Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 3110015 **Date: 01/01/2020**

Subject Name: Mathematics –2

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Find the length of curve of the portion of the circular helix $\vec{r}(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+t\,\hat{k}$ from $t=0$ to $t=\pi$	**03**
	(b)	$\int_{(1,2)}^{(3,4)}\left(xy^2+y^3\right)dx+\left(x^2y+3xy^2\right)dy$ is independent of path joining the points (1, 2) and (3,4). Hence, evaluate the integral.	**04**
	(c)	Verify tangential form of Green's theorem for $\vec{F}=\left(x-\sin y\right)\hat{i}+\left(\cos y\right)\hat{j}$, where C is the boundary of the region bounded by the lines $y=0, x=\pi/2$ and $y=x$.	**07**
Q.2	**(a)**	Find the Laplace transform of $f(t)$ defined as $f(t)=\frac{t}{k}\quad 0<t<k$; $\quad =1\quad t>k$	**03**
	(b)	Find the inverse Laplace transform of $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$	**04**
	(c)	(i) Calculate the curl of the vector $xyz\,\hat{i}+3x^2y\,\hat{j}+\left(xz^2-y^2z\right)\hat{k}$ (ii) The temperature at any point in space is given by $T=xy+yz+zx$. Determine the derivative of T in the direction of the vector $3\hat{i}-4\hat{k}$ at the point (1, 1, 1).	**07**
		OR	
	(c)	Let $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$, $r=\left\lvert\bar{r}\right\rvert$, and $\bar{a}$ is a constant vector. Find the value of $div\left(\frac{\bar{a}\times\bar{r}}{r^n}\right)$	**07**
Q.3	**(a)**	Find constants a, b and c such that $\vec{V}=\left(x+2y+a\,z\right)\hat{i}+\left(bx-3y-z\right)\hat{j}+\left(4x+cy+2z\right)\hat{k}$ is irrotational.	**03**
	(b)	Using Fourier cosine integral representation show that $\int_0^\infty \frac{\cos\omega x}{k^2+\omega^2}d\omega=\frac{\pi e^{-kx}}{2k}$	**04**
	(c)	Solve the following differential equations: (i) $\cos(x+y)\,dy=dx$ (ii) $\sec^2 y\frac{dy}{dx}+x\tan y=x^3$	**07**
		OR	

Q.3 **(a)** Find the Laplace transform of (i) $\int_0^t \frac{\sin t}{t} dt$ (ii) $t^2 u(t-3)$ **03**

(b) Using Convolution theorem obtain $L^{-1}\left(\frac{1}{s(s^2+a^2)}\right)$ **04**

(c) Find the power series solution of $\frac{d^2y}{dx^2} + xy = 0$ **07**

Q.4 **(a)** Find the Laplace transform of the waveform **03**

$f(t) = \left(\frac{2t}{3}\right), 0 \le t \le 3$

(b) Using the Laplace transforms, find the solution of the initial value problem $y'' + 25y = 10\cos 5t \quad y(0) = 2, y'(0) = 0$ **04**

(c) Using variation of parameter method solve $(D^2+1)y = x\sin x$ **07**

OR

Q.4 **(a)** Solve $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$ **03**

(b) Solve $y''' - 3y'' + 3y' - y = 4e^t$ **04**

(c) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ using method of undetermined coefficients. **07**

Q.5 **(a)** Classify the singular points of the equation $x^3(x-2)y'' + x^3y' + 6y = 0$ **03**

(b) Solve $(D^2+4)y = \cos 2x$ **04**

(c) Solve (i) $ye^x dx + (2y + e^x)dy = 0$ (ii) $\frac{dy}{dx} + 2y\tan x = \sin x$ **07**

OR

Q.5 **(a)** Solve $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$ **03**

(b) If $y_1 = x$ is one of solution of $x^2y'' + xy' - y = 0$ find the second solution. **04**

(c) Using Frobenius method solve $x^2y'' + 4xy' + (x^2+2)y = 0$ **07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2021

Subject Code:3110015 Date:21/03/2022

Subject Name:Mathematics - 2

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find $L\{t^3 e^{-4t}\}$.	**03**
	(b)	Find $L^{-1}\left\{\dfrac{6e^{-2s}}{s^2+4}\right\}$.	**04**
	(c)	Verify Green's theorem for the function $\overline{F}=(x+y)i+2xyj$ and C is the rectangle in the *xy*-plane bounded by $x=0, y=0, x=a, y=b.$	**07**
Q.2	**(a)**	Find $L\{te^{4t}\cos 2t\}$.	**03**
	(b)	Find the Fourier cosine integral of $f(x)=\dfrac{\pi}{2}e^{-x}$, $x \ge 0$.	**04**
	(c)	(i) Find the directional derivative of $f(x,y,z)=2x^2+3y^2+z^2$ at the point (2,1,3) in the direction of $\overline{a}=(1,0,-2)$.	**03**
		(ii) If $\overline{F}=(2y+3)i+xzj+(yz-x)k$, evaluate $\int_C \overline{F}.d\overline{r}$ along the path C: $x=2t^2, y=t, z=t^3$ from *t*=0 to *t*=1.	**04**
		OR	
	(c)	Solve in series $3xy''+2y'+y=0$ using Frobeneous method.	**07**
Q.3	**(a)**	Find the arc length of the curve (semi-circular) $x(t)=\cos t, y(t)=\sin t, z(t)=0$; $0 \le t \le \pi.$	**03**
	(b)	A vector field is given by $\overline{F}=(x^2+xy^2)i+(y^2+x^2y)j.$ Show that $\overline{F}$ is irrotational and find its scalar potential.	**04**
	(c)	Use divergence theorem for $\overline{F}=(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$ over the surface of rectangular parallelepiped, $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$ to evaluate $\iint_S \overline{F}.\hat{n}ds$.	**07**
		OR	
Q.3	**(a)**	Solve $\dfrac{dy}{dx}-y\cot x=2x\sin x.$	**03**
	(b)	Solve $y''+y'-12y=e^{6x}$.	**04**
	(c)	Solve $\dfrac{dy}{dt}-4y=2e^{2t}+e^{4t}$ by Laplace transformation.	**07**

Q.4	**(a)**	Solve $\frac{dy}{dx}+\frac{y}{x}=y^3$.	**03**
	(b)	Solve $(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy=0$.	**04**
	(c)	Solve $y''+9y'=2x^2$ using the method of undetermined coefficients.	**07**

OR

Q.4	**(a)**	Solve $4xp^2=(3x-a)^2$.	**03**
	(b)	Solve $x^2y''+xy'-4y=x^2$.	**04**
	(c)	(i) Express $2-3x+4x^2$ in terms of Legendre's polynomial.	**03**
		(ii) Find ordinary and singular points of $2x^2y''+6xy'+(x+3)y=0$.	**04**

Q.5	**(a)**	Solve $(y-px)(p-1)=p$.	**03**
	(b)	Solve $(D^3+D)y=\cos x$.	**04**
	(c)	Solve $y''+4y=\sec 2x$ by using the method of variation of parameters.	**07**

OR

Q.5	**(a)**	Solve $(D^3-6D^2+11D-6)y=0$.	**03**
	(b)	Solve $(2x+3)^2y''-2(2x+3)y'-12y=6x$.	**04**
	(c)	Find the series solution of $(1+x^2)y''+xy'-9y=0$ near the ordinary point $x=0$.	**07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019

Subject Code: 3110015 **Date: 01/06/2019**

Subject Name: Mathematics –2

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Find the Fourier integral representation of $f(x) = \begin{cases} x\,; x \in (0,a) \\ 0\,;\ x \in (a,\infty) \end{cases}$	03
	(b)	Define: Unit step function. Use it to find the Laplace transform of $f(t) = \begin{cases} (t-1)^2\,;\ t \in (0,1] \\ 1 \quad ;\ t \in (1,\infty) \end{cases}$	04
	(c)	Use the method of undetermined coefficients to solve the differential equation $y'' - 2y' + y = x^2 e^x$.	07
Q.2	**(a)**	Evaluate $\oint_{\boldsymbol{C}} \bar{F} \cdot d\bar{r}$; where $\bar{F} = (x^2 - y^2)\hat{\imath} + 2xy\,\hat{\jmath}$ and $\boldsymbol{C}$ is the curve given by the parametric equation $\boldsymbol{C} : r(t) = t^2\,\hat{\imath} + t\,\hat{\jmath}\,;\ 0 \le t \le 2$.	03
	(b)	Apply Green's theorem to find the outward flux of a vector field $\bar{F} = \frac{1}{xy}(x\,\hat{\imath} + y\,\hat{\jmath})$ across the curve bounded by $y = \sqrt{x}$, $2y = 1$ and $x = 1$.	04
	(c)	Integrate $f(x,y,z) = x - yz^2$ over the curve $C = C_1 + C_2$, where C_1 is the line segment joining (0,0,1) to (1,1,0) and C_2 is the curve $y=x^2$ joining (1,1,0) to (2,4,0).	07
		OR	
	(c)	Check whether the vector field $\bar{F} = e^{y+2z}\,\hat{\imath} + x\,e^{y+2z}\,\hat{\jmath} + 2x\,e^{y+2z}\,\hat{k}$ is conservative or not. If yes, find the scalar potential function $\varphi(x,y,z)$ such that $\bar{F} = \text{grad}\,\varphi$.	07
Q.3	**(a)**	Write a necessary and sufficient condition for the differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact differential equation. Hence check whether the differential equation $[(x+1)e^x - e^y]dx - xe^y dy = 0$ is exact or not.	03
	(b)	Solve the differential equation $(1+y^2)dx = (e^{-\tan^{-1} y} - x)dy$	04
	(c)	By using Laplace transform solve a system of differential equations $\frac{dx}{dt} = 1 - y$, $\frac{dy}{dt} = -x$, where $x(0) = 1, y(0) = 0$.	07
		OR	
Q.3	**(a)**	Solve the differential equation $(2x^3 + 4y)dx - xdy = 0$.	03

(b) Solve: $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$. **04**

(c) By using Laplace transform solve a differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = e^{-t}$, where $y(0) = 0,\ y'(0) = -1$. **07**

Q.4 **(a)** Find the general solution of the differential equation $e^{-y}\frac{dy}{dx} + \frac{e^{-y}}{x} = \frac{1}{x^2}$ **03**

(b) Solve : $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} + 6y = e^x$ **04**

(c) Find a power series solution of the differential equation $y'' - xy = 0$ near an ordinary point x=0. **07**

OR

Q.4 **(a)** Find the general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} - \sqrt{y} = 0$. **03**

(b) Solve : $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = x$ **04**

(c) Find a Frobenius series solution of the differential equation $2x^2y'' + xy' - (x+1)y = 0$ near a regular-singular point x=0. **07**

Q.5 **(a)** Write Legendre's polynomial $P_n(x)$ of degree-n and hence obtain $P_1(x)$ and $P_2(x)$ in powers of x. **03**

(b) Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $y'' + xy' = 0$. **04**

(c) Solve the differential equation **07**

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3\cos x$$

by using the method of variation of parameters.

OR

Q.5 **(a)** Write Bessel's function $J_p(x)$ of the first kind of order-p and hence show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **03**

(b) Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $xy'' + y' = 0$. **04**

(c) Solve the differential equation $y'' + 25y = \sec 5x$ by using the method of variation of parameters. **07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE – SEMESTER 1&2 EXAMINATION – SUMMER 2020

Subject Code: 3110015 **Date:09/11/2020**

Subject Name: Mathematics II

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Evaluate $\int_c \bar{F}\cdot d\,\bar{r}$ along the parabola $y^2 = x$ between the points (0, 0) and (1, 1) where $\bar{F} = x^2\hat{i} + xy\,\hat{j}$	**03**
	(b)	Find the work done in moving particle from A (1, 0, 1) to B (2,1,2) along the straight-line AB in the force field $\bar{F} = x^2\hat{i} + (x-y)\,\hat{j} + (y+z)\hat{k}$	**04**
	(c)	Verify green`s theorem for $\oint_c \left(2xydx - y^2dy\right)$ where C is the boundary of the region bounded by the ellipse $3x^2 + 4y^2 = 12$	**07**
Q.2	**(a)**	Find the Laplace transform of $te^{-4t}\sin 3t$.	**03**
	(b)	Find the inverse Laplace transform of $\dfrac{5s+3}{(s-1)\left(s^2+2s+5\right)}$.	**04**
	(c)	Show that the vector field $\bar{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\,\hat{j} + \left(xy\cos z + y^2\right)\hat{k}$ is conservative and find the corresponding scalar potential.	**07**
		OR	
	(c)	Show that $\bar{F} = 2xyz\,\hat{i} + \left(x^2z + 2y\right)\hat{j} + x^2y\,\hat{k}$ is irrotational and find a scalar function ϕ such that $\bar{F} = grad\,\phi$.	**07**
Q.3	**(a)**	Find the directional derivative of $f(x,y) = xy + xe^y + \cos(xy)$ at the point $P(1,0)$ in the direction of $\bar{u} = 3\hat{i} - 4\hat{j}$.	**03**
	(b)	Find the inverse Laplace transform of $\log\left(1+\dfrac{1}{s^2}\right)$.	**04**
	(c)	Find the singular solution and general solution of $y + px = x^4p^2$	**07**
		OR	
Q.3	**(a)**	Find the Laplace transform of $\dfrac{\cos at - \cos bt}{t}$.	**03**
	(b)	Show that $\int_0^\infty \dfrac{\omega^3\sin\omega x}{\omega^4+4}\,d\omega = \dfrac{\pi}{2}e^{-x}\cos x; x>0.$	**04**
	(c)	Find the power series solution of $y' - 2xy = 0;\ y(0) = 1$ near $x = 0$.	**07**

Q.4	**(a)**	Find the Laplace transform of $e^{-t}\left\{1-u(t-2)\right\}$.	**03**
	(b)	Solve $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$ with $x=2, \frac{dx}{dt}=-1$ at $t=0$.	**04**
	(c)	Solve $\left(D^2-1\right)y=xe^x\sin x$	**07**

OR

Q.4	**(a)**	Solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$	**03**
	(b)	Using method of variation of parameter, solve $\frac{d^2y}{dx^2}+4y=\tan 2x$.	**04**
	(c)	Using method of undetermined coefficients solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^x$.	**07**
Q.5	**(a)**	Classify the singular points of $x^2y''+xy'-2y=0$.	**03**
	(b)	Solve $\frac{d^2y}{dx^2}+9y=\sin 2x\sin x$.	**04**
	(c)	Solve (i) $\left(x^3+3xy^2\right)dx+\left(3x^2y+y^3\right)dy=0$. (ii) $\frac{dy}{dx}+y\cot x=2\cos x$.	**07**

OR

Q.5	**(a)**	Solve $\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}$.	**03**
	(b)	Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+y=\cos(\ln x)$.	**04**
	(c)	Using Frobenius method solve $2x^2y''+xy'-(x+1)y=0$.	**07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II(NEW)EXAMINATION – SUMMER 2022

Subject Code:3110015 **Date:22-08-2022**

Subject Name:Mathematics - 2

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find the Laplace transform of t^2e^{-3t}.	**03**
	(b)	Define conservative vector field and potential function.	**04**
	(c)	Solve $y''' - 3y'' + 3y' - y = 4e^x$ using the method of undetermined coefficients.	**07**
Q.2	**(a)**	Find the divergence of $\boldsymbol{F} = (x^2 - y)\boldsymbol{i} + (xy - y^2)\boldsymbol{j}$.	**03**
	(b)	Find Fourier cosine integral of $f(x) = e^{-kx}(x > 0, k > 0)$	**04**
	(c)	Integrate $f(x, y, z) = 3x^2 - 2y + z$ over the line segment C joining the origin to the point (2,2,2).	**07**
		OR	
	(c)	Write Green's theorem. Evaluate the integral $\oint_C \{xydy - y^2dx\}$ where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.	**07**
Q.3	**(a)**	Obtain convolution of t and e^t.	**03**
	(b)	Find the Laplace transform of $\frac{cosat - \cos bt}{t}$.	**04**
	(c)	Solve the initial value problem $y'' - y' - 2y = 0, y(0) = 1, y'(0) = 0$ using Laplace transform.	**07**
		OR	
Q.3	**(a)**	Find the inverse Laplace transform of $\frac{s-4}{s^2-4}$.	**03**
	(b)	State second shifting theorem and find the inverse Laplace transform of the function $\frac{se^{-\pi s}}{s^2+1}$.	**04**
	(c)	State convolution theorem and using it obtain the inverse Laplace transform of $\frac{1}{s(s^2+4)}$.	**07**
Q.4	**(a)**	Solve $\frac{dy}{dx} - 2y = 4 - x$.	**03**
	(b)	Solve $p^2 + 2pycotx = y^2$.	**04**
	(c)	Solve $y'' + 4y = 4\ tan2x$ using the method of variation of parameters.	**07**
		OR	
Q.4	**(a)**	Find particular solution of $y'' - 2y' + y = cos3x$.	**03**
	(b)	Solve $x^2y'' - 3xy' + 4y = 0$	**04**

(c) Solve the initial value problem **07**
$y''' + y' = 0,$
$y(0) = 0, y'(0) = 1, y''(0) = 2$

Q.5 (a) Write Legendre's and Bessel's differential equations. **03**

(b) Solve the differential equation **04**

$$(ycosx + 2xe^y) + (sinx + x^2e^y - 1)y' = 0$$

(c) Find the power series solution of the equation $(x^2 + 1)y'' + xy' - xy = 0$ in powers of x. **07**

OR

Q.5 (a) Write Legendre polynomials of degree one and two. **03**

(b) Solve $y = 2px + p^2y$. **04**

(c) Solve $x(x - 1)y'' + (3x - 1)y' + y = 0$ about $x = 0$ using Frobenius method. **07**

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