Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (New) EXAMINATION – WINTER 2019

Subject Code: 3130006 Date: 26/11/2019

Subject Name: Probability and Statistics

Time: 02:30 PM TO 05:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** In how many different ways can 4 of 15 laboratory assistants be chosen to assist with an experiment? **03**

(b) If 5 of 20 tires in storage are defective and 5 of them are randomly chosen for inspection (that is, each tire has the same chance of being selected), what is the probability that the two of the defective tires will be included? **04**

(c) The following are the data on the drying time of a certain varnish and the amount of an additive that is intended to reduce the drying time? **07**

Amount of varnish additive(grams)"x"	0	1	2	3	4	5	6	7	8
Drying time(hr) "y"	12.0	10.5	10.0	8.0	7.0	8.0	7.5	8.5	9.0

(i) Fit a second degree polynomial by the method of least square.
(ii) Use the result of (i) to predict the drying time of the varnish when 6.5 gms of the additive is being used.

Q.2 **(a)** If 3 balls are "randomly drawn" from a bowl containing 6 white and 5 black balls. What is the probability that one of the balls is white and the other two black? **03**

(b) The article "A Thin-Film Oxygen Uptake Test for the Evaluation of Automotive Crankcase Lubricants" reported the following data on oxidation-induction time (min) for various commercial oils: **04**
87, 103, 130, 160, 180, 195, 132, 145, 211, 105, 145, 153, 152, 138, 87, 99, 93, 119, 129
(i) Calculate the sample variance and standard deviation.
(ii) If the observations were re-expressed in hours, what would be the resulting values of the sample variance and sample standard deviation?

(c) In an examination, minimum 40 marks for passing and 75 marks for distinction are required. In this examination 45% students passed and 9% obtained distinction. Find average marks and standard deviation of this distribution of marks. **07**
[P(z=0.125)=0.05 and P(z=1.34)=0.41]

OR

(c) Distribution of height of 1000 students is normal with mean 165 cms and standard deviation 15 cms. How many soldiers are of height **07**
(i) less than 138 cms (ii) more than 198 cms (iii) between 138 and 198 cms.
[P(z=1.8)=0.4641, P(z=2.2)=0.4861]

Q.3 **(a)** Compute the coefficient of correlation between X and Y using the following data: **03**

X	2	4	5	6	8	11
Y	18	12	10	8	7	5

(b) An analysis of monthly wages paid to workers in two firms A and B belong to the same industry gave the following results. **04**

	Firm A	Firm B
No. of wages earners	986	548
Average monthly wages	Rs. 52.5	Rs. 47.5
Variance of distribution of wages	100	121

(a) Which firm pays out large amounts as wage bill?
(b) In which firm there is greater variability in individual wages?

(c) Obtain the two lines of regression for the following data: **07**

Sales (No. of tablets)	190	240	250	300	310	335	300
Advertising expenditure (Rs.)	5	10	15	20	20	30	30

OR

Q.3 **(a)** A sample of 20 items has mean 42 units and standard deviation 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.[t at 5% level for 19 d.f. is 2.09.] **03**

(b) A university warehouse has received a shipment of 25 printers, of which 10 are laser printers and 15 are inkjet models. If 6 of these 25 are selected at random to be checked by a particular technician, what is the probability that exactly 3 of those selected are laser printers (so that the other 3 are inkjets)? **04**

(c) Find the regression equation showing the capacity utilization on production from the following data: **07**

	Average	Standard deviation
Production (in lakh units)	35.6	10.5
Capacity utilization (in %)	84.8	8.5
Correlation coefficient	r = 0.62	

Estimate the production when capacity utilization is 70%.

Q.4 **(a)** Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, at least 4 samples contain the pollutant. **03**

(b) Goal scored by two teams A and B in a football season were as follows: **04**

No. of goals scored in a match	0	1	2	3	4
No. of matches played by team A	27	9	8	5	4
No. of matches played by team B	17	9	6	5	3

Find out which team is more consistent.

(c) Out of 800 families with 4 children each, how many families would be expected to have (i) 2 girls and 2 boys (ii) at least one boy (iii) no girl (iv) at most two girls? Assume equal probabilities for boys and girls. **07**

OR

Q.4 **(a)** Assume that the probability that a wafer contains a large particle of contamination is 0.01 and that the wafers are independent; that is, the probability that a wafer contains a large particle is not dependent on the characteristics of any of the other wafers. If 15 wafers are analyzed, what is the probability that no large particles are found? **03**

(b) A microchip company has two machines that produce the chips. Machine I produces 65% of the chips, but 5% of its chips are defective. Machine II produces 35% of the chips and 15% of its chips are defective. A chip is selected at random and found to be defective. What is the probability that it came from Machine I? **04**

(c) If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is .005 and errors are independent from page to page, what is the probability that one of its 400-page novels will contain (i) exactly one page with errors? (ii)At most three pages with errors? **07**

Q.5 **(a)** Samples of sizes 10 and 14 were taken from two normal populations with standard deviation 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at 5% level. [$t_{0.05}$=2.0739]. **03**

(b) Two independent samples of 8 and 7 items respectively had the following values of the variable (weight in kg): **04**

Sample I :	9	11	13	11	15	9	12	14
Sample II:	10	12	10	14	9	8	10	

Do the two estimates of population variance differ significantly? Given that for (7,6) d.f. the value of F at 5% level of significance is 4.20 nearly.

(c) Records taken of the number of male and female births in 830 families having four children are as follows: **07**

Number of male births	0	1	2	3	4
Number of female births	4	3	2	1	0
Number of families	32	178	290	236	94

Test whether the data are consistent with the hypothesis that the Binomial law holds and the chance of male birth is equal to that of female birth, namely p = q = 1/2. [χ^2 at 5% level of significance for 4 df is 9.49]

OR

Q.5 **(a)** Two samples of size 9 and 8 give the sum of squares of deviations from their respective means equal 160 inches and 91 inches square respectively. Can they be regarded as drawn from two normal populations with the same variance? **03**
(F for 8 and 7 d.f. = 3.73).

(b) A die is thrown 276 times and the results of these throws are given below: **04**

Number appeared on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not. [χ^2 at 5% level of significance for 5 df is 11.09]

(c) The following figures refer to observations in live independent samples: **07**

Sample I:	25	30	28	34	24	20	13	32	22	38
Sample II:	40	34	22	20	31	40	30	23	36	17

Analyse whether the samples have been drawn from the population of equal means.
[t at 5% level of significance for 18 d.f is 2.1] Test whether the means of two population are same at 5% level (t at 0.05=2.0739)

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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–III (NEW) EXAMINATION – WINTER 2020

Subject Code:3130006 Date:09/03/2021

Subject Name:Probability and Statistics

Time:10:30 AM TO 12:30 PM Total Marks:56

Instructions:

1. **Attempt any FOUR questions out of EIGHT questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Statistical Tables are required.**

Q.1 **(a)** Find the mean, median and Mode for the following frequency distribution: **03**

x	1	2	3	4	5	6	7	8	9	10
f	4	7	8	10	6	6	4	2	2	1

(b) An insurance company insured 2000 bike drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a bike driver, a car driver and a truck driver is 0.10, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a bike driver? **04**

(c) (i) A manufacturer of external hard drives claims that only 10 % of his drives require repairs within the warranty period of 12 months. If 5 of 20 of his drives required repairs within the first year, does this tend to support or refute the claim? **03**

(ii) The actual amount of instant coffee that a filling machine puts into "4 −ounce" jars may be looked upon as a random variable having a normal distribution with $\sigma = 0.04$ ounce. If only 2 % of the jars are to contain less than 4 ounces, what should be the mean fill of these jars? Out of 10000 jars sold, how many are expected to contain more than 4.2 ounces? **04**

Q.2 **(a)** If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Find the probability that out of 2000 individuals, (i) more than 2 individuals; (ii) exactly 3 individuals will suffer a bad reaction. **03**

(b) A stenographer claims that she can type at the rate of 120 words per minute. She demonstrated, on the basis of 100 trials, an average speed of 116 words with a standard deviation of 15 words. Does this enable us to reject the null hypothesis $\mu = 120$ against the alternative hypothesis $\mu < 120$ at the 0.05 level of significance? **04**

(c) (i) The time to check out and process payment information at an office supplies Web site can be modeled as a random variable with mean $\mu = 63$ seconds and variance $\sigma^2 = 81$ seconds. What is the probability that a random sample of size 36 has mean greater than 66.75? **03**

(ii) If two random variables X and Y have the joint density **04**

$$f(x,y) = \begin{cases} k(x+y^2), & for\ 0 < x < 1, 0 < y < 1 \\ 0, & elsewhere \end{cases},$$

find k and the mean of the conditional density $f_1(x \mid 0.5)$ where $f_1(x)$ is the marginal probability density of X.

Q.3 **(a)** If A and B are independent events with $P(A) = 0.26$, and $P(B) = 0.45$, find $(a)\ P(A \cap B)$; $(b)\ P(A \cap \bar{B})$; $(c)\ P(\bar{A} \cap \bar{B})$. **03**

(b) Compute Karl Pearson's coefficient of correlation between X and Y for the following data: **04**

X	100	98	78	85	110	93	80
Y	85	90	70	72	95	81	74

(c) (i) The arithmetic means of runs scored by three batsmen A, B and C, in the same series of 10 innings, are 50, 48 and 12 respectively. The standard deviations of their runs are 15, 12 and 2 respectively. Who is the most consistent of the three? **03**

(ii) Calculate the first four moments about the mean of the following data: **04**

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

Q.4 **(a)** The following table gives the probabilities that a certain computer will malfunction $0, 1, 2, 3, 4, 5$ or 6 times on any one day: **03**

Number of malfunctions x :	0	1	2	3	4	5	6
Probability $f(x)$:	0.17	0.29	0.27	0.16	0.07	0.03	0.01

Find the mean and variance of this probability distribution.

(b) The coefficient of rank correlation of marks obtained by 10 students in English and Economics was found to be 0.6. It was later discovered that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 7 instead of 1. Find the correct coefficient of rank correlation. **04**

(c) (i) Find out mean deviation about median for the following series: **03**

Size	4	6	8	10	12	14	16
Freq.	2	1	3	6	4	3	1

(ii) Find Karl Pearson's coefficient of skewness for the following data: **04**

x	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$
f	13	20	30	25	12

Q.5 **(a)** The life in hours of a certain kind of radio tube has the probability density **03**

$$f(x) = \begin{cases} {}^{100}/_{x^2}, & for\ x \geq 100 \\ 0, & elsewhere \end{cases},$$

find the distribution function and use it to determine the probability that the life of tube is more than 150 hrs.

(b) The number of flaws in a fiber optic cable follows a Poisson process with an average of 0.6 per 100 feet. **04**

(i) Find the probability of exactly 2 flaws in a 200 foot cable.

(ii) Find the probability of exactly 1 flaw in the first 100 feet and exactly 1 flaw in the second 100 feet.

(c) The population (p) of a small community on the outskirts of a city grows rapidly over a $20-$year period: **07**

t	0	5	10	15	20
p	100	200	450	950	2000

As an engineer working for a utility company, you must forecast the population 5 years into the future in order to anticipate the demand for power. Employ an exponential model and linear regression to make this prediction.

Q.6 **(a)** The joint probability density of two random variables is given by **03**

$$f(x_1, x_2) = \begin{cases} 6e^{-2x_1-3x_2}, & for\ x_1 > 0, x_2 > 0 \\ 0, & elsewhere \end{cases}.$$

Find the marginal densities of both the random variables and hence show that the two random variables are independent.

(b) The probability that an electronic component will fail in less than 1000 hours of continuous use is 0.25. Use the normal approximation to find the probability that among 200 such components fewer than 45 will fail in less than 1000 hours of continuous use. **04**

(c) Fit a parabola $y = a + bx + cx^2$ to the following data: **07**

x	1	2	3	5	6
y	1.1	5.8	17.5	55.9	86.7

Q.7 **(a)** In a study of automobile collision insurance costs, a random sample of 80 body repair costs for a particular kind of damage had a mean of 33065 $Rs.$ and a standard deviation of 4364 $Rs.$ If $\bar{x} = 33065\ Rs.$ is used as a point estimate of the true average repair cost of this kind of damage, with what confidence can one assert that the error does not exceed 700 $Rs.$? **03**

(b) In a certain city, the daily consumption of electric power (in millions of kilowatt-hours) can be treated as a random variable having a gamma distribution with $\alpha = 2$ and $\beta = 3$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day? Also, find the mean of this probability density. **04**

(c) (i) Ten bearings made by a certain process have a mean diameter of 0.506 cm and a standard deviation of 0.004 cm. Assuming that the data may be looked upon as a random variable from a normal population, construct a 95 % confidence interval for the actual average diameter of bearings made by this process. **03**

(ii) A consumer protection agency wants to test a paint manufacturer's claim that the average drying time of his new paint is 20 minutes. It instructs a member of its research staff to paint each of 36 boards using a different 1 −gallon can of the paint, with the intention of rejecting the claim if the mean of the drying times exceeds 20.75 minutes. Otherwise, it will accept the claim. Find the probability of a Type I error. Also, find the probability of a Type II error when $\mu = 21$ minutes. Assume that $\sigma = 2.4$ minutes. **04**

Q.8 **(a)** The dean of a college wants to use the mean of a random sample to estimate the average amount of time students take to get from one class to the next, and she wants to be able to assert with 99 % confidence that the error is at most 0.25 minute. If it can be presumed from experience that $\sigma = 1.40$ minutes, how large a sample will she have to take? **03**

(b) How exponential distribution is useful in real applications? Find the mean and variance of the exponential distribution **04**

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & for\ x > 0, \beta > 0 \\ 0, & elsewhere \end{cases}.$$

(c) A random sample from a company's very extensive files shows that orders for a certain piece of machinery were filled, respectively, in 10, 12, 19, 14, 15, 18, 11 and 13 days. Use the level of significance $\alpha = 0.01$ to test the claim that on average such orders are filled in 10.5 days. Choose the alternative hypothesis so that rejection of the null hypothesis $\mu = 10.5$ implies that it takes longer than indicated. Assume normality. **07**

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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – WINTER 2021

Subject Code:3130006 Date:17-02-2022

Subject Name:Probability and Statistics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Define a term random variable and explain different types of random variable. **03**

(b) A card is drawn at random from a pack of 52 cards. What is the probability that the card is a spade or a king? **04**

(c) State Baye's theorem. There are three bags; first containing 1 white, 2 red and 3green balls; second 2 white, 3 red and 1green balls and third 3 white, 1 red and 2 green balls. Two balls are drawn from a beg chosen at random. These are found to be 1 white and 1 red. Find the probability that the balls so drawn came from the second bag. **07**

Q.2 **(a)** Two judges in a beauty contest rank the 12 contestants as follows: **03**

x	1	2	3	4	5	6	7	8	9	10	11	12
y	12	9	6	10	3	5	4	7	6	2	11	1

Calculate rank correlation coefficient.

(b) A book contains 100 misprints distributed randomly throughout its 100 pages. What is the probability that a page observed at random contains at least 2 misprints. **04**

(c) A die is thrown six times. If getting an odd number is a success, find the probability of (i) 5 success (ii) at least five success and (iii) at most five success. **07**

OR

(c) If a random variable x is Gamma distribution with parameter $\lambda = 3$, compute the value of (i) P(x≤1) and (ii) P(1≤x≤2). **07**

Q.3 **(a)** Calculate the coefficient of variance for the following data: **03**

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	7	2	3	3

(b) Calculate the median for the following data: **04**

Class Interval	0-30	30-60	60-90	90-120	120-150	150-180
Frequency	8	13	22	27	18	7

(c) Compute the correlation coefficients between X and Y using following data: **07**

X	2	4	5	6	8	11
Y	18	12	10	8	7	5

OR

Q.3 **(a)** Obtain correlation coefficient between x and y if two regression lines are 4x-5y+33=0 and 20x-9y-107=0. **03**

(b) Calculate the mode for the following data: **04**

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	10	14	19	7	13

(c) Obtain the regression line of y on x for the following data: **07**

x	100	98	78	85	110	93	80
y	85	90	70	72	95	81	74

Q.4 **(a)** Explain the term related to testing of hypothesis: (i) Null hypothesis (ii) Alternate hypothesis and (iii) Errors while accepting or rejecting a hypothesis. **03**

(b) The mean of 35 sample of the thermal conductivity of a certain kind of cement brick is 0.343 with standard deviation of 0.010. Test the hypothesis that the population mean is 0.340 at 5% level of significance. **04**

(c) Fit a binomial distribution for the following data showing the survey of 800 families with 4 children and test the goodness of fit. **07**

No. of boys	0	1	2	3	4
No. of girls	4	3	2	1	0
No. of families	32	178	290	238	64

OR

Q.4 **(a)** A random sample of size 15 from bivariate normal distribution gave a correlation coefficient r=0.5. Is this indicate the existence of correlation in the population? **03**

(b) A tire company is suspicious to claim that the average lifetime of certain tires is at least 28000 km. To check the claim, the company takes the sample of 40 tires and gets a mean life time of 27463 km with standard deviation of 1348 km. Test the hypothesis at 1% level of significance. **04**

(c) Fit a Poisson distribution for the following data and test the goodness of fit. **07**

x	0	1	2	3	4
f	112	73	30	4	1

Q.5 **(a)** In $y = a + bx$ if $\sum x = 50$, $\sum y = 80$, $\sum xy = 1030$, $\sum x^2 = 750$ and $n = 10$, then find a and b. **03**

(b) Fit a curve $y = ae^{bx}$ for the following data: **04**

x	1	2	3	4
y	7	11	17	27

(c) State properties of the normal distribution. Suppose the marks of 800 students are normally distributed with mean 66 and standard deviation 5. Find number of students getting marks (i) between 65 and 70 (ii) greater than or equal to 72 (Given that P(0≤z≤0.20)=0.0793, that P(0≤z≤0.80)=0.2881 and that P(0≤z≤1.2)=0.3849) **07**

OR

Q.5 **(a)** A random variable x has the following probability distribution: **03**

x_i	0	1	2	3
p_i	1/6	3/8	3/8	1/8

Find the standard deviation of x for the given distribution.

(b) With usual notations, find the value of p for a binomial random variable x when n=6 and 9P(x=4)=P(x=2). **04**

(c) Fit a parabola $y = ax^2 + bx + c$ for the following data: **07**

x	-1	0	1	2
y	-2	1	2	4

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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III EXAMINATION – SUMMER 2020

Subject Code: 3130006 Date:28/10/2020

Subject Name: PROBABILITY AND STATISTICS

Time: 02:30 PM TO 05:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Statistical Tables are required.**

Q.1 **(a)** An insurance company obtained the following data for accident claims (in thousand rupees) from a particular region. Find its mean, median and Mode. **03**

Amount	$1-3$	$3-5$	$5-7$	$7-9$	$9-11$	$11-13$
Frequency	6	47	75	46	18	8

(b) A market survey was conducted in four cities to find out the preference for brand A soap. The responses are shown below: **04**

	Delhi	Kolkata	Chennai	Mumbai
Yes	45	55	60	50
No	35	45	35	45
No opinion	5	5	5	5

(a) What is the probability that a consumer preferred brand A, given that he was from Chennai?

(b) Given that a consumer preferred brand A, what is the probability that he was from Mumbai?

(c) (i) The number of monthly breakdowns of a computer is a random variable having Poisson distribution with mean 1.8. Find the probability that the computer will function for a month (a) without a breakdown (b) with at least one breakdown. **03**

(ii) Assume that 5 % of the apples weigh less than 150 gm and 20 % of the apples weigh more than 225 gm. If the distribution of the weight of the apples is normal, find the mean and standard deviation of the distribution. **04**

Q.2 **(a)** The probability that one of the ten telephone lines is busy at an instant is 0.2. **03**

(a) What is the probability that 5 of the lines are busy?

(b) What is the probability that all the lines are busy?

(b) An auto company claims that the mean petrol consumption of its new six cylinder car is 9.5 km per liter which is lower than the existing auto engine. It was found that the mean petrol consumption of a sample of 50 of these cars was 10 km per liter with a standard deviation of 3.5 km per liter. Test the claim at 5 % level of significance. **04**

(c) (i) The life of batteries manufactured by a battery manufacturer can be modelled as a random variable having approximately a normal distribution with $\mu = 50$ months and $\sigma = 6$ months. Find the probability that the mean of a random sample of 36 such batteries will be less than 48 months. **03**

(ii) If two random variables X and Y have the joint density **04**

$$f(x,y) = \begin{cases} k(x^2+y), & for\ 0<x<1, 0<y<1 \\ 0, & elsewhere \end{cases},$$

find k and the mean of the conditional density $f_1(x \mid 0.5)$ where $f_1(x)$ is the marginal probability density of X.

OR

(c) (i) A process for making certain bearings is under control if the diameters of **03**

the bearings have a mean of 0.5000 cm. What can we say about this process if a sample of 10 of these bearings has a mean diameter of 0.5060 cm and a standard deviation of 0.0040 cm?

(ii) Three balanced coins are tossed. Let X denote the number of heads on the first two coins and Y denote the number of tails on the last two coins. Find the joint distribution of X and Y. **04**

Q.3 **(a)** Show that A and B are independent events if $P(A) = 0.25,\ P(B) = 0.40$, and $P(A \cup B) = 0.50$. **03**

(b) Given that $n = 25,\ \sum X = 125,\ \sum X^2 = 650,\ \sum Y = 100,\ \sum Y^2 = 460$ and $\sum XY = 508$. Later on it was found that two of the points $(8, 12)$ and $(6, 8)$ were wrongly entered as $(6, 14)$ and $(8, 6)$. Prove that $r = {}^{2}/_{3}$. **04**

(c) (i) The runs scored by two batsmen A and B in 10 matches are given in the following table: **03**

A	14	13	26	53	17	29	79	36	84	49
B	37	22	56	52	14	10	37	48	20	4

Who is more consistent?

(ii) Calculate the first four moments about the mean of the following data: **04**

x	5	10	15	20	25
f	6	10	14	6	4

OR

Q.3 **(a)** The number of page requests that arrive at a Web server is a Poisson random variable. Its probability distribution is as follows: **03**

Number of requests/sec.	x	0	1	2	3	4	5	6
Probability	$f(x)$	0.368	0.368	0.184	0.061	0.015	0.003	0.001

Find the mean and variance of this probability distribution.

(b) From the following data of the marks obtained by 8 students in Computer Networking (CN) and Complier Design (CD) papers, compute rank coefficient of correlation. **04**

CN	15	20	28	12	40	60	20	80
CD	40	30	50	30	20	10	30	60

(c) (i) Find out mean deviation about median for the following series: **03**

Size	4	6	8	10	12	14	16
Freq.	1	2	4	5	4	3	1

(ii) Find the coefficient of skewness based on the Method of Moments for the following data: **04**

Class	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$
f	13	20	30	25	12

Q.4 **(a)** In a certain district, the proportion of highway sections requiring repairs in any given year is a random variable having the probability density **03**

$$f(x) = \begin{cases} 12\,x^2(1-x), & for\ 0 < x < 1 \\ 0, & elsewhere \end{cases}.$$

Find the distribution function and use it to determine the probability that at least half of the highways sections will require repairs in any given year.

(b) At a checkout counter customers arrive at an average of 1.5 per minute. Find the probabilities that (i) at most 4 will arrive in any given minute; (ii) one customer will arrive in the first one minute and two customers will arrive in the next one minute. **04**

(c) Fit a parabola $y = a + bx + cx^2$ to the following data: **07**

x	0	1	2	3	4
y	1	4	10	17	30

OR

Q.4 **(a)** The joint probability density of two random variables X_1 and X_2 is given by **03**

$$f(x_1, x_2) = \begin{cases} \frac{1}{5}(x_1 + 2x_2), & for\ 0 < x_1 < 1, 0 < x_2 < 2 \\ 0, & elsewhere \end{cases}.$$

Find the marginal densities of both the random variables and check whether the two random variables are independent.

(b) A safety engineer feels that 30 % of all industrial accidents in her plant are caused by failure of employees to follow instructions. If this figure is correct, find approximately, the probability that among 84 industrialized accidents in this plant anywhere from 20 to 30 (inclusive) will be due to failure of employees to follow instructions. **04**

(c) The following show the gain in reading speed of 3 students in a speed-reading program, and the number of weeks they have been in the program: **07**

No. of weeks	3	5	2	8	6	9	3	4
Speed gain	86	118	49	193	164	232	73	109

Fit a straight line by the method of least squares.

Q.5 **(a)** Suppose that the time it takes to get service in a restaurant follows a gamma distribution with mean 8 minutes and variance 32 minutes. Suppose that you went to this restaurant at 6: 30 p.m. What is the probability that you will receive service before 6: 36 p.m.? **03**

(b) If 57 out of 150 patients suffering from certain disease are cured by allopathy and 33 out of 100 patients with the same disease are cured by homeopathy, is there reason to believe that allopathy is better than homeopathy at 0.05 level of significance? **04**

(c) (i) If two independent random samples of size $n_1 = 7$ and $n_2 = 13$ are taken from a normal population, what is the probability that the variance of the first sample will be at least three times as large as that of the second sample? **03**

(ii) A courier service advertises that its average delivery time is less than 5 hours for local deliveries. A random sample of 10 for the amount of time this courier service takes to deliver packages to an addressee across town produced the following times: $8, 3, 4, 7, 10, 5, 6, 4, 5, 8$. Is this evidence support the claim of the courier service at 5 % level of significance? **04**

OR

Q.5 **(a)** A power supply unit for a computer component is assumed to follow an exponential distribution with a mean life of 1200 hours. What is the probability that the component will survive more than 1500 hours? **03**

(b) Twenty people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate if attacked by this disease is 85 % in favour of the hypothesis that it is more at 5 % level. **04**

(c) (i) The mean life of a random sample of 10 light bulbs was found to be 1456 hours with a standard deviation of 423 hours. A second sample of 17 bulbs chosen at random from a different batch showed a mean life of 1280 hours with a standard deviation of 398 hours. Is the difference between the mean life of the two batches significant at 5 % level of significance? **03**

(ii) The manager of a theatre complex with four theaters wanted to see whether there was difference in popularity of the four movies currently showing for Saturday afternoon with the following results: $86, 77, 84, 81$ custormers viewed movies $1, 2, 3$, and 4 respectively. Complete the test to see whether there is a difference at the 5 % level of significance. **04**

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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2021

Subject Code:3130006 Date:06/09/2021

Subject Name:Probability and Statistics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	An MBA applies for a job in two firms X and Y. The probability of his being selected in firm X is 0.7 and being rejected in Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will selected in one the firms?	**03**
	(b)	A study showed that 65% of managers had some business education and 50% had some engineering education. Furthermore 20% the managers had some business education but no engineering education. What is the probability that a manager had some business education, given that he has some engineering education?	**04**
	(c)	A manufacturing firm produces steel pipes in three plants with daily production volume of 500, 1000, 2000 units respectively. According to past experience it is know that the fractions of defective output produced the three plants are respectively 0.05, 0.08, and 0.10. If a pipe is selected from a day's total production and found to be defective. What is the probability that it came from the first plant? Also find out from which plant the defective pipe comes.	**07**
Q.2	**(a)**	The probability that an item produced by a machine will be defective is $\frac{1}{10}$. If 12 such items are produced, find the probability that (i) Exactly one will be defective, (ii) at least two will be defective (iii) None of the item is defective.	**03**
	(b)	A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean $\mu = 1.5$. Calculate the proportion of days on which neither car is used and proportion of days on which some demand is refused. $(e^{-1.5} = 0.2231)$	**04**
	(c)	The average daily sales of 500 branch offices was Rs. 150 thousand and the standard deviation Rs. 15 thousand. Assuming the distribution to be normal indicate how many branches have sales between a) Rs. 120 thousand and Rs. 145 thousand b) Rs. 140 thousand and Rs. 165 thousand. P(0<z<2)=0.4772, P(0<z<0.33)=0.1293, P(0<z<1)=0.2486	
		OR	
	(c)	Accidents occur with a Poisson distribution at an average 2 per week. Then a) Obtain the probability of more than 3 accidents during a week. b) What is the probability that at least two weeks will elapse between accidents.	**07**
Q.3	**(a)**	A fair die is thrown 300 times. Find the lower bound for the probability of getting 30 to 60 sixes.	**03**

(b) Find the quartile deviation and its coefficients. Also find inter quartile range and coefficient of variations. **04**

Marks	< 35	35-37	38-40	41-43	> 43
Students	8	16	13	8	5

(c) The following data relate to the profits of 1,000 companies: **07**

Profits Rs. in thousands	100-120	120-140	140-160	160-180	180-200	200-220	220-240
No. of companies	17	53	199	194	327	208	02

Calculate the coefficient of skewness.

OR

Q.3 **(a)** Following is the table showing number of visitors in 180 days to a zoo. Obtain average number of visitors per day. **03**

Marks	1-10	11-20	21-30	31-40	41-50	51-60
Students	22	28	35	45	30	20

(b) Define moments about the assumed mean A. Obtain fist four moments about arbitrary origin from the following table, **04**

Scorers	50-60	60-70	70-80	80-90	90-100
Players	8	11	18	09	04

(c) Find the mean, median and mode from the following table. **07**

class	50-53	53-56	56-59	59-62	62-65	65-68	68-71	71-74	74-77
frequency	3	8	14	30	36	28	16	10	3

Q.4 **(a)** Airscrew escape systems powered by a solid propellant. The burning rate of this propellant is an important product characteristics. Specifications require that the mean burning rate must be $\mu = 50$ centimeters per second and standard deviation of burning rate $\sigma = 2$ centimeters per second. The experimenter choose $\alpha = 0.05$ level of significance and selects random sample of $n = 25$ and obtain a sample average of $\bar{x} = 51.3$ centimeters per second. What conclusions should be drawn? ($z_{0.025} = \pm 1.96$) **03**

(b) Psychological tests of intelligence and of engineering ability were applied to 10 students as per the following data. Find the coefficient of correlation. **04**

Intelligence ration	105	104	102	101	100	99	98	96	93	92
Engineering ability	101	103	100	98	95	96	104	92	97	94

(c) The following table gives the aptitude test scores and productivity indices of 10 workers selected at random **07**

Aptitude scores	60	62	65	70	72	48	53	73	65	82
Productivity index	68	60	62	80	85	40	52	62	60	81

Estimate (i) the productivity index of a worker whose test score is 0.92 (ii) the test score of a worker whose productivity index is 0.75.

OR

Q.4 **(a)** You are working as a purchase manager for a company. The following information has been supplied to you by two manufactures of electric bulbs. **03**

	Company A	Company B
Mean life in hours	1300	1248
Standard deviation	82	93

Sample size	100	100

Which brand of bulbs are you going to purchase if you desire to take a risk at 5%? ($Z_{0.05} = \pm 1.96$)

(b) An examination of eight applicants for a critical post was given by a firm. From the marks obtained by the applicants in the Accountancy and Statistics papers, compute rank coefficient correlations. **04**

Marks in accountancy	15	20	28	12	40	60	20	80
Marks in statistics	40	30	50	30	20	10	30	60

(c) In partially destroyed laboratory record of an analysis of correlation data, the following results are eligible. **07**

- Variance of *x*, $\sigma_x^2 = 9$
- Two line of regressions: 8x-10y+66=0, 40x-18y=214.

From the above obtain mean values of *x* and *y*, the standard deviation of *y* and correlation coefficient.

Q.5 (a) 500 units from a factory are inspected and 12 are found to be defective, 800 units from another factory are inspected and 12 are found to be defective. Can it be concluded at 5% level of significance that production at second factory is better that in first factory. ($Z_{0.05} = \pm 1.96$) **03**

(b) Two salesmen A and B are working in a certain district. From a sample survey conducted by the head office, the following results were obtained. Whether is there any significant difference in the average sales between two salesmen's? **04**

	A	**B**
Number of sales	20	18
Average sales in Rs.	170	205
Standard deviation in Rs.	20	25

The critical value of t at 5% level of significance and 36 degree of freedom is more than 36.

(c) A simply supported beam carries a concentrated load P(lb) at its mid-point. Corresponding to various values of P the maximum deflection y (in) is measured. The corresponding table is given below. Find the low of the $y = a + bP$. **07**

P	100	120	140	160	180	200
y	0.45	0.55	0.60	0.70	0.80	0.85

OR

Q.5 (a) A random sample of size 15 from a bivariate normal population gave correlation coefficient *r*=0.5. Is this indicate the correlation in the population? Choose $\alpha = 0.05$ as level of significance, $\left[t_{(\alpha/2,13)} = 2.160\right]$ **03**

(b) By the method of least square fit a curve of the form $y = ax^b$ to the following data. **04**

x	2	3	4	5
y	27.8	62.1	110	161

(c) The number of defects in printed circuit board is hypothesized to follow Poisson distribution. A random sample of *n*=60 printed board has been collected and the following number of defects observed. **07**

Number of defects	0	1	2	3
Observed frequency	32	15	09	04

Use chi-square distribution to test the claim that the number defects follows the Poisson distribution. $\chi^2_{(0.05,1)} = 3.84$

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Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (NEW) EXAMINATION – SUMMER 2022

Subject Code:3130006 Date:11-07-2022

Subject Name:Probability and Statistics

Time:02:30 PM TO 05:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Define and give the example of: (i) Random variable, (ii) Independent Events. **03**

(b) Two fair six-sided dice are tossed independently. Let M be the maximum of the two tosses. What is the probability mass function (pmf) of M? **04**

(c) Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared. **07**
(i) If it has an emergency locator, what is the probability that it will not be discovered?
(ii) If it does not have an emergency locator, what is the probability that it will be discovered?

Q.2 **(a)** State the probability function of Exponential and Gamma distribution. **03**

(b) A dice is thrown 264 times with the following results. Show that the dice is biased. **04**
[Use $\chi^2_{0.05} = 11.07$ for 5 degree of freedom].

No. appeared on dice	1	2	3	4	5	6
Frequency	40	32	28	58	54	52

(c) Fit a straight line to the following data. Also, estimate the value of y at x=72. **07**

x:	65	66	67	67	68	69	71	73
y:	67	68	64	68	72	70	69	70

OR

(c) Fit the second degree parabola using the least square method to the following data: **07**

x:	1	2	3	4	5
y:	5	12	26	60	97

Also, estimate y at x=6.

Q.3 **(a)** State the properties of the Normal Distribution **03**

(b) If a random variable has a Poisson distribution such that P(X=1)=P(X=2), find **04**
(i) the mean of the distribution, (ii) P(X=5), (iii) P(X>1), and (iv) P(1<X<4).

(c) Define Binomial Distribution. **07**
A particular telephone number is used to receive both voice calls and fax messages. Suppose that 25% of the incoming calls involve fax messages, and consider a sample of 10 incoming calls. What is the probability that (i)At most 3 (ii) Exactly 3 (iii) At least 3 (iv) More than 3, of the calls involve a fax message?

OR

Q.3 **(a)** The mean and variance of a binomial distribution are 4 and 2. Find $P(X \geq 2)$. **03**

(b) A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean of 1.5. Calculate the proportion of days on which (i) neither car is used, (ii) some demand is refused, (iii) only one car is used. **04**

(c) Define Standard normal variate. **07**
The lifetime of a certain kind of batteries has a mean life of 400 hours and the standard deviation as 45 hours. Assuming the distribution of lifetime to be normal.
Find The percentage of batteries with lifetime (i) at least 490 hours, (ii) between 385 and 490 hours. Also, find the minimum life of the best 5% of batteries.
[Use: P(0< z < 2) = 0.4772, P(0 < z < 0.33) = 0.1293 and P(0< z < 1.65) = 0.45]

Q.4 **(a)** Explain the term related to testing of hypothesis: (i) Type I Error, (ii) Type II Error, (iii) Level of Significance. **03**

(b) A coin was tossed 960 times and returned heads 183 times. Test the hypothesis that the coin is unbiased. Use 5% level of significance. [use $Z_{0.05} = 1.96$]. **04**

(c) Two types of batteries are tested for their length of life and the following data are obtained: **07**

	No. of samples	Mean Life in hours	Variance
Type A	9	600	121
Type B	8	640	144

Is there a significant difference in the two means? [Use $t_{0.05,15} = 2.132$]

OR

Q.4 **(a)** Ten objects are chosen at random from a large population and their weights are found to be in grams:61,63,64,65,68,69,69,70,71,71. Discuss the suggestion that the mean is 65 g. [Use $t_{0.05} = 2.262\ at\ v = 9$]. **03**

(b) The means of simple samples of sizes 1000 and 2000 are 67.5 cm and 68 cm respectively. Can the samples be regarded as drawn from the same population of standard deviation 2 cm. [use $Z_{0.05} = 1.96$] **04**

(c) Two random samples are drawn from two populations and the following results were obtained: **07**

Sample I	21	24	25	26	27	
Sample II	22	27	28	30	31	36

Find the variances of the two samples and test whether the two populations have the same variances.[Use $F_{0.05}(5,4) = 6.26$.]

Q.5 **(a)** The probability distribution of a random variable X is given below. **03**
Find (i) E(X), (ii) V(X)

X:	-2	-1	0	1	2
P(x=X)	0.2	0.1	0.3	0.3	0.1

(b) The following are the lines of regression $9y = x + 288\ and\ 4y = x + 38$. Estimate y when x = 99 and x when y = 30. Also, find the means of x and y. **04**

(c) Ten competitors in a test are ranked by three judges in the following order: **07**

Rank by First Judge:	6	10	2	9	8	1	5	3	4	7
Rank by Second Judge:	5	4	10	1	9	3	8	7	2	6
Rank by Third Judge:	4	8	2	10	7	5	9	1	3	6

Use the method of rank correlation to gauge which pairs of judges has nearest common approach.

OR

Q.5 **(a)** For a group of 10 items, $\Sigma x = 452, \Sigma x^2 = 24270, and\ mode = 43.7$. Find Karl Pearson's coefficient of Skewness. **03**

(b) Find the correlation coefficient for the following data: **04**

X:	-3	-2	-1	0	1	2	3
Y:	9	4	1	0.5	1	4	9

(c) Calculate the regression coefficients and find the two lines of regression for the following data: **07**

x:	57	58	59	59	60	61	62	64
y:	67	68	65	68	72	72	69	71

Find the value of y when x=65.

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