.array([[-1, 0], [0, -1]])
4:
5: C41 = np.array([[0, -1],      #(matrix)
6:                  [1,  0]])
7:
8: print(np.dot(C41, C41))          #(dot)
9:
10: print np.dot(C41, C41) == C21    #(equal)
11: print np.all(np.dot(C41, C41) == C21) #(all)

```

```

[[-1  0]
 [ 0 -1]]
[[ True  True]
 [ True  True]]
True

```

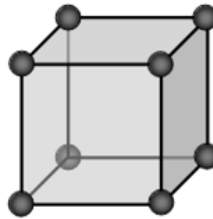
3.1.4 Symmetry in 3D

- Real surfaces are formed by slicing planes through 3D object.
- If the 3D solid has some periodicity or is crystalline then the surface that is produced by the cut will also have periodicity and symmetry.
- In 3D there are 14 possible Bravais lattices that will fill space with translational periodicity.
- The symmetry operations that can exist in 3D are :
 - mirror planes
 - rotations
 - inversion
 - rotation translation (screw axes)
- Combination of the Bravais lattices with these symmetry elements

gives 230 possible 3D space groups.

- The only metal with the simple cubic structure is Polonium.

Simple Cubic Lattice - “no-brainer”



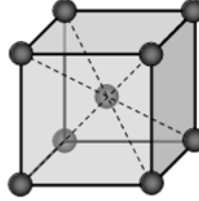
primitive lattice -
conventional lattice -

$|a_x| = |a_y| = |a_z|$, and $\alpha = \beta = \gamma = 90^\circ$
same

- Most metals have bulk structures that fall into the classes: face centered cubic, body centered cubic or hexagonal close packed.

- Both fcc and hcp are close packed structures of spheres.

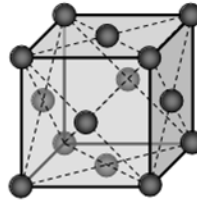
Body Centered Cubic - bcc



primitive lattice -
conventional lattice -

$|a| = |b| = |c|$, and $\alpha = 90^\circ, \beta = \gamma = 54.7^\circ$
 $|a'_x| = |a'_y| = |a'_z|$, and $\alpha' = \beta' = \gamma' = 90^\circ$
two identical atoms - non-primitive

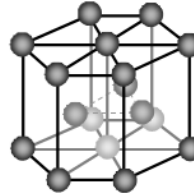
Face Centered Cubic - fcc



primitive lattice -
conventional lattice -

$|a| = |b| = |c|$, and $\alpha = 90^\circ, \beta = \gamma = 120^\circ$
 $|a'_x| = |a'_y| = |a'_z|$, and $\alpha' = \beta' = \gamma' = 90^\circ$
four identical atoms, non-primitive

Hexagonal Close Packed - hcp



primitive lattice -
conventional lattice -

hexagonal
 $|a| = |b| \neq |c|$, and $\alpha = 120^\circ, \beta = \gamma = 90^\circ$
same

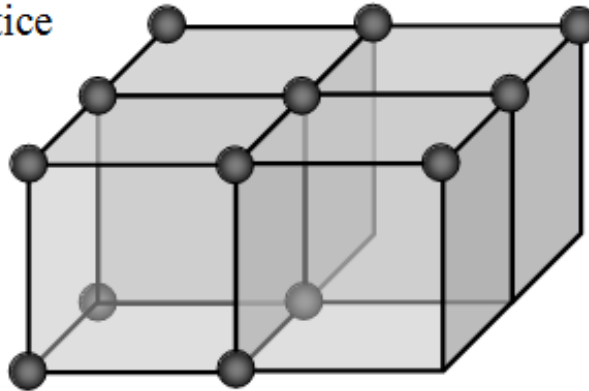
- Note that the two atoms in the hcp lattice are not identical. They are positioned above triangles of different orientation. As a result the hcp lattice has two atoms per unit cell but is still primitive.

3.2 Surfaces of the Simple Cubic Lattice

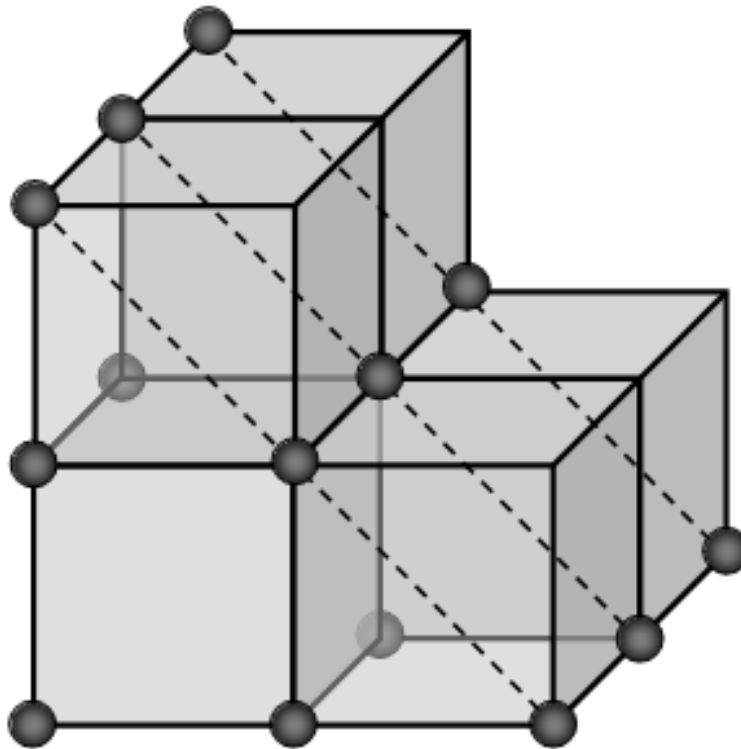
- Start by considering these just to get the idea of taking slices of three dimensional structures. Depending on how you cut the surface, you can

get square, rectangular, or even hexagonal arrangements of atoms at the surface.

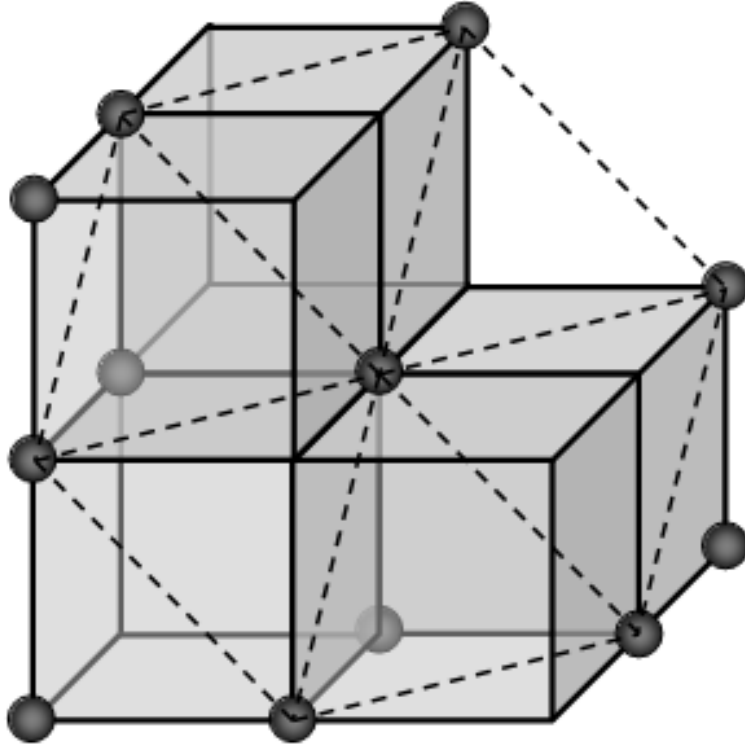
Square Lattice



Rectangular Lattice



Hexagonal Lattice



3.3 Miller Indices

- There are an infinite number of planes that can be cut through a 3D lattice. We need a way to name the planes.
- For every vector direction that one defines in the lattice there is a set of planes perpendicular to that vector.
- In a lattice that has some symmetry to it there will be equivalent directions that expose identical faces.

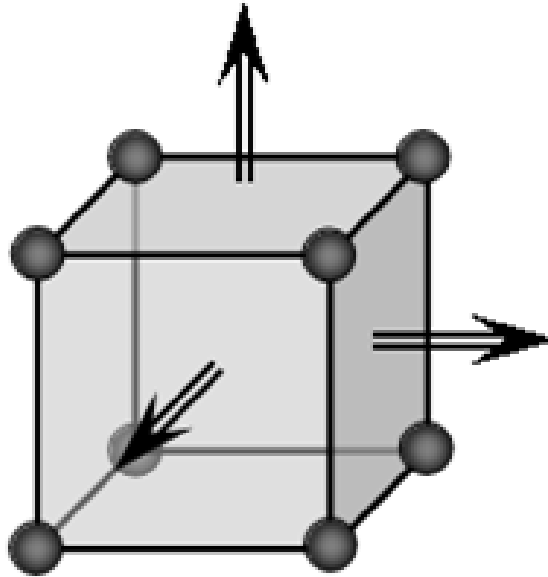
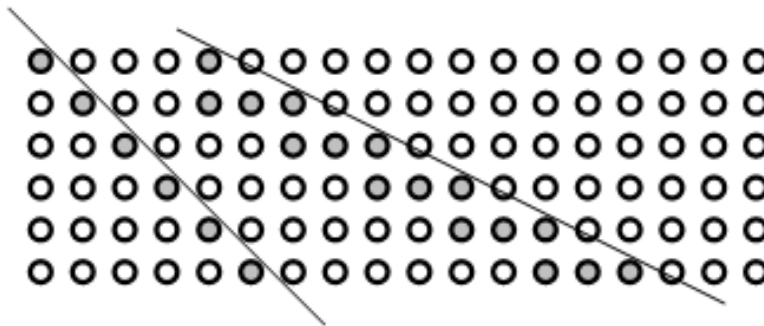
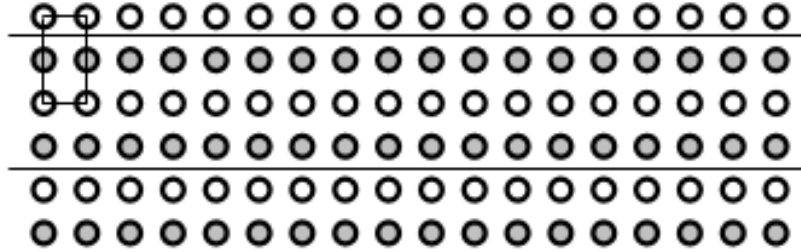


Figure 6: The arrows indicate symmetrically equivalent surfaces.

- For atomic lattices or even molecular lattices one does not usually think of cutting “through” atoms or molecules. Cuts pass between atoms and expose those whose centers lie to one side of the cutting plane.



- If one considers just the lattice then it doesn't matter at what position along a given direction that the cut is made. However, if one considered the contents of the unit cell then it does matter.



3.3.1 Miller Index Formulation

- For a plane cut through a lattice:
 1. Find the intersections (b_x , b_y , b_z) with $b_i > 1$ along each of the three lattice vectors (in units of the vector length).
 2. Find the lowest common denominator of the three lengths.

$$n = \text{lcd} \left\{ \frac{1}{b_x}, \frac{1}{b_y}, \frac{1}{b_z} \right\}$$

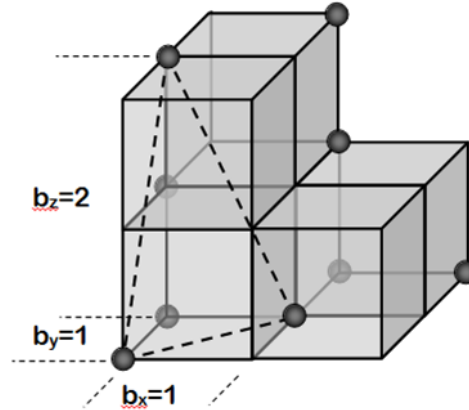
1. Define the Miller indices of the plane as:

$$(h, k, l) = \left(\frac{n}{b_x}, \frac{n}{b_y}, \frac{n}{b_z} \right).$$

1. If $b_x = \infty$ then $h = 0$

Examples :

$$\begin{aligned} (b_x, b_y, b_z) &= (1, 1, 2) \\ \text{lcd} \left(\frac{1}{1}, \frac{1}{1}, \frac{1}{2} \right) &= 2 \\ (h, k, l) &= \left(\frac{2}{1}, \frac{2}{1}, \frac{2}{2} \right) \\ &= (2, 2, 1) \end{aligned}$$



That example is pretty easy to work out in your head. Here is how to compute the lowest common denominator in Python. Note that the lowest common denominator is the same as the lowest common multiple of the numbers in the divisor, and is related to the greatest common divisor.³ Also, the lowest common denominator of three numbers is the same as the lowest common

³http://en.wikipedia.org/wiki/Least_common_multiple#Computing_the_least_common_multiple

denominator of one number with the lowest common denominator of the other two numbers.

```

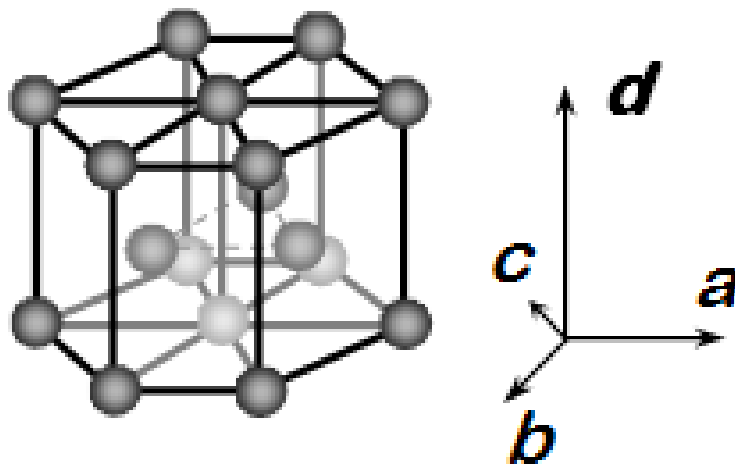
1 1: from fractions import gcd
2 2:
3 3: def LCM(a, b):
4 4:     'returns the least common multiple of a and b'
5 5:     return a * b / gcd(a, b)
6 6:
7 7: print(LCM(1, LCM(1, 2)))

```

2

In line 1 we import the `gcd` function from the `fractions` module. Line 3 illustrates a new python concept: the definition of a function. The function is called `LCM` and it takes two arguments. Line 4 is simply a documentation string that tells us what the function does. Note that the body of the function *must* be indented! This is a critical difference between python and Matlab. The standard indentation is 4 spaces. Finally, in line 7, we use the functional form of the `print` command to output the result.

- Note that for the hexagonal lattice it is quite common to use four indices (h, k, l, m) as though there were four vectors needed to define the unit cell. This is illustrated below.



- In this case the four indices are not unique and the relationship between them is that $h + k = -l$.

3.4 Common Low Miller Index Surface Structures

- The most common and important crystal structures for metals are the fcc, bcc, and hcp.
- The most common and important structures for the semiconductors are the diamond and zincblende structures.
- Of these, the most important surfaces are usually the low Miller index surfaces. These tend to be closely packed arrays that are thermodynamically stable.
- Metal crystallites in catalysts usually expose low Miller index surfaces.

3.4.1 The most common fcc surfaces

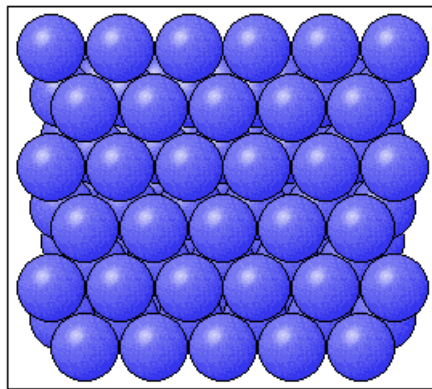


Figure 7: fcc(111)

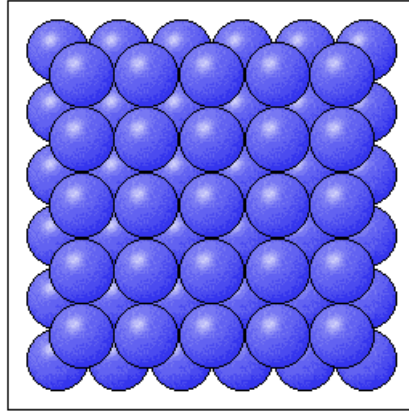


Figure 8: fcc(100)

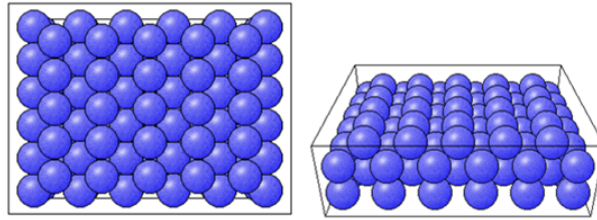


Figure 9: fcc(110)

3.4.2 The most common bcc surfaces

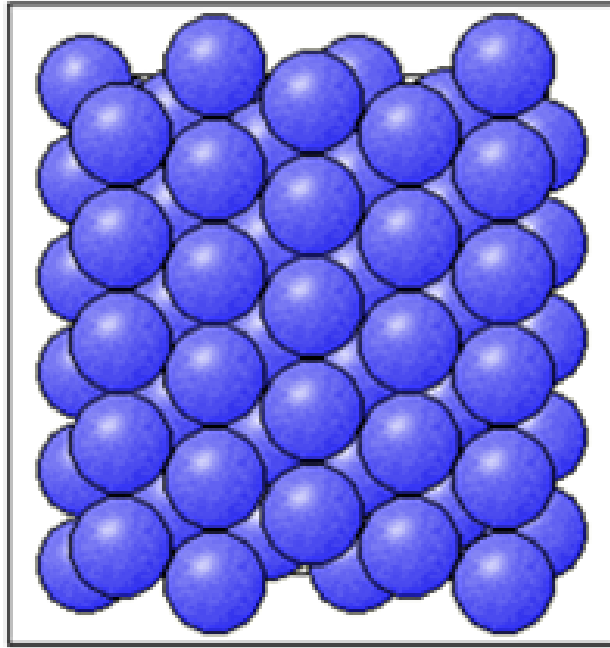


Figure 10: bcc(110)

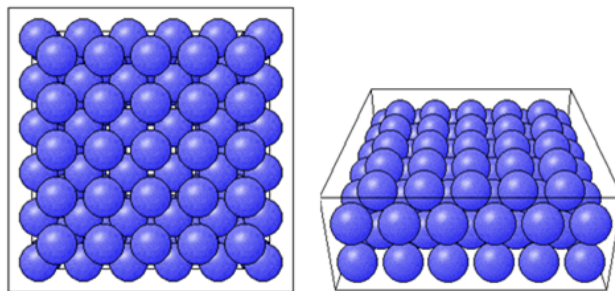


Figure 11: bcc(100)

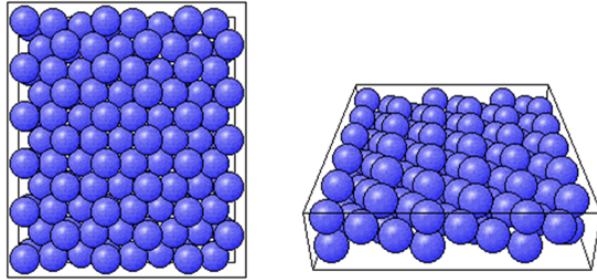


Figure 12: bcc(111)

3.4.3 hcp surfaces

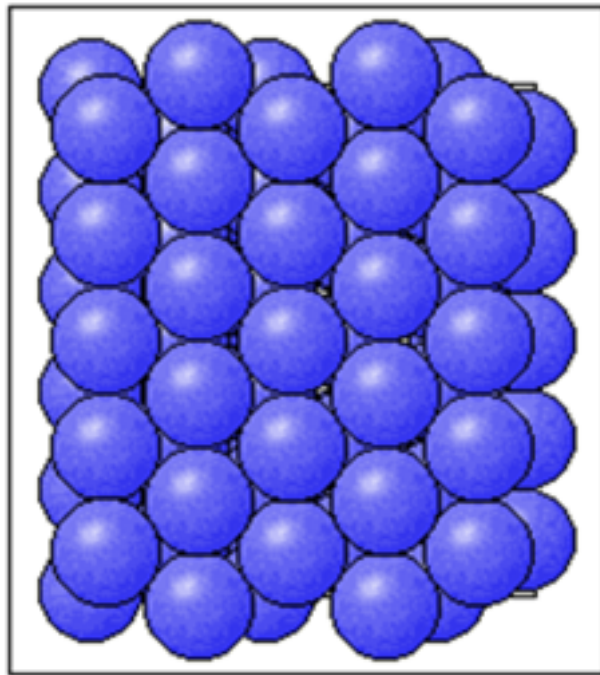


Figure 13: hcp(001)

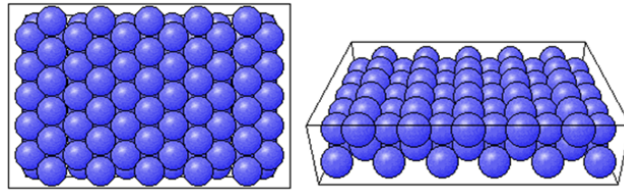
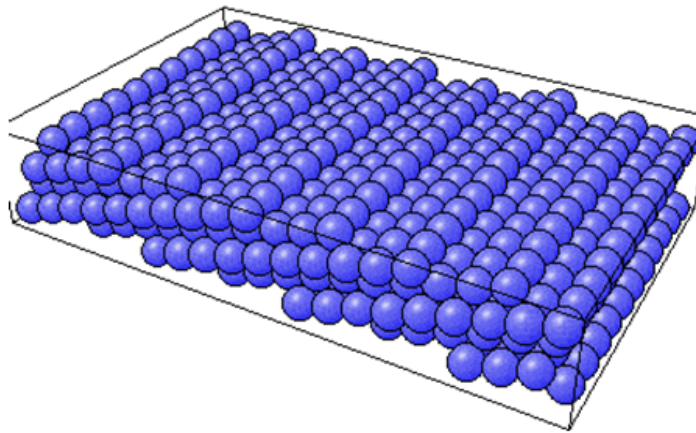


Figure 14: hcp(100)

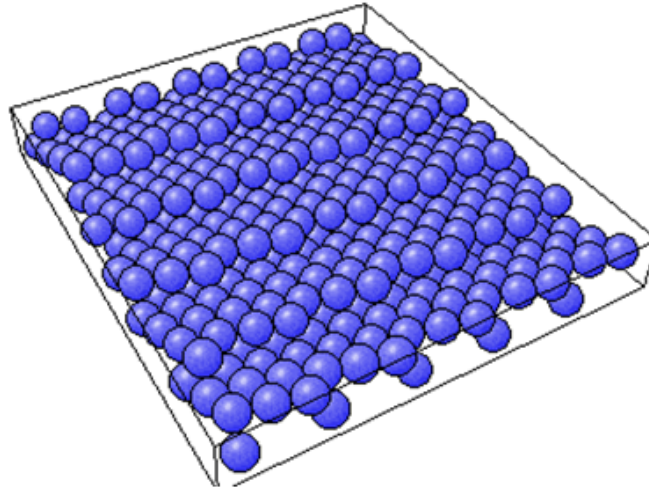
3.5 High Miller Index Surfaces

- Surfaces cut along planes that are not low Miller index planes can have very complex structures and are certainly not close packed.
- The structures that are exposed by high Miller index cuts are :
 - monoatomic steps, and
 - kinked steps.

fcc(511)



fcc(643)



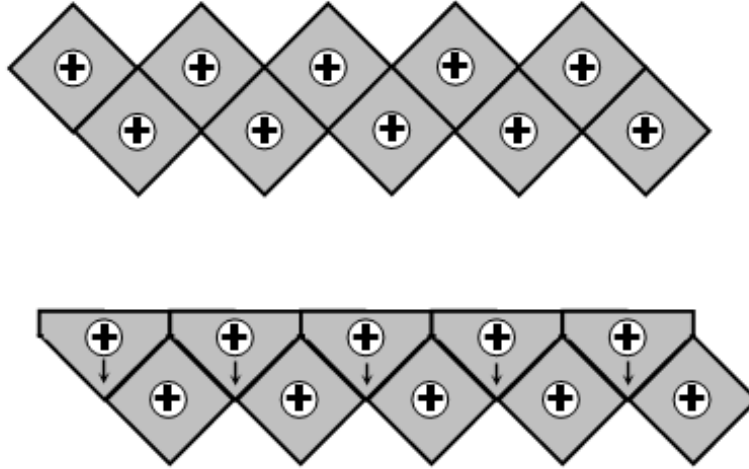
- All possible structure can be viewed on the web at the site:

<http://surfexp.fhi-berlin.mpg.de/>

- It is important to realize that the steps have certain structure to them also and that they can be thought of as planes that project up through the surface.
- The fcc(511) surface can be thought of as (111) steps combined with the (100) terraces.
- When thinking about the step structure it is important to remember the atoms within the original unit cell.

3.6 Surface Relaxation

- Cleavage of a 3D solid removes one degree of translational periodicity. The semi-infinite solid is now only periodic in the plane parallel to the cut.
- Clearly this must have some effect on the positions of atoms at the surface. They are now only bonded to atoms on one side and must react to minimize total energy.
- The universally observed effect is that there is a contraction of the layers at the surface.
- The contraction is due to the contraction of electron density into the bulk of the solid.

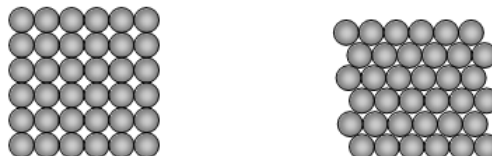


- For the close packed surfaces the contraction is minimal; 1-3% of the plane spacing.
- For non-close packed low Miller index surfaces the contraction can be 5-10% of the plane spacing.
- For rough surfaces the contraction is greater as a percentage of the surface plane spacing because the surface plane spacing is lower. The absolute contraction is not necessarily much different from that of a low Miller index surface.
- Relaxation preserves the 2D translational periodicity of the bulk.

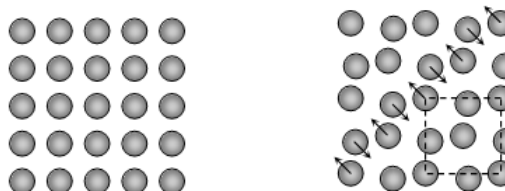
3.7 Reconstruction

- The need to minimize the total energy of the surface can drive the surface to much more drastic rearrangements than relaxation.
- Reconstruction involves an atomic rearrangement that changes the 2D translational periodicity of the surface.

Pt(100) - hex



W(100) - c(2x2)



Au(110) - (2x1)



- The tendency is that the close packed or low Miller index surfaces of metals are thermodynamically stable.
- A number of the reconstructions that have been observed can be rationalized in terms of a surface taking on close packed structures. The Au(110)-(2 × 1) forms facets of (111) surfaces.
- One of the types of reconstruction that should be obvious is the coalescence of steps. Steps are high energy features but are prevented from coalescence by dipole repulsion.
- At high enough step density, however, these become unstable and do coalesce into polyatomic steps.

4 Computational crystallography

In this section we show some general tools for computing crystallographic properties, such as unit cell volume, distances, distance between planes, etc. . . We will only consider the linear algebra approach to these problems.

First, we consider that every vector has a set of basis vectors it is associated with. For example, the vector $2\vec{a} + 3\vec{y} + 4\vec{z}$ can be compactly represented by $[2, 3, 4]$ with the implied cartesian vector basis set:

$$\hat{e}_1 = 1\vec{x} + 0\vec{y} + 0\vec{z} \quad (4)$$

$$\hat{e}_2 = 0\vec{x} + 1\vec{y} + 0\vec{z} \quad (5)$$

$$\hat{e}_3 = 0\vec{x} + 0\vec{y} + 1\vec{z} \quad (6)$$

This special set of vectors is orthonormal (each vector has unit length and is orthogonal to the other vectors).

4.1 Computing distances between two points

We know that the length of a vector is simply the square root of the dot-product of the vector with itself, so we can express the distance between two points as the length of the vector connecting them. Let point 1 be defined by a position vector \vec{p} and point 2 by a position vector \vec{q} . We know the distance between point 1 and point 2 is:

$$D = \sqrt{(\vec{q} - \vec{p}) \cdot (\vec{q} - \vec{p})}$$

```

1 import numpy as np
2 p = np.array([0,0,0])
3 q = np.array([1,1,0])
4
5 D = np.sqrt(np.dot((q-p),(q-p)))
6 print D

```

1.41421356237

This only works, however, when the basis vectors are orthonormal! In many unit cells, the lattice vectors are not orthonormal, and it is more difficult to compute distances. To generalize the formula, we need to introduce the metric tensor. [Graef and McHenry \[2007\]](#)

$$g = \begin{bmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{bmatrix}$$

Note that for an orthonormal basis, this matrix is simply the identity matrix. With this definition, we define the distance simply as:

$$D = \sqrt{(q - p) \cdot g \cdot (q - p)}$$

Here is an example of computing the distance between two points defined in a unit cell basis, i.e. $[1/2, 1/3, 1/4]$ means $1/2\vec{a} + 1/3\vec{b} + 1/4\vec{c}$ where each lattice vector is in turn defined, e.g. $\vec{a} = 2\vec{x}$, $\vec{b} = 2\vec{y}$ and $\vec{c} = 3\vec{z}$.

```

1 import numpy as np
2
3 # define lattice vectors
4 a = np.array([2, 0, 0])
5 b = np.array([0, 2, 0])

```

```

6  c = np.array([0, 0, 3])
7
8  # define metric tensor
9  g = np.array([[np.dot(a,a), np.dot(a,b), np.dot(a,c)],
10                [np.dot(b,a), np.dot(b,b), np.dot(b,c)],
11                [np.dot(c,a), np.dot(c,b), np.dot(c,c)]])
12
13 # define points
14 p = np.array([0.5, 1./3., 0.25])
15 q = np.array([1./3., 0.5, 0.75])
16
17 # compute distance
18 D = np.sqrt(np.dot((p-q), np.dot(g, (p-q))))
19
20 print('The distance between the points is {0:1.2f}'.format(D))

```

The distance between the points is 1.57.

4.2 Computing the angle between two vectors

When vectors are given in cartesian coordinates, we can easily calculate the length and angles between vectors using the `Scientific.Geometry.Vector` class. Here is an example:

```

1  import numpy as np
2  from Scientific.Geometry import Vector
3
4  A = Vector([ 4, 0, 0])
5  B = Vector([ 0, 6, 0])
6  C = Vector([-2.5, 0, 5*np.sqrt(3)/2.])
7
8  print('a = {0} angstroms'.format(A.length()))
9  print('b = {0} angstroms'.format(B.length()))
10 print('c = {0} angstroms'.format(C.length()))
11 print('alpha = {0:1.1f} degrees'.format(B.angle(C)*180./np.pi))
12 print('beta  = {0:1.1f} degrees'.format(A.angle(C)*180./np.pi))
13 print('gamma = {0:1.1f} degrees'.format(A.angle(B)*180./np.pi))

```

```

a = 4.0 angstroms
b = 6.0 angstroms
c = 5.0 angstroms
alpha = 90.0 degrees
beta  = 120.0 degrees
gamma = 90.0 degrees

```

In the example above, we show how to construct the $a, b, c, \alpha, \beta, \gamma$ representation of a 3D unit cell.

When the vectors are not given in cartesian coordinates, but rather in some basis set of vectors that is not orthonormal, the example above does not work. We need a different approach to calculate angles between vectors. We define [Graef and McHenry \[2007\]](#) without derivation, that

$$\theta = \cos^{-1} \frac{\vec{a} \cdot \vec{g} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

Let us recalculate the angles between the vectors in the previous example with this equation.

```

1 import numpy as np
2 A = [1, 0, 0]
3 B = [0, 1, 0]
4 C = [0, 0, 1]
5
6 # these are the basis vectors
7 a = np.array([ 4, 0, 0])
8 b = np.array([ 0, 6, 0])
9 c = np.array([-2.5, 0, 5*np.sqrt(3)/2.])
10
11 # define metric tensor
12 g = np.array([[np.dot(a,a), np.dot(a,b), np.dot(a,c)],
13               [np.dot(b,a), np.dot(b,b), np.dot(b,c)],
14               [np.dot(c,a), np.dot(c,b), np.dot(c,c)]])
15
16 alpha = np.arccos(np.dot(B, np.dot(g, C))
17                  /(np.sqrt(np.dot(B, B)) * np.sqrt(np.dot(C, C))))*180./np.pi
18
19 print alpha

```

90.0

As an example, we consider the vectors $[1, 0, 1]$ and $[-2, 0, 1]$ in the basis vectors $[4, 0, 0]$, $[0, 6, 0]$, and $[-2.5, 0, 5\sqrt{3}/2]$. What is the angle between these vectors?

```

1 import numpy as np
2
3 a = np.array([ 4, 0, 0])
4 b = np.array([ 0, 6, 0])
5 c = np.array([-2.5, 0, 5*np.sqrt(3)/2.])
6
7 # define metric tensor
8 g = np.array([[np.dot(a,a), np.dot(a,b), np.dot(a,c)],
9               [np.dot(b,a), np.dot(b,b), np.dot(b,c)],
10              [np.dot(c,a), np.dot(c,b), np.dot(c,c)]])
11
12 p = np.array([1, 0, 1])
13 q = np.array([-2, 0, 1])
14
15 lp = np.sqrt(np.dot(p, np.dot(g, p)))
16 lq = np.sqrt(np.dot(q, np.dot(g, q)))
17
18 theta = np.arccos(np.dot(p, np.dot(g, q))/(lp * lq))
19 print 'The angle between the vectors is {0:1.2f} degrees.'.format(theta * 180. / np.pi)

```

The angle between the vectors is 86.70 degrees.

4.3 Computing unit cell volumes

The volume of a unit cell is particularly simple to compute from the unit cell vectors: $V = |\det U|$ where U is the unit cell vector matrix. We must take the absolute value because the order of vectors is arbitrary, and some orders will result in negative determinants.

Let us consider a trivial example of the volume of a cube first.

```

1 import numpy as np
2
3 a = np.array([5, 0, 0])
4 b = np.array([0, 5, 0])
5 c = np.array([0, 0, 5])
6
7 U = np.vstack([a, b, c])
8 print('V = {0:1.3f} ang^3'.format(np.abs(np.linalg.det(U))))

```

V = 125.000 ang³

Now, consider the primitive lattice vectors of an fcc lattice with a lattice constant of 5 and ask what is the volume?

```

1 import numpy as np
2
3 U = 5.0 * np.array([[0.5, 0.5, 0.0],
4                    [0.5, 0.0, 0.5],
5                    [0.0, 0.5, 0.5]])
6
7 print('V = {0:1.3f} ang^3'.format(np.abs(np.linalg.det(U))))

```

V = 31.250 ang³

No geometry required! Note that the volume of the primitive cell is 1/4 of the conventional cubic cell. Naturally, the primitive cell has one atom in it, whereas the conventional cell has four atoms in it.

4.4 The reciprocal lattice in crystallography

The reciprocal lattice is a convenient, alternative set of basis vectors that are defined by:

$$A \cdot A^{*T} = I$$

Or,

$$A^* = (A^{-1})^T$$

Let us consider a brief example:

```

1 import numpy as np
2
3 # primitive fcc lattice
4 A = 3.61*np.array([[0.0, 0.5, 0.5],
5                  [0.5, 0.0, 0.5],
6                  [0.5, 0.5, 0.0]])
7
8 Astar = np.linalg.inv(A).T
9
10 print Astar

```

```

[[-0.27700831  0.27700831  0.27700831]
 [ 0.27700831 -0.27700831  0.27700831]
 [ 0.27700831  0.27700831 -0.27700831]]

```

It is of some interest that the reciprocal lattice of the fcc lattice has the same symmetry as the bcc lattice!

The reciprocal lattice is of particular significance because a vector in the reciprocal lattice basis set with components (h, k, l) is normal to the surface plane with Miller indices (h, k, l). Furthermore, the distance between the planes with Miller indices is related inversely to the length of the reciprocal lattice vector. Thus, we find ourselves needing to conveniently calculate the length of a reciprocal lattice vector, which may be defined in a non-orthonormal set of basis vectors. As we did in real space, we define the reciprocal metric tensor:

$$g^* = \begin{bmatrix} a^* \cdot a^* & a^* \cdot b^* & a^* \cdot c^* \\ b^* \cdot a^* & b^* \cdot b^* & b^* \cdot c^* \\ c^* \cdot a^* & c^* \cdot b^* & c^* \cdot c^* \end{bmatrix}$$

As we saw in the real space computational crystallography section, the reciprocal metric tensor is useful in computing quantities in the reciprocal basis vector space.

For example, given a cubic crystal with a lattice constant of 2, compute the distance between the (110) planes. It was stated that the distance between two planes is inversely related to the length of the reciprocal lattice vector with components of the Miller indices that defines the plane of interest.

```

1  import numpy as np
2
3  A = np.array([[2, 0, 0],
4               [0, 2, 0],
5               [0, 0, 2]])
6
7  Astar = np.linalg.inv(A).T
8
9  astar = Astar[0, :]
10 bstar = Astar[1, :]
11 cstar = Astar[2, :]
12
13 gstar = np.array([[np.dot(astar, astar), np.dot(astar, bstar), np.dot(astar, cstar)],
14                  [np.dot(bstar, astar), np.dot(bstar, bstar), np.dot(bstar, cstar)],
15                  [np.dot(cstar, astar), np.dot(cstar, bstar), np.dot(cstar, cstar)]])
16
17 # MUCH more compact definition of g*
18 gstar = np.dot(Astar, Astar.T)
19
20 v = [1, 1, 0]
21
22 d_110 = np.sqrt(np.dot(v, np.dot(gstar, v)))
23
24 spacing = 1./d_110
25 print 'The spacing between the planes is {0}.'.format(spacing)

```

The spacing between the planes is 1.41421356237.

Let us consider another example, with a more complex unit cell. Find the spacing between (111) planes for the following unit cell.

```

1 import numpy as np
2
3 # these are the basis vectors
4 a = np.array([ 3, 0, 0])
5 b = np.array([-2, 0, 4*np.sqrt(3)/2.])
6 c = np.array([ 0, 6, 0])
7
8
9 A = np.vstack([a, b, c])
10
11 Astar = np.linalg.inv(A).T
12
13 # define reciprocal metric tensor
14 gstar = np.dot(Astar, Astar.T)
15
16 v = [1, 1, 1]
17 dstar_111 = np.sqrt(np.dot(v, np.dot(gstar, v)))
18 print 'd = ', 1./dsstar_111

```

d = 1.64316767252

The linear algebra notation we use here is very compact, and avoids the need to memorize the special equations that are valid for specific crystal systems.

4.5 Coordinate system transformations

It is frequently convenient to work in different coordinate systems. For example, the cartesian coordinate system is intuitive, but it can be difficult to tell if a position is inside a unit cell that if the unit cell vectors are not orthogonal. In this case it can be convenient to work in fractional coordinates, where the point uses the unit cell vectors as a basis. Alternatively, if there is a lattice, it may be convenient to use the primitive lattice vectors as a basis, because then all points will have vectors with integer components. It is not difficult to interchange between these different basis sets if we understand some linear algebra.

Let us consider computing the fractional coordinates of a position in the unit cell basis set. Given a point at $\vec{p} = p_1\hat{x} + p_2\hat{y} + p_3\hat{z}$, and the unit cell vectors

$\underline{\underline{A}} = \begin{bmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{bmatrix}$, we can express the fractional coordinates as

$$\vec{s} = (\underline{\underline{A}}^T)^{-1}\vec{p}^T$$

\vec{s} conventionally has components between 0 and 1, and anything less than zero or greater than one is outside the unit cell.

Let us consider another example. Let us compute the unit cell vectors of the conventional fcc unit cell in the primitive lattice vector basis set.

```

1: import numpy as np
2:
3: # conventional vectors
4: A = np.array([[1, 0, 0],
5:               [0, 1, 0],
6:               [0, 0, 1]])
7:

```

```

8  8: P = np.array([[0.0, 0.5, 0.5],
9  9:                [0.5, 0.0, 0.5],
10 10:                [0.5, 0.5, 0.0]])
11 11:
12 12: a = np.dot(np.linalg.inv(P.T), A.T).T
13 13:
14 14: print a

```

```

[[-1.  1.  1.]
 [ 1. -1.  1.]
 [ 1.  1. -1.]]

```

The vectors should have components that are all integers (every lattice point must be an integer linear combination of the primitive lattice vectors). There is an important note to consider about line 12 and that is the final transpose of the dot product. Without this, the components of the transformed vectors are in *columns*, and we have constructed everything in *rows*. In this specific case, it does not matter because the matrix is highly symmetric, and the transpose does not change it. In less symmetric cases, it can matter a lot, and it is important to keep the linear algebra in mind. The vector of the point being transformed must be a column vector, or a matrix with the components going column-wise.

5 Low Energy Electron Diffraction (LEED)

- As mentioned the structures of surface are not just what they would appear to be from cleaving of the bulk crystal. Reconstructions can occur that change the periodicity of the surface lattice. Relaxations cause changes in the layer spacings at solid surfaces. Furthermore, adsorbed atoms and molecules can create new surface lattices, induce reconstructions or even remove the reconstructions of the clean surfaces.
- There are several types of structural experiments that have been developed. The most common are Low Energy Electron Diffraction (LEED) and Scanning Tunneling Microscopy (STM).
- LEED is the primary way in which the structures of surfaces and the adsorbed layers have been determined quantitatively and is quite commonly done by most research groups in the field of surface science.
- LEED can be used in one of two modes :
 - determination of unit cells of surfaces and adsorbed overlayers, and
 - true quantitative structure determination.
- The quantitative determination of structure is very difficult and highly computationally intensive.

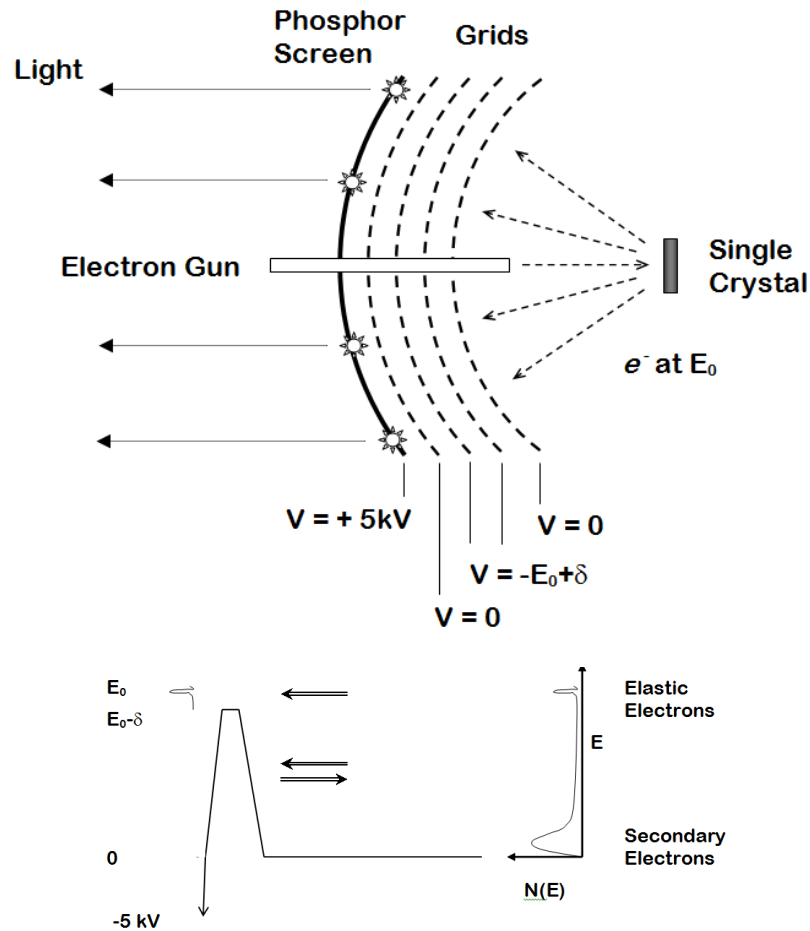


Figure 15: Potential energy profile seen by the electron beam.

- The phosphorus screen allows easy and cheap visualization of the LEED pattern. Usually a photograph is taken of the back side of the screen. This projects the diffraction pattern onto a plane.
- More complex methods of detection allow measurement of the diffraction spot profiles and the intensities.

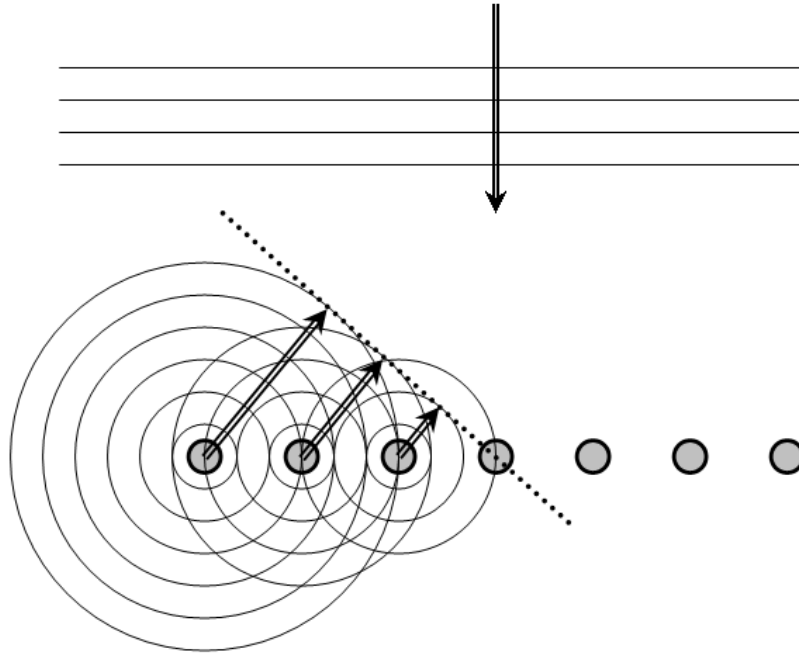
5.1 Diffraction

- Diffraction requires wavelike behavior of the electron. This was postulated by de Broglie.

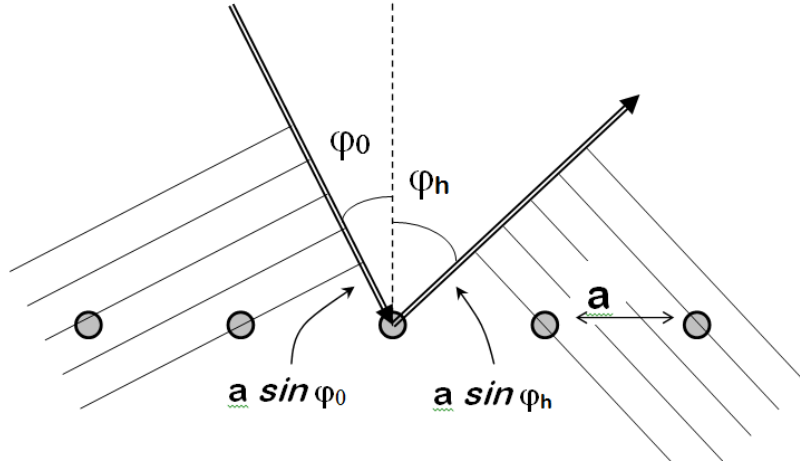
$$\lambda = \frac{h}{p} = \sqrt{\frac{150}{E_{kin}}}$$

This gives the wavelength in Å for kinetic energy in eV.

- Let us review the rules of diffraction from a lattice in 1-D. Basically diffraction is a process in which a wave is incident on an array of atoms. To first order the waves scattered by each atom are spherical waves propagating with the same wavelength as the incident wave. At long distances from the array the scattered waves can be considered to be plane waves from each source atom.



- Diffraction occurs when the distances traveled by the scattered plane waves all differ by some integral multiple of the wavelength. A wave with wavelength λ is incident on a 1D lattice with spacing a at an angle ϕ_0 .

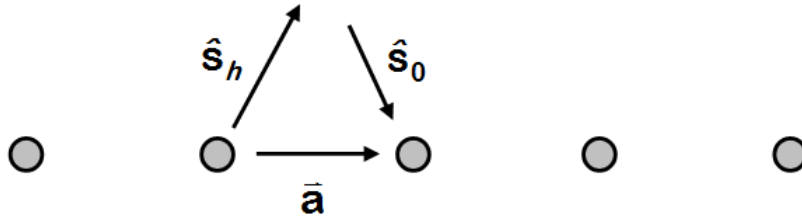


- In order to have constructive interference and obtain a diffracted beam the waves scattering off adjacent lattice points must travel distances which differ by integral multiples of the wavelength.
- The diffraction condition must be that

$$a(\sin \phi_h - \sin \phi_0) = h\lambda$$

for $h \in I$. For a given incident angle ϕ_0 this determines the angles ϕ_h at which diffracted beams appear.

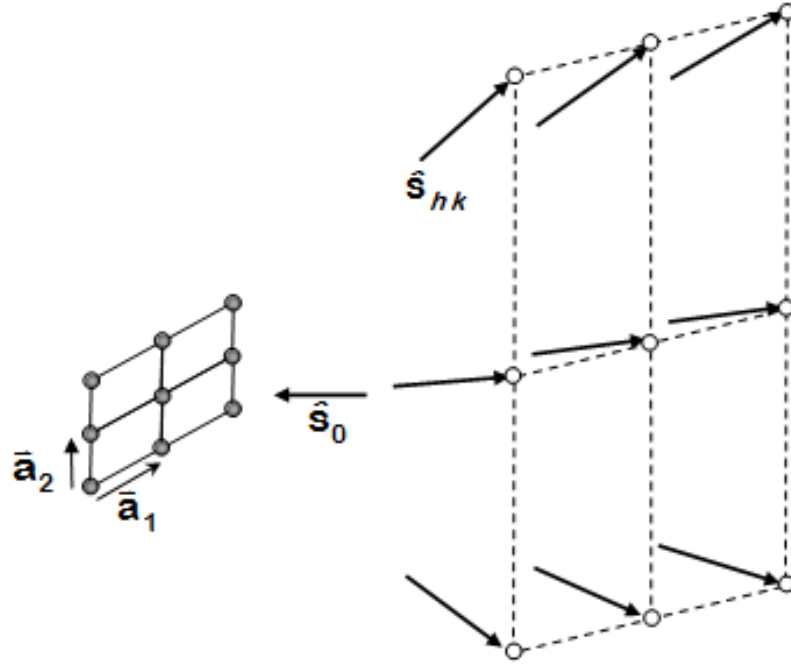
- Another way of writing this equation is to think of the lattice spacing as a vector \vec{a} and the incident and scattered waves as traveling along directions given by the unit vectors \hat{s}_0 and \hat{s}_h .



- Then the diffraction condition can be written as

$$\vec{a} \cdot (\hat{s}_h - \hat{s}_0) = h\lambda$$

Consider the 2D diffraction problem from a lattice with vectors \vec{a}_1 and \vec{a}_2 . The beam is incident along a vector \hat{s}_0 and diffracting in a direction \hat{s}_{hk} .



- Now the diffraction conditions are given by

$$\vec{a}_1 \cdot (\hat{s}_{hk} - \hat{s}_0) = h\lambda$$

and

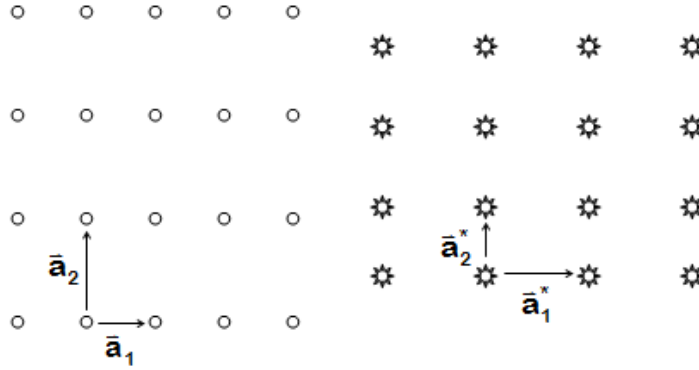
$$\vec{a}_2 \cdot (\hat{s}_{hk} - \hat{s}_0) = k\lambda$$

Both must be satisfied in order to get diffraction

5.2 The reciprocal lattice

For any lattice vectors, (\vec{a}_1, \vec{a}_2) , we can define a new set of vectors, which we call the reciprocal lattice $(\vec{a}_1^*, \vec{a}_2^*)$ based on these definitions:

$$\begin{aligned}\bar{\mathbf{a}}_1 \cdot \bar{\mathbf{a}}_1^* &= 1 & \bar{\mathbf{a}}_1 \cdot \bar{\mathbf{a}}_2^* &= 0 \\ \bar{\mathbf{a}}_2 \cdot \bar{\mathbf{a}}_1^* &= 0 & \bar{\mathbf{a}}_2 \cdot \bar{\mathbf{a}}_2^* &= 1\end{aligned}$$



- Note that the reciprocal lattice is periodic and has symmetry just like the real space lattice.
- The power of this construction is that it provides us with the solutions to the diffraction problem. The directions in which diffraction occurs are given by

$$\hat{s} - \hat{s}_0 = h\lambda\bar{\mathbf{a}}_1^* - k\lambda\bar{\mathbf{a}}_2^*$$

Let us consider a general approach to computing the reciprocal lattice. First, we consider how to represent the vectors in the form of a matrix. For the two lattice vectors \vec{a}_1 and \vec{a}_2 , we express them in a matrix as: $A = [\vec{a}_1 \vec{a}_2]$. We next consider the reciprocal lattice vectors, $A^* = \begin{bmatrix} \vec{a}_1^* \\ \vec{a}_2^* \end{bmatrix}$. Now, we have the following equation: $A \cdot A^* = I$, where I is the identity matrix. We can readily solve for the reciprocal lattice vectors as $A^* = I \cdot A^{-1} = A^{-1}$.

Let us consider a specific example where $a_1 = 1x + 0y$ and $a_2 = 0x + 2y$.

```

1: import numpy as np
2:
3: a1 = [1, 0]           #(a1)
4: a2 = [0, 2]
5:
6: A = np.vstack([a1, a2]) #(vstack)
7: print('A = \n{0}'.format(A))
8:
9: Astar = np.linalg.inv(A) #(inv)
10:
11: a1star= Astar[0]        #(ref:a1*)
12: a2star = Astar[1]
13: print 'a1* = {0}'.format(a1star)
14: print 'a2* = {0}'.format(a2star)
15:
16: # Verify the requirements.
```

```

17 17: print np.dot(a1, a1star)    #(dot)
18 18: print np.dot(a1, a2star)
19 19: print np.dot(a2, a1star)
20 20: print np.dot(a2, a2star)

```

```

A =
[[1 0]
 [0 2]]
a1* = [ 1.  0.]
a1* = [ 0.  0.5]
1.0
0.0
0.0
1.0

```

This method works generally for 2 and 3 dimensional lattices. Some notes about the code above:

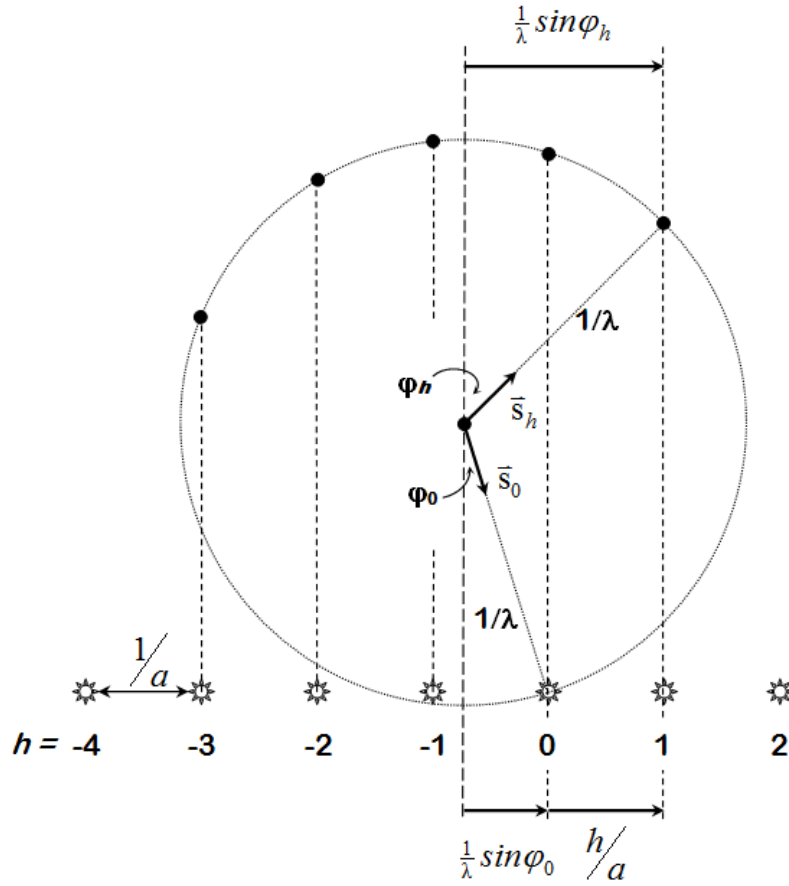
1. In line 3 we represent the components of the vector as a *list*.
2. In line 6 we create an array by using `numpy.vstack`. This makes the columns of the array the components of the vector. An array is not the same as a matrix!
3. In line 9 we calculate the inverse of the array. Note that now the components of the reciprocal lattice vectors are in the *rows* of the resulting array.
4. In line 11 we use indexing to extract the first row. In Python, indexing starts at 0.
5. In line 17 we must use the `numpy.dot` function to get the matrix multiplication, or dot product of the two vectors. The `*` operator will perform element-wise multiplication.

5.3 The LEED Pattern

- It will turn out that the LEED pattern is actually an image of the reciprocal lattice.
 - Interpretation of the LEED pattern requires transforming the image of the reciprocal lattice back into the real lattice.

5.3.1 Ewald Sphere

- At the crystal surface there is a lattice of real atoms and a reciprocal lattice.
 - Define an origin for the reciprocal lattice and then imagine a set of rods drawn perpendicular to the surface through the reciprocal lattice points.
 - Finally, draw a sphere of radius $r = \frac{1}{\lambda}$ over the lattice centered at a point $\frac{-\hat{s}_0}{\lambda}$ from the origin of the reciprocal lattice.



- The directions in which one gets diffraction beams are given by the intersections of the reciprocal lattice rods with the sphere.

$$\frac{1}{\lambda} \sin \varphi_h = \frac{1}{\lambda} \varphi_0 + \frac{h}{a} \quad (7)$$

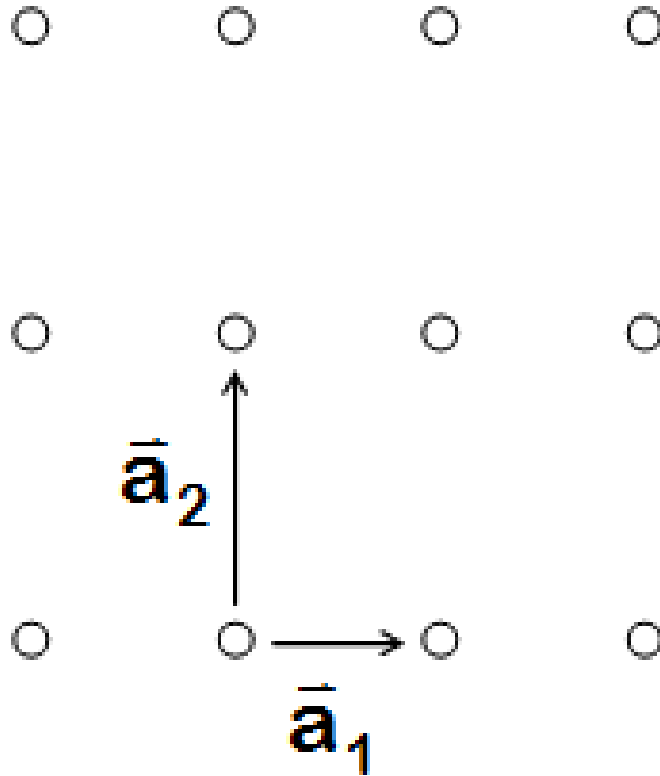
$$a \cdot (\sin \varphi_h - \sin \varphi_0) = h\lambda \quad (8)$$

- If you imagine a much larger sphere (the LEED) screen, then the diffracted beams are intersecting the screen and the image that one obtains is the projection of the screen onto a plane. The screen is a magnified version of the Ewald sphere and the image is then a magnified view of the reciprocal lattice.

- As the crystal is rotated the image on the screen will rotate also.
- As the energy of the beam increases, $\frac{1}{\lambda}$ increases and the number of beams intersecting the screen increases. What one sees is that the reflected beam stays put but the diffracted beams converge onto the reflected beam.
- From the image one can determine the real space lattice but nothing more. True structural information comes from the beam intensities.

5.3.2 LEED Patterns from Simple Clean Surfaces

- LEED patterns are simply images of the reciprocal lattice of the real space surface lattice.
 - Given the rectangular lattice below, what does the LEED pattern look like?



The lengths of the real space lattice vectors are:

$$|\vec{a}_1| = 1 \text{ and } |\vec{a}_2| = 2$$

Let us use python to compute the reciprocal lattice vectors.

```

1 import numpy as np
2
3 A = np.matrix([[1, 0],
4               [0, 2]])
5
6 Astar = A.T.I           #(ati)
7 print Astar

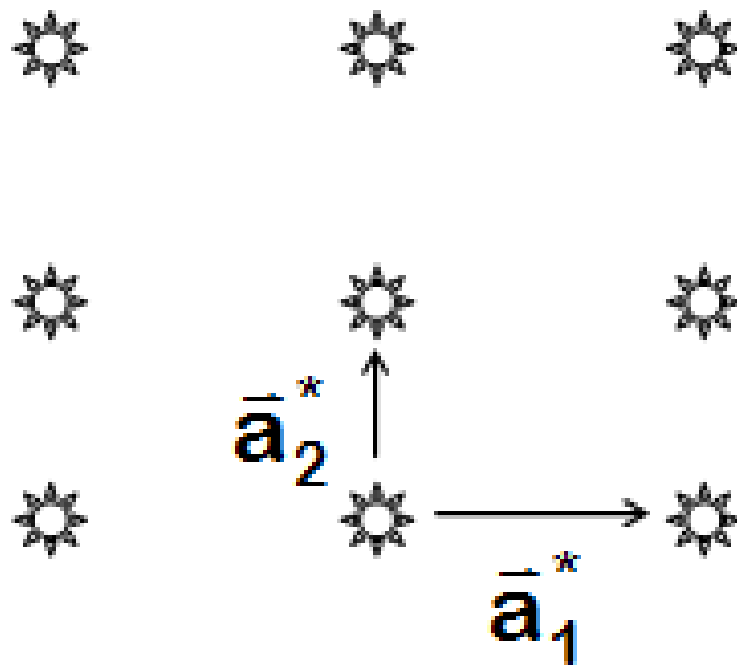
```

```

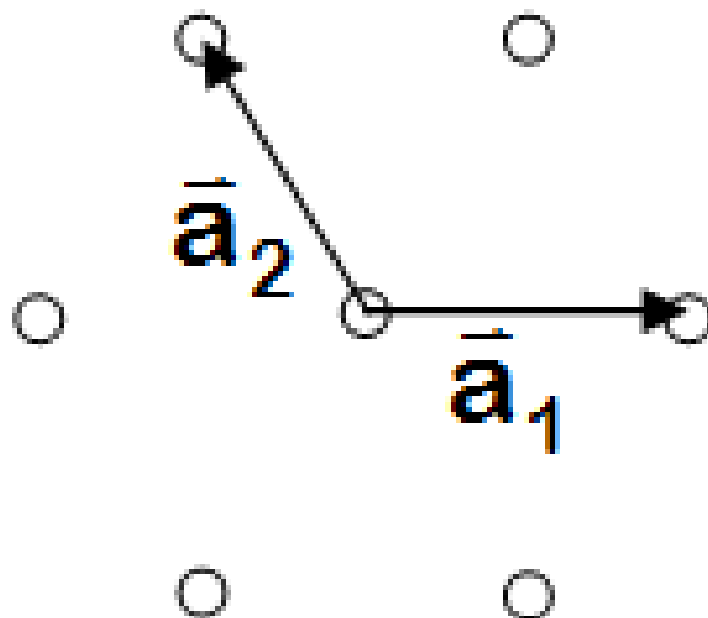
[[ 1.  0. ]
 [ 0.  0.5]]

```

The reciprocal lattice and therefore the LEED pattern look like:



- Given a hexagonal lattice below, what does the LEED pattern look like?



```

1 import numpy as np
2
3 A = np.matrix([[1, 0],
4                [-0.5, np.sqrt(3)/2]]) # note the vectors are in row form
5
6 Astar = A.T.I
7 print Astar

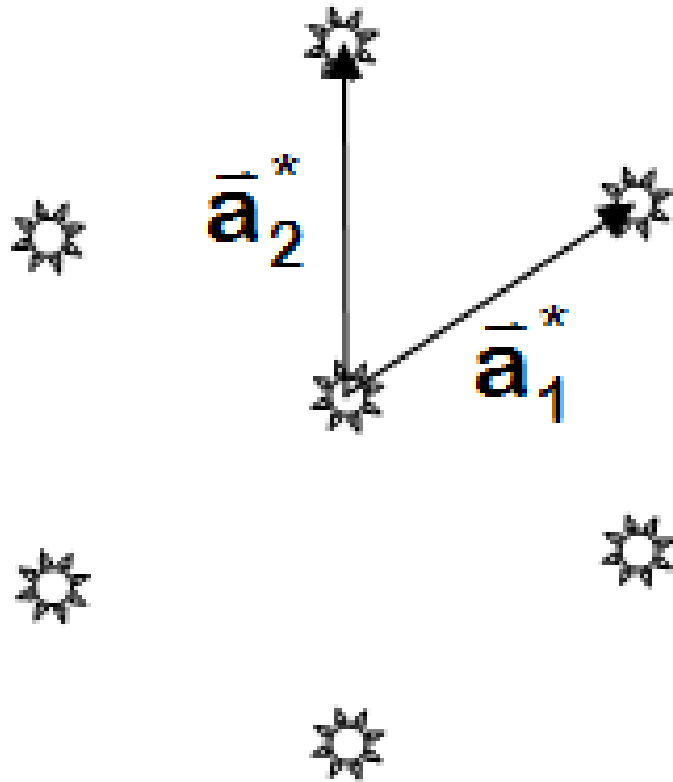
```

```

[[ 1.          0.57735027]
 [ 0.          1.15470054]]

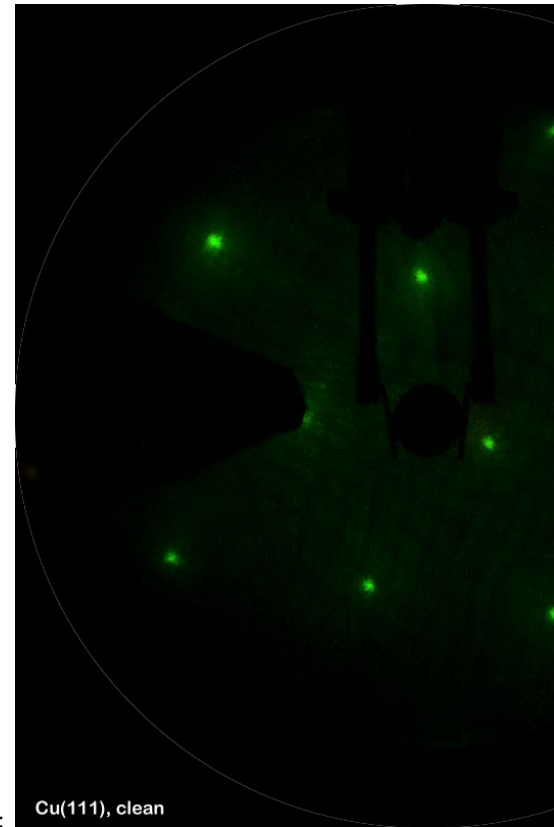
```

The reciprocal lattice and therefore the LEED pattern look like



- Note that the reciprocal lattice is a true lattice. It extends infinitely in both directions, is periodic, and all points are equivalent.

A real LEED image of a clean Cu(111) surface looks like this:



5.4 Overlayers and Superlattices

- As noted earlier some surfaces can reconstruct such that the periodicity of the surface is different from that of the bulk termination plane.
 - When atoms or molecules are adsorbed on surfaces they tend to form well defined structures. They will adsorb in well defined site on the surface and often with long range order or periodicity which is commensurate with the substrate but with longer periodicity.
 - Adsorption sites are often high symmetry sites which optimize the bonding between adsorbate and the surface: on-top, bridging, fcc hollow, hcp hollow etc.
 - The ordered overlayers are actually lattices of atoms or molecules. If they are commensurate with the substrate this means that they have periodicity or lattice vectors that are linear combinations of the surface lattice vectors.

5.4.1 Overlayer Notations

- Adsorbed atoms also produce ordered overlayers with lattice parameters different from those of the substrate.

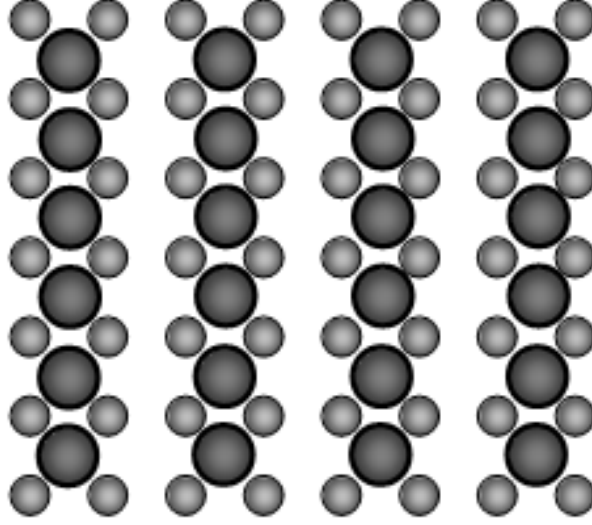


Figure 16: p(2x1) overlayer on a square lattice.

- The superlattice will have its own lattice vectors (\vec{b}_1, \vec{b}_2) which may or may not have some simple relationship with the substrate lattice vectors (\vec{a}_1, \vec{a}_2) .
- Those superlattices with lattice vectors that are separated by the same angle as the substrate lattice vectors can be referred to by Wood's notation.
- If the lengths are related by $|\vec{b}_1| = x|\vec{a}_1|$ and $|\vec{b}_2| = y|\vec{a}_2|$ and the angle between the vectors is φ , then the notation for the overlayer is $(x \times y)R\varphi$.

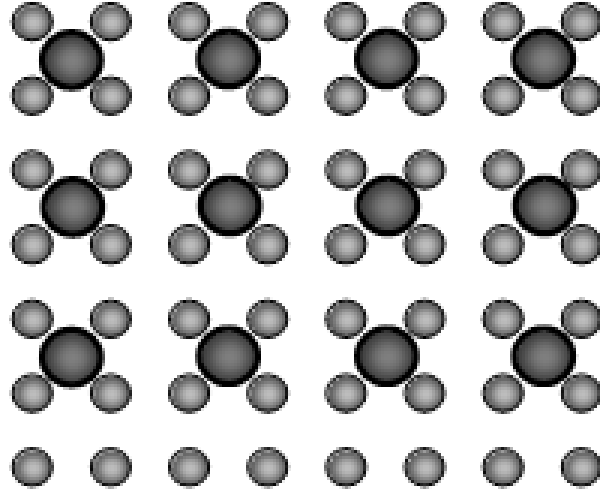


Figure 17: Example of $p(2 \times 2)$ on a square lattice.

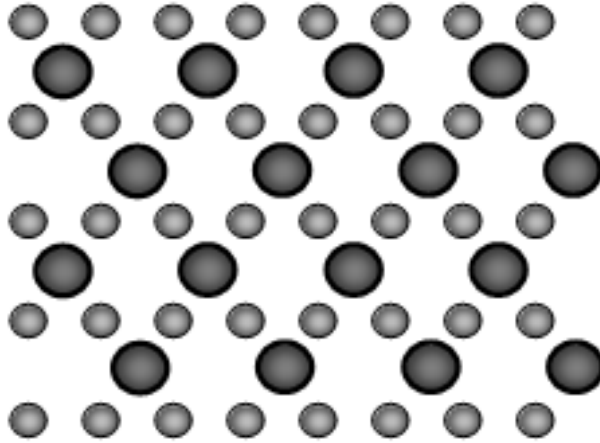


Figure 18: Example of a $c(2 \times 2)$ on a rectangular lattice.

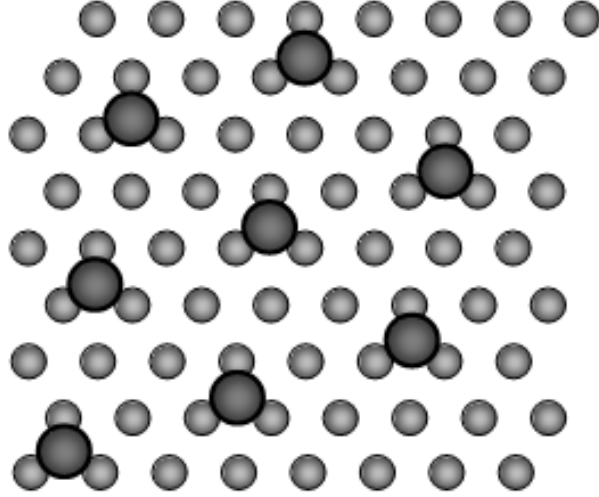


Figure 19: Example of a $(\sqrt{7} \times \sqrt{7})$ R19° on a hexagonal lattice.

- If the angle between the two sets of lattice vectors is not the same then it is not possible to define a Wood's notation for the superlattice.
- The relationship between the two lattices can always be given as

$$\vec{b}_1 = m_{11}\vec{a}_1 + m_{12}\vec{a}_2 \quad (9)$$

$$\vec{b}_2 = m_{21}\vec{a}_1 + m_{22}\vec{a}_2 \quad (10)$$

- The overlayer can then be denoted by a simple matrix which transforms the overlayer lattice into the substrate lattice.

$$\begin{pmatrix} \vec{b}_1 \\ \vec{b}_2 \end{pmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{pmatrix} \vec{a}_1 \\ \vec{a}_2 \end{pmatrix} = \underline{\underline{M}} \begin{pmatrix} \vec{a}_1 \\ \vec{a}_2 \end{pmatrix}$$

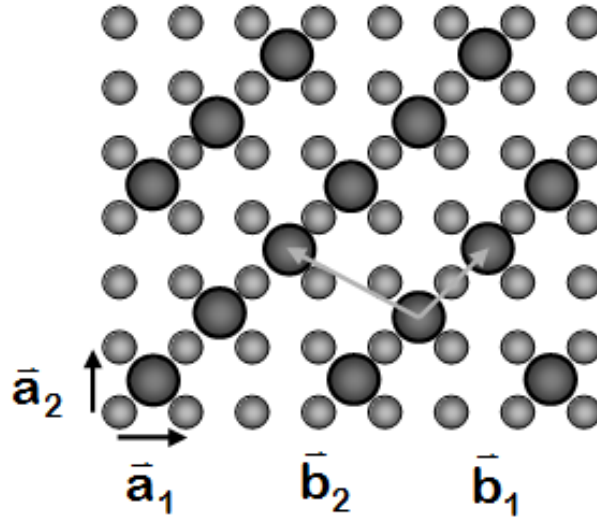


Figure 20: Example of a lattice with a structure requiring matrix notation.

In this example: $\vec{b}_1 = 1 \cdot \vec{a}_1 + 1 \cdot \vec{a}_2$
 and $\vec{b}_2 = -2 \cdot \vec{a}_1 + 1 \cdot \vec{a}_2$
 so:

$$M = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

- Notice that there are several different choices of lattice vectors that can be used. All describe the same lattice.

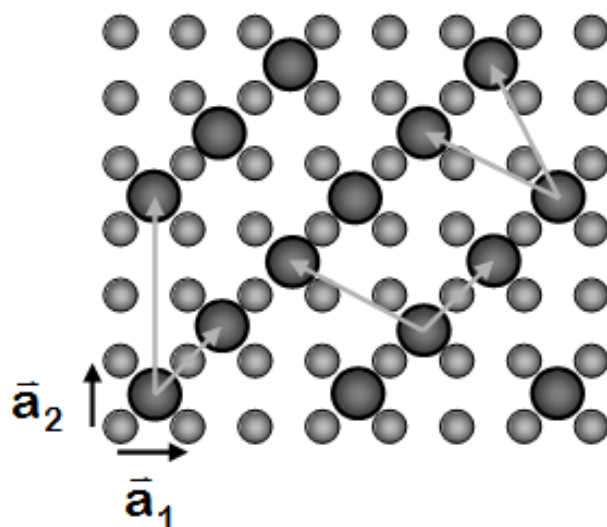
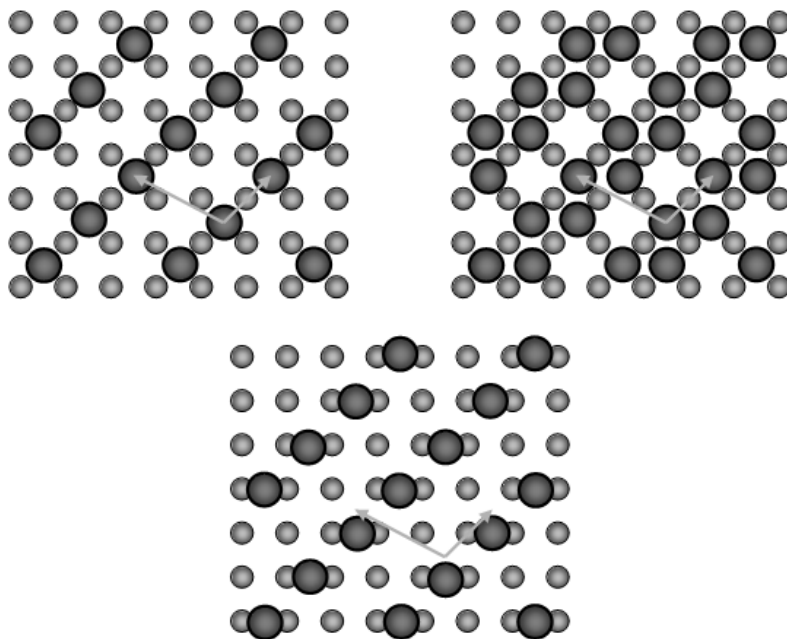


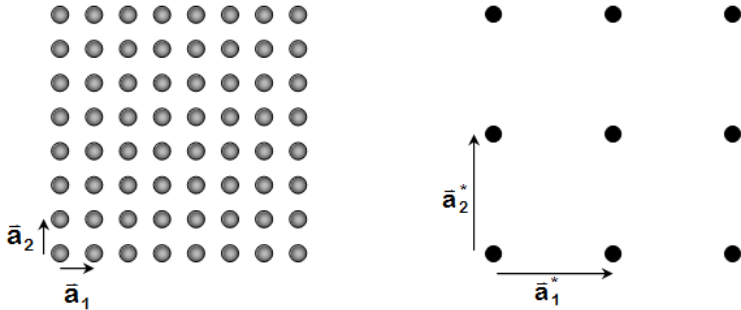
Figure 21: Equivalent lattice vectors for describing the same lattice.

- Also note that there are several different adsorbate overlayers that yield the same lattice. They may have different bases however.



5.4.2 LEED Patterns from superlattices

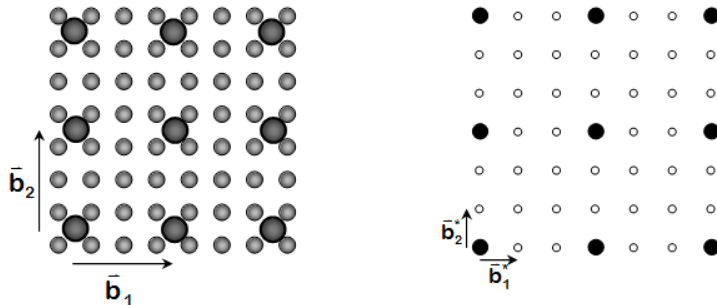
- LEED patterns are produced by diffraction from lattices at surfaces. The clean unreconstructed surface is said to give a (1×1) pattern. The LEED pattern would be the reciprocal lattice of the surface lattice vectors determined by termination of the bulk lattice.
- The LEED pattern is actually a picture of the reciprocal lattice.
- One of the primary objectives of the LEED experiment is to identify the periodicity of the overlayer lattice.
- Since there is a well defined relationship between the reciprocal lattice and the real lattice it is not hard to obtain the real space lattice parameters from the reciprocal space lattice.
- Example - $p(3 \times 3)$ on an fcc(100) surface.
 - The fcc(100) lattice is a square array and thus gives a diffraction pattern



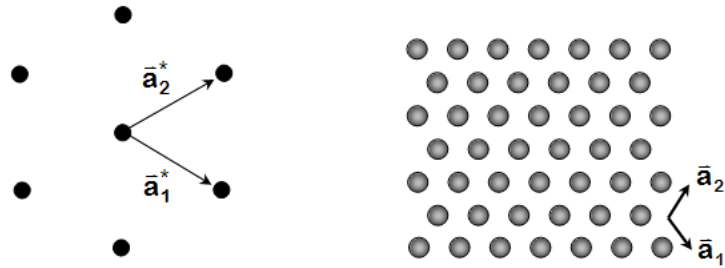
which is square.

$$\bar{b}_1 = 3\bar{a}_1$$

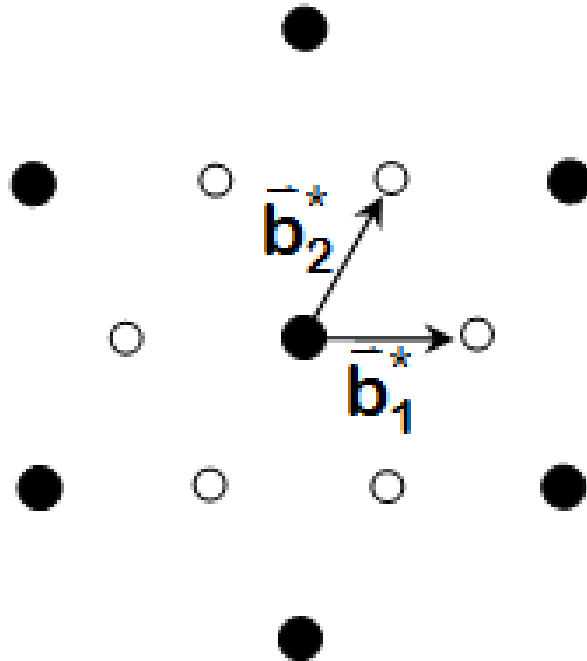
$$|\bar{b}_1^*| = \frac{1}{|\bar{b}_1|} = \frac{1}{3|\bar{a}_1|} = \frac{1}{3}|\bar{a}_1^*|$$



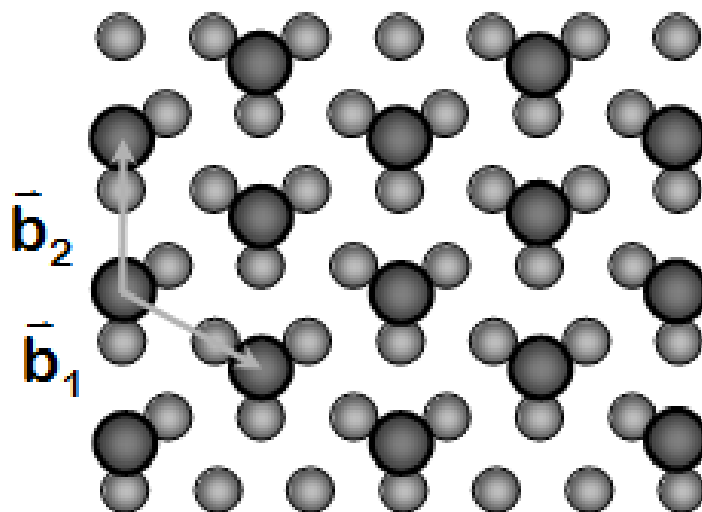
- Example - $(\sqrt{3} \times \sqrt{3})R30^\circ$ on an fcc(111) lattice.
- The fcc(111) lattice is hexagonal so its diffraction pattern has hexagonal symmetry.



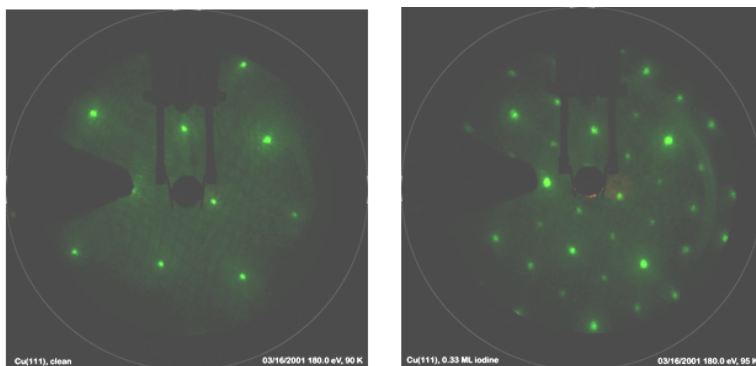
- An adsorbed layer of atoms on the surface gives the LEED pattern shown below. Note that the spots would all appear the same except that there are some new ones due to diffraction from the overlayer.



- This gives a structure such as the following for the real space lattice. It is a $(\sqrt{3} \times \sqrt{3})R30^\circ$ lattice.



Here is what an actual LEED pattern looks like for a clean surface of Cu(111) and a $(\sqrt{3} \times \sqrt{3})R30^\circ$ adsorbate overlayer.

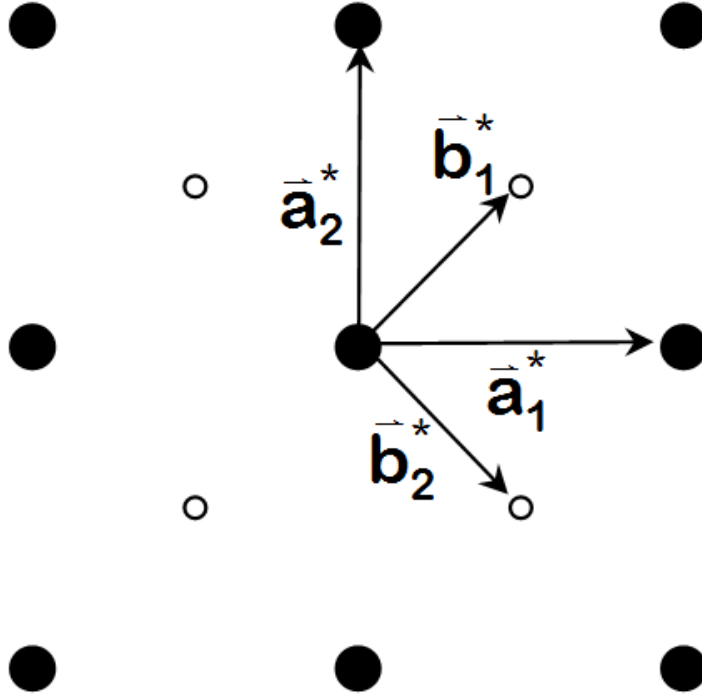


- Remember that the reciprocal lattice only gives you the real space lattice and not the positions of the atoms in the unit cells or the registry of the lattice with respect to the substrate.

5.5 General Reciprocal Space to Real Space Conversion

- There is a general means for converting a reciprocal lattice into a real space lattice.
 - The usual type of problem is one in which the substrate lattice (1x1) is known and what one is trying to do is to determine the unit cell of the overlayer in terms of the substrate lattice vectors.
 - Basically the problem is to find M , the matrix which relates the overlayer lattice to the surface lattice.

5.5.1 Example - LEED of $(\sqrt{2} \times \sqrt{2})R45^\circ$ on an fcc(100) lattice.



- Find the reciprocal lattice vectors from the LEED pattern. You can usually tell which pattern belongs to the surface atoms; they are the ones with the longest spacings (because they are the closest together).

$$\vec{b}_1^* = \frac{1}{2}\vec{a}_1^* + \frac{1}{2}\vec{a}_2^*$$

$$\vec{b}_2^* = \frac{1}{2}\vec{a}_1^* - \frac{1}{2}\vec{a}_2^*$$

- Find the reciprocal space overlayer matrix.

$$M^* = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- Find the real space overlayer matrix from the reciprocal space overlayer matrix.

$$M = [(M^*)^T]^{-1}$$

- Use the real space matrix to find the unit cell of the overlayer lattice.

$$b = M \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix}$$

```

1 import numpy as np
2
3 Mstar = 0.5 * np.array([[1, 1], [1, -1]])
4
5 M = np.linalg.inv(Mstar.T)
6 print M

```

```

[[ 1.  1.]
 [ 1. -1.]]

```

- Note that all we know is the scaling and the orientation of the overlayer unit cell with respect to the substrate lattice. We do not know the registry or the contents of the unit cell.

- Note that the area of the unit cell is $A = 2$. In other words it contains two substrate atoms.

- In terms of the area of the substrate lattice the superlattice area is

$area = |\vec{b}_1 \times \vec{b}_2|$ —

- The coverage of the adsorbate must be some multiple of the inverse unit cell area.

$\theta = n/A$

- In the case of the $(\sqrt{2} \times \sqrt{2})R45^\circ$ lattice the coverage is most probably

$\theta = 1/2$

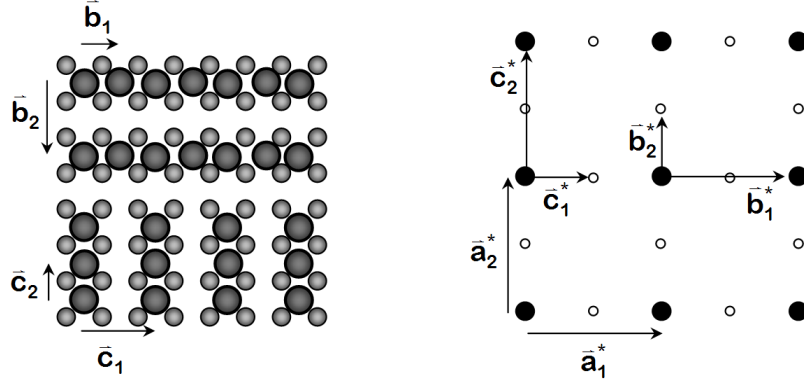
although some independent means is needed to determine this.

5.6 Domains

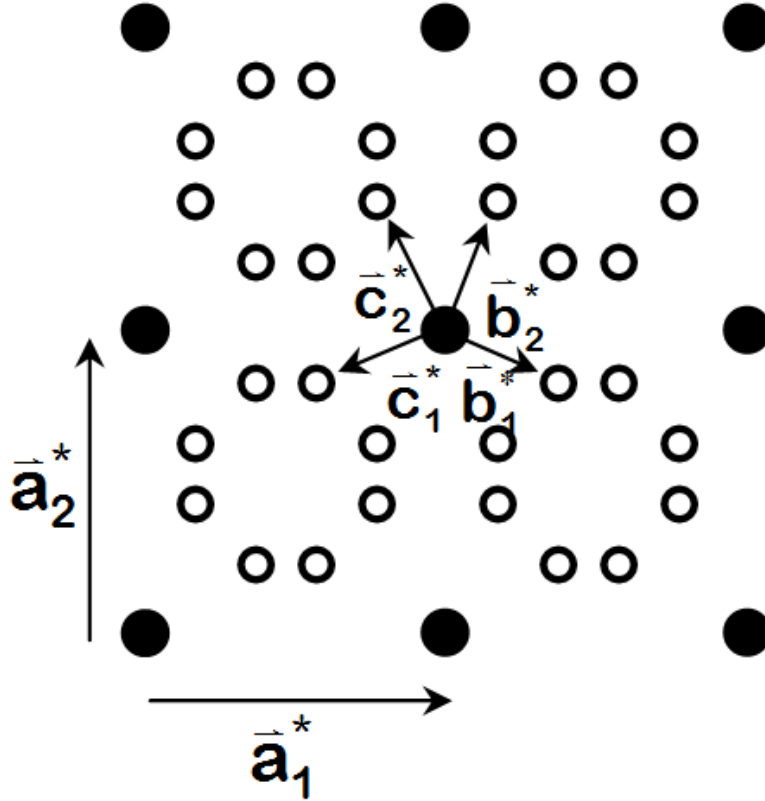
- It is very often the case that because of symmetry of the substrate multiple orientations of the superlattice are energetically equivalent.

- The presence of one or more domains can give two superimposed LEED patterns. This complicates the analysis of the LEED pattern to get the reciprocal lattice.

5.6.1 Example - (2×1) overlayer on a square fcc(100) lattice.



5.6.2 Example - $(\sqrt{5} \times \sqrt{5})R26^\circ$ on square lattice. Oxygen on Mo(100) at $\theta = 0.6$.



The reciprocal lattice vectors are given by:

$$\vec{b}_1^* = \frac{2}{5}\vec{a}_1^* - \frac{1}{5}\vec{a}_2^*$$

$$\vec{b}_2^* = \frac{1}{5}\vec{a}_1^* + \frac{2}{5}\vec{a}_2^*$$

This gives the reciprocal lattice matrix as:

```

1 import numpy as np
2
3 Mstar = 1. / 5. * np.array([[2, -1],
4                             [1,  2]])
5
6 M = np.linalg.inv(Mstar.T)
7 print M

```

```

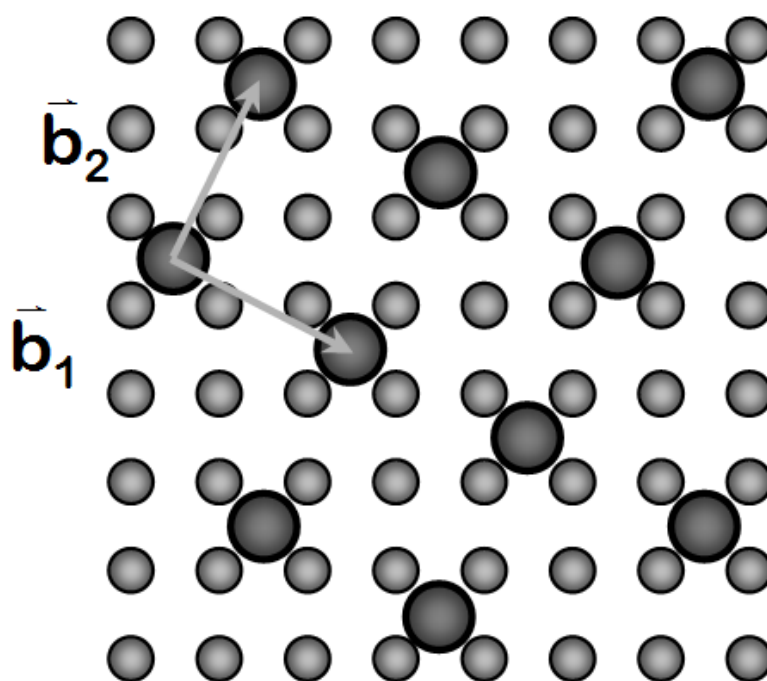
[[ 2. -1.]
 [ 1.  2.]]

```

The real space lattice vectors are:

$$\vec{b}_1 = 2\vec{a}_1 - \vec{a}_2$$

$$\vec{b}_2 = \vec{a}_1 + 2\vec{a}_2$$



6 References

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