Functions

2019

New Expressions and Statements

- Extended expressions

$$\mathscr{E} += \mathscr{X}\,\mathscr{E}^*$$

Call $f(e_1, \ldots, e_k)$

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$$\mathscr{S} += \texttt{\textbf{return}}\ \mathscr{E}^?$$

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- Extended configuration

$$\mathscr{C} = \Sigma \times \mathbb{Z}^* \times \mathbb{Z}^*$$

State $\mathscr{X} \to \mathbb{Z}$ Input Output

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$$\mathscr{S} += \texttt{\textbf{return}}\ \mathscr{E}^?$$

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Optional result

$$\mathscr{C} = \Sigma \times \mathbb{Z}^* \times \mathbb{Z}^* \times \mathbb{Z}^?$$

State $\mathscr{X} \to \mathbb{Z}$ Input Output

Big-Step Semantics for Expressions

$$\Phi \vdash \langle \sigma, i, o, - \rangle \xRightarrow{n} \langle \sigma, i, o, n \rangle \qquad [\mathsf{Const}^{\mathscr{E}}_{bs}]$$

$$\Phi \vdash \langle \sigma, i, o, - \rangle \xRightarrow{x} \langle \sigma, i, o, \sigma\, x \rangle \qquad [\mathsf{Var}^{\mathscr{E}}_{bs}]$$

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New Semantics for Statements

- return

$$\frac{}{c \xRightarrow{\textbf{return } e} c'}$$

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Suppose we fill it

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- **return** e ; S

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New Semantics for Statements

- return

$$\frac{[\![e]\!] \Downarrow \quad \textcolor{red}{???}}{c \xRightarrow{\textbf{return } e} c'}$$

Suppose we fill it

- $\texttt{return}\ e\ ;\ S$

$$\frac{c \xRightarrow{\textbf{return } e} c'' \qquad c'' \xRightarrow{S} c'}{c \xRightarrow{\textbf{return } e\ ;\ S} c'}$$

Always executes S!

I.e. **return** is not a local construction (from the control flow point of view; as **break**, **throw**, **continue**, ...)

CPS Semantics for Statements

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- New component in the environment

$$K, \Phi \vdash c \xRightarrow{s} c'$$

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Lack of *locality*

CPS Semantics for Statements

- New component in the environment

$$K, \Phi \vdash c \overset{s}{\Longrightarrow} c'$$

Lack of *locality*

- New meta-operator $\diamond$

$$\begin{array}{rcrcl} S & \diamond & \mathbf{skip} & = & S \\ S_1 & \diamond & S_2 & = & S_1; S_2 \end{array}$$

$$\texttt{skip}, \Phi \vdash c \xRightarrow{\texttt{skip}} c \qquad [\text{SkipSkip}]$$

CPS Rules — Basic Stmts

$$\texttt{\textbf{skip}}, \Phi \vdash c \xrightarrow{\texttt{\textbf{skip}}} c \qquad \text{[SkipSkip]}$$

$$\frac{\texttt{\textbf{skip}}, \Phi \vdash c \xrightarrow{K} c' \qquad K \neq \texttt{\textbf{skip}}}{K, \Phi \vdash c \xrightarrow{\texttt{\textbf{skip}}} c'} \qquad \text{[Skip]}$$

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CPS Rules — Seq and If

$$\frac{}{K, \Phi \vdash c \xRightarrow{s_1\,;\,s_2} c'} \quad [\mathsf{Seq}]$$

CPS Rules — Seq and If

$$\frac{s_2 \diamond K,\, \Phi \vdash c \xRightarrow{s_1} c'}{K,\, \Phi \vdash c \xRightarrow{s_1;\, s_2} c'} \quad [\text{Seq}]$$

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CPS Rules — While

$$\frac{}{K, \Phi \vdash c \xRightarrow{\texttt{while}\ e\ \texttt{do}\ s} c'} \quad [\text{WhileTrue}]$$

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CPS Rules — Call and Return

$$\frac{}{K, \Phi \vdash c \xRightarrow{f(\overline{e_k})}} \quad [\text{Call}]$$

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$$K, \Phi \vdash c \xrightarrow{\texttt{return}} \quad [\text{ReturnEmpty}]$$

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$$\frac{\begin{array}{c}\text{for } j \in [1..k] \;.\; \Phi \vdash c_{j-1} \xRightarrow{e_j}_{\mathscr{E}} c_j = \left\langle \sigma_j, i_j, o_j, v_j \right\rangle \\ \Phi\, f = \texttt{fun}\, f\ \ (\overline{a})\ \texttt{local}\, \overline{l}\ \ \{s\} \\ \texttt{skip}, \Phi \vdash \left\langle \textbf{enter}\, \sigma_k\ (\overline{a}@\overline{l})\ [\overline{a_j \leftarrow v_j}], i_k, o_k, \textbf{—} \right\rangle \xRightarrow{s} \langle \sigma', i', o', n \rangle \\ \texttt{skip}, \Phi \vdash {\color{red}???} \xRightarrow{K} c'' \end{array}}{K, \Phi \vdash c_0 = \langle \sigma_0, \_, \_, \_ \rangle \xRightarrow{f(\overline{e_k})}} \quad [\text{Call}]$$

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Functions X86-32

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$\ldots$
env_f
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...
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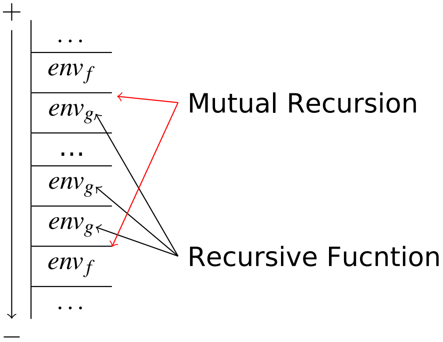

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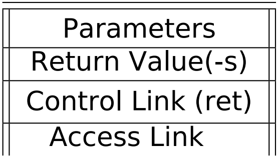

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Temporal Data

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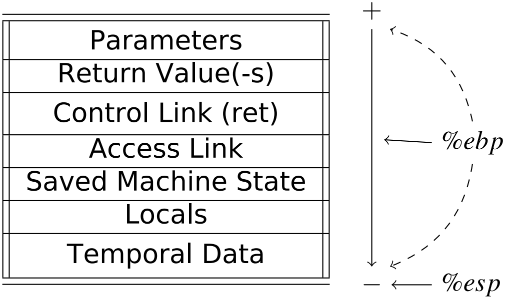

Application Binary Interface

- ABI
- EABI
- Calling convention

Standard prologue
X86-32:

pushl $\%ebp \leftarrow$ Save Callers ebp
movl $\%esp, \%ebp \leftarrow$ Set up our ebp
subl S$, \%esp \leftarrow$ Set up our esp

Locals Size

Epilogue

Standard Epilogue X86-32:

movl $\%ebp, \%esp$
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Registers

- EAX, EDX, ECX — caller-saved registers
- EBX, EDI, ESI (, and EBP) — callee-saved registers
- EIP, ESP (, and EBP) — special purpose registers

Compiling SM to x86-32

- Call in SM: $\textbf{call}_{SM}$ f
  How many arguments we have to copy from symbolic stack?

  $addl$ S. $\%esp$ How to compute S?

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- See **class env** for details

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5. Save registers on x86 stack before the call and restore after

   look into **env#live_registers**