Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019**

**Subject Code: 3110015** **Date: 01/01/2020**

**Subject Name: Mathematics –2**

**Time: 10:30 AM TO 01:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

| | | | Marks |
|---|---|---|---|
| **Q.1** | **(a)** | Find the length of curve of the portion of the circular helix $\vec{r}(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+t\,\hat{k}$ from $t=0$ *to* $t=\pi$ | **03** |
| | **(b)** | $\int_{(1,2)}^{(3,4)}\left(xy^2+y^3\right)dx+\left(x^2y+3xy^2\right)dy$ is independent of path joining the points (1, 2) and (3,4). Hence, evaluate the integral. | **04** |
| | **(c)** | Verify tangential form of Green's theorem for $\vec{F}=\left(x-\sin y\right)\hat{i}+\left(\cos y\right)\hat{j}$, where C is the boundary of the region bounded by the lines $y=0, x=\pi/2$ and $y=x$. | **07** |
| **Q.2** | **(a)** | Find the Laplace transform of $f(t)$ defined as $f(t)=\frac{t}{k}\quad 0<t<k$; $=1\quad t>k$ | **03** |
| | **(b)** | Find the inverse Laplace transform of $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$ | **04** |
| | **(c)** | (i) Calculate the curl of the vector $xyz\,\hat{i}+3x^2y\,\hat{j}+\left(xz^2-y^2z\right)\hat{k}$ <br> (ii) The temperature at any point in space is given by $T=xy+yz+zx$. Determine the derivative of T in the direction of the vector $3\hat{i}-4\hat{k}$ at the point (1, 1, 1). | **07** |
| | | **OR** | |
| | **(c)** | Let $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$, $r=\left\lvert\bar{r}\right\rvert$, and $\bar{a}$ is a constant vector. Find the value of $div\left(\frac{\bar{a}\times\bar{r}}{r^n}\right)$ | **07** |
| **Q.3** | **(a)** | Find constants a, b and c such that $\vec{V}=\left(x+2y+a\,z\right)\hat{i}+\left(bx-3y-z\right)\hat{j}+\left(4x+cy+2z\right)\hat{k}$ is irrotational. | **03** |
| | **(b)** | Using Fourier cosine integral representation show that $\int_0^\infty\frac{\cos\omega x}{k^2+\omega^2}d\omega=\frac{\pi e^{-kx}}{2k}$ | **04** |
| | **(c)** | Solve the following differential equations: <br> (i) $\cos(x+y)\,dy=dx$ <br> (ii) $\sec^2 y\frac{dy}{dx}+x\tan y=x^3$ | **07** |
| | | **OR** | |

| | | | |
|---|---|---|---|
| **Q.3** | **(a)** | Find the Laplace transform of (i) $\int_0^t \frac{\sin t}{t}\,dt$ (ii) $t^2 u(t-3)$ | **03** |
| | **(b)** | Using Convolution theorem obtain $L^{-1}\left(\frac{1}{s(s^2+a^2)}\right)$ | **04** |
| | **(c)** | Find the power series solution of $\frac{d^2y}{dx^2}+xy=0$ | **07** |
| **Q.4** | **(a)** | Find the Laplace transform of the waveform $f(t)=\left(\frac{2t}{3}\right), 0\le t\le 3$ | **03** |
| | **(b)** | Using the Laplace transforms, find the solution of the initial value problem $y''+25y=10\cos 5t \quad y(0)=2, y'(0)=0$ | **04** |
| | **(c)** | Using variation of parameter method solve $(D^2+1)y=x\sin x$ | **07** |

**OR**

| | | | |
|---|---|---|---|
| **Q.4** | **(a)** | Solve $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\frac{dy}{dx}$ | **03** |
| | **(b)** | Solve $y'''-3y''+3y'-y=4e^t$ | **04** |
| | **(c)** | Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=2x^2+3e^{-x}$ using method of undetermined coefficients. | **07** |
| **Q.5** | **(a)** | Classify the singular points of the equation $x^3(x-2)y''+x^3y'+6y=0$ | **03** |
| | **(b)** | Solve $(D^2+4)y=\cos 2x$ | **04** |
| | **(c)** | Solve (i) $ye^x dx+(2y+e^x)dy=0$ (ii) $\frac{dy}{dx}+2y\tan x=\sin x$ | **07** |

**OR**

| | | | |
|---|---|---|---|
| **Q.5** | **(a)** | Solve $\frac{dy}{dx}=\frac{y^3}{e^{2x}+y^2}$ | **03** |
| | **(b)** | If $y_1=x$ is one of solution of $x^2y''+xy'-y=0$ find the second solution. | **04** |
| | **(c)** | Using Frobenius method solve $x^2y''+4xy'+(x^2+2)y=0$ | **07** |

************

Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019**

**Subject Code: 3110015** **Date: 01/06/2019**

**Subject Name: Mathematics –2**

**Time: 10:30 AM TO 01:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

| | | | Marks |
|---|---|---|---|
| **Q.1** | **(a)** | Find the Fourier integral representation of $f(x)=\begin{cases} x\,; x\in(0,a) \\ 0\,;\ x\in(a,\infty)\end{cases}$ | **03** |
| | **(b)** | Define: Unit step function. Use it to find the Laplace transform of $f(t)=\begin{cases}(t-1)^2\,;\ t\in(0,1] \\ 1\quad ;\ t\in(1,\infty)\end{cases}$ | **04** |
| | **(c)** | Use the method of undetermined coefficients to solve the differential equation $y''-2y'+y=x^2e^x$ . | **07** |
| **Q.2** | **(a)** | Evaluate $\oint_{\boldsymbol{C}} \bar{F}\cdot d\bar{r}$ ; where $\bar{F}=(x^2-\ y^2)\hat{\imath}+2xy\,\hat{\jmath}$ and $\boldsymbol{C}$ is the curve given by the parametric equation $\boldsymbol{C}: r(t)=t^2\,\hat{\imath}+t\,\hat{\jmath};\ \ 0\ \le\ t\ \le\ 2$ . | **03** |
| | **(b)** | Apply Green's theorem to find the outward flux of a vector field $\bar{F}=\frac{1}{xy}(x\,\hat{\imath}\ +\ y\,\hat{\jmath})$ across the curve bounded by $y=\sqrt{x},\ 2y=1$ and $x=1$. | **04** |
| | **(c)** | Integrate $f(x,y,z)=\ x-yz^2$ over the curve $C=C_1+\ C_2$ , where $C_1$ is the line segment joining (0,0,1) to (1,1,0) and $C_2$ is the curve $y=x^2$ joining (1,1,0) to (2,4,0). | **07** |
| | | **OR** | |
| | **(c)** | Check whether the vector field $\bar{F}=e^{y+2z}\,\hat{\imath}+x\,e^{y+2z}\ \hat{\jmath}\ +2x\,e^{y+2z}\ \hat{k}$ is conservative or not. If yes, find the scalar potential function $\varphi(x,y,z)$ such that $\bar{F}=\ \text{grad}\,\varphi$ . | **07** |
| **Q.3** | **(a)** | Write a necessary and sufficient condition for the differential equation $M(x,y)dx+\ N(x,y)dy=0$ to be exact differential equation. Hence check whether the differential equation $[(x+1)e^x-\ e^y]dx-xe^y dy=0$ is exact or not. | **03** |
| | **(b)** | Solve the differential equation $(1+y^2)dx=\ (e^{-\tan^{-1}y}-x)dy$ | **04** |
| | **(c)** | By using Laplace transform solve a system of differential equations $\frac{dx}{dt}=1-y\,,\ \frac{dy}{dt}=\ -x\,,$ where $x(0)=1, y(0)=0.$ | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Solve the differential equation $(2x^3+4y)dx-xdy=0.$ | **03** |

| | | | |
|---|---|---|---|
| | **(b)** | Solve: $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$. | **04** |
| | **(c)** | By using Laplace transform solve a differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = e^{-t}$, where $y(0) = 0,\ y'(0) = -1$. | **07** |
| **Q.4** | **(a)** | Find the general solution of the differential equation $e^{-y}\frac{dy}{dx} + \frac{e^{-y}}{x} = \frac{1}{x^2}$ | **03** |
| | **(b)** | Solve : $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} + 6y = e^x$ | **04** |
| | **(c)** | Find a power series solution of the differential equation $y'' - xy = 0$ near an ordinary point $x$=0. | **07** |
| | | **OR** | |
| **Q.4** | **(a)** | Find the general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} - \sqrt{y} = 0$. | **03** |
| | **(b)** | Solve : $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = x$ | **04** |
| | **(c)** | Find a Frobenius series solution of the differential equation $2x^2y'' + xy' - (x+1)y = 0$ near a regular-singular point $x$=0. | **07** |
| **Q.5** | **(a)** | Write Legendre's polynomial $P_n(x)$ of degree-$n$ and hence obtain $P_1(x)$ and $P_2(x)$ in powers of $x$. | **03** |
| | **(b)** | Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $y'' + xy' = 0$. | **04** |
| | **(c)** | Solve the differential equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3\cos x$ by using the method of variation of parameters. | **07** |
| | | **OR** | |
| **Q.5** | **(a)** | Write Bessel's function $J_p(x)$ of the first kind of order-$p$ and hence show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ . | **03** |
| | **(b)** | Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $xy'' + y' = 0$. | **04** |
| | **(c)** | Solve the differential equation $y'' + 25y = \sec 5x$ by using the method of variation of parameters. | **07** |

**************

Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE – SEMESTER 1&2 EXAMINATION – SUMMER 2020**

**Subject Code: 3110015** **Date:09/11/2020**

**Subject Name: Mathematics II**

**Time: 10:30 AM TO 01:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

| | | | Marks |
|---|---|---|---|
| **Q.1** | **(a)** | Evaluate $\int_c \bar{F}\cdot d\,\bar{r}$ along the parabola $y^2 = x$ between the points (0, 0) and (1, 1) where $\bar{F} = x^2\hat{i} + xy\,\hat{j}$ | **03** |
| | **(b)** | Find the work done in moving particle from A (1, 0, 1) to B (2,1,2) along the straight-line AB in the force field $\bar{F} = x^2\hat{i} + (x-y)\,\hat{j} + (y+z)\hat{k}$ | **04** |
| | **(c)** | Verify green\`s theorem for $\oint_c (2xydx - y^2dy)$ where C is the boundary of the region bounded by the ellipse $3x^2 + 4y^2 = 12$ | **07** |
| **Q.2** | **(a)** | Find the Laplace transform of $te^{-4t}\sin 3t$. | **03** |
| | **(b)** | Find the inverse Laplace transform of $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$. | **04** |
| | **(c)** | Show that the vector field $\bar{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\,\hat{j} + (xy\cos z + y^2)\hat{k}$ is conservative and find the corresponding scalar potential. | **07** |
| | | **OR** | |
| | **(c)** | Show that $\bar{F} = 2xyz\,\hat{i} + (x^2z + 2y)\,\hat{j} + x^2y\,\hat{k}$ is irrotational and find a scalar function $\phi$ such that $\bar{F} = grad\,\phi$. | **07** |
| **Q.3** | **(a)** | Find the directional derivative of $f(x,y) = xy + xe^y + \cos(xy)$ at the point $P(1,0)$ in the direction of $\bar{u} = 3\hat{i} - 4\hat{j}$. | **03** |
| | **(b)** | Find the inverse Laplace transform of $\log\left(1+\dfrac{1}{s^2}\right)$. | **04** |
| | **(c)** | Find the singular solution and general solution of $y + px = x^4p^2$ | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Find the Laplace transform of $\dfrac{\cos at - \cos bt}{t}$. | **03** |
| | **(b)** | Show that $\int_0^\infty \dfrac{\omega^3 \sin\omega x}{\omega^4+4}\,d\omega = \dfrac{\pi}{2}e^{-x}\cos x; x>0.$ | **04** |
| | **(c)** | Find the power series solution of $y' - 2xy = 0;\ y(0) = 1$ near $x = 0$. | **07** |

| | | | |
|---|---|---|---|
| **Q.4** | **(a)** | Find the Laplace transform of $e^{-t}\left\{1-u\left(t-2\right)\right\}$. | **03** |
| | **(b)** | Solve $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$ with $x=2, \frac{dx}{dt}=-1$ at $t=0.$ | **04** |
| | **(c)** | Solve $\left(D^2-1\right)y=xe^x\sin x$ | **07** |

**OR**

| | | | |
|---|---|---|---|
| **Q.4** | **(a)** | Solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ | **03** |
| | **(b)** | Using method of variation of parameter, solve $\frac{d^2y}{dx^2}+4y=\tan 2x$. | **04** |
| | **(c)** | Using method of undetermined coefficients solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^x.$ | **07** |
| **Q.5** | **(a)** | Classify the singular points of $x^2y''+xy'-2y=0$. | **03** |
| | **(b)** | Solve $\frac{d^2y}{dx^2}+9y=\sin 2x\sin x.$ | **04** |
| | **(c)** | Solve (i) $\left(x^3+3xy^2\right)dx+\left(3x^2y+y^3\right)dy=0.$<br>(ii) $\frac{dy}{dx}+y\cot x=2\cos x.$ | **07** |

**OR**

| | | | |
|---|---|---|---|
| **Q.5** | **(a)** | Solve $\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}$. | **03** |
| | **(b)** | Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+y=\cos\left(\ln x\right).$ | **04** |
| | **(c)** | Using Frobenius method solve $2x^2y''+xy'-\left(x+1\right)y=0.$ | **07** |

************************

Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE- SEMESTER–I & II(NEW)EXAMINATION – SUMMER 2022**

**Subject Code:3110015** **Date:22-08-2022**

**Subject Name:Mathematics - 2**

**Time:10:30 AM TO 01:30 PM** **Total Marks:70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

| | | | Marks |
|---|---|---|---|
| **Q.1** | **(a)** | Find the Laplace transform of $t^2e^{-3t}$. | **03** |
| | **(b)** | Define conservative vector field and potential function. | **04** |
| | **(c)** | Solve $y''' - 3y'' + 3y' - y = 4e^x$ using the method of undetermined coefficients. | **07** |
| **Q.2** | **(a)** | Find the divergence of $\boldsymbol{F} = (x^2 - y)\boldsymbol{i} + (xy - y^2)\boldsymbol{j}$. | **03** |
| | **(b)** | Find Fourier cosine integral of $f(x) = e^{-kx}(x > 0, k > 0)$ | **04** |
| | **(c)** | Integrate $f(x, y, z) = 3x^2 - 2y + z$ over the line segment $C$ joining the origin to the point (2,2,2). | **07** |
| | | **OR** | |
| | **(c)** | Write Green's theorem. Evaluate the integral $\oint_C \{xydy - y^2dx\}$ where $C$ is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$. | **07** |
| **Q.3** | **(a)** | Obtain convolution of $t$ and $e^t$. | **03** |
| | **(b)** | Find the Laplace transform of $\frac{cosat - \cos bt}{t}$. | **04** |
| | **(c)** | Solve the initial value problem $y'' - y' - 2y = 0, y(0) = 1, y'(0) = 0$ using Laplace transform. | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Find the inverse Laplace transform of $\frac{s-4}{s^2-4}$. | **03** |
| | **(b)** | State second shifting theorem and find the inverse Laplace transform of the function $\frac{se^{-\pi s}}{s^2+1}$. | **04** |
| | **(c)** | State convolution theorem and using it obtain the inverse Laplace transform of $\frac{1}{s(s^2+4)}$. | **07** |
| **Q.4** | **(a)** | Solve $\frac{dy}{dx} - 2y = 4 - x$. | **03** |
| | **(b)** | Solve $p^2 + 2pycotx = y^2$. | **04** |
| | **(c)** | Solve $y'' + 4y = 4\ tan2x$ using the method of variation of parameters. | **07** |
| | | **OR** | |
| **Q.4** | **(a)** | Find particular solution of $y'' - 2y' + y = cos3x$. | **03** |
| | **(b)** | Solve $x^2y'' - 3xy' + 4y = 0$ | **04** |

| | | | |
|---|---|---|---|
| | **(c)** | Solve the initial value problem<br>$y''' + y' = 0,$<br>$y(0) = 0, y'(0) = 1, y''(0) = 2$ | **07** |
| **Q.5** | **(a)** | Write Legendre's and Bessel's differential equations. | **03** |
| | **(b)** | Solve the differential equation<br>$(ycosx + 2xe^y) + (sinx + x^2e^y - 1)y' = 0$ | **04** |
| | **(c)** | Find the power series solution of the equation$(x^2 + 1)y'' + xy' - xy = 0$ in powers of x. | **07** |

**OR**

| | | | |
|---|---|---|---|
| **Q.5** | **(a)** | Write Legendre polynomials of degree one and two. | **03** |
| | **(b)** | Solve $y = 2px + p^2y$. | **04** |
| | **(c)** | Solve $x(x-1)y'' + (3x-1)y' + y = 0$ about $x = 0$ using Frobenius method. | **07** |

*************