Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE –SEMESTER 1&2(NEW SYLLABUS)EXAMINATION- WINTER 2018

Subject Code: 3110014 **Date: 07-01-2019**

Subject Name: Mathematics - I

Time: 10:30 am to 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	State Cayley– Hamilton theorem. Find eigen values of A and A^{-1}, where $A=\begin{bmatrix}1 & 2\\ 0 & 4\end{bmatrix}$	**03**
	(b)	State L' Hospital's Rule. Use it to evaluate $\lim\limits_{x\to 0}\left[\frac{1}{x^2}-\frac{1}{\sin^2 x}\right]$	**04**
	(c)	Investigate convergence of the following integrals: (i) $\int_5^{\infty}\frac{5x}{\left(1+x^2\right)^3}dx$ (ii) $\int_0^{\infty}\frac{x^{10}\left(1+x^5\right)}{\left(1+x\right)^{27}}dx$	**07**
Q.2	**(a)**	Test the convergence of series $\sum\limits_{n=1}^{\infty}\frac{4^n+5^n}{6^n}$	**03**
	(b)	State the p-series test. Discuss the convergence of the series $\sum\limits_{n=2}^{\infty}\frac{n+1}{n^3-3n+2}$	**04**
	(c)	State D'Alembert's ratio test and Cauchy's root test. Discuss the convergence of the following series: (i) $\sum\limits_{n=1}^{\infty}\frac{4^n\left(n+1\right)!}{n^{n+1}}$ (ii) $\sum\limits_{n=2}^{\infty}\frac{n}{\left(\log n\right)^n}$	**07**
		OR	
	(c)	Test the convergence of the series $\frac{1}{1\cdot2\cdot3}+\frac{x}{4\cdot5\cdot6}+\frac{x^2}{7\cdot8\cdot9}+\frac{x^3}{10\cdot11\cdot12}+\cdots;$ $x\ge 0$	**07**
Q.3	**(a)**	Reduce matrix $A=\begin{bmatrix}1 & 5 & 3 & -2\\ 2 & 0 & 4 & 1\\ 4 & 8 & 9 & -1\end{bmatrix}$ to row echelon form and find its rank.	**03**
	(b)	Derive half range sine series of $f\left(x\right)=\pi-x,\ 0\le x\le\pi$	**04**
	(c)	Find the eigen values and corresponding eigen vectors for the matrix A where $A=\begin{bmatrix}1 & 2 & 2\\ 0 & 2 & 1\\ -1 & 2 & 2\end{bmatrix}$	**07**

OR

Q.3 **(a)** Expand $e^{x\sin(x)}$ in power of x up to the terms containing x^6. **03**

(b) Solve system of linear equation by Gauss Elimination method, if solution exists. **04**

$x + y + 2z = 9;\ 2x + 4y - 3z = 1;\ 3x + 6y - 5z = 0$

(c) Find Fourier series of $f(x) = \begin{cases} x, & -1 < x < 0 \\ 2, & 0 < x < 1 \end{cases}$ **07**

Q.4 **(a)** Discuss the continuity of the function f defined as **03**

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}; & (x, y) \neq (0,0) \\ 0 & (x, y) = (0,0) \end{cases}$$

(b) Define gradient of a function. Use it to find directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P(1,1,0)$ in the direction of $\bar{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. **04**

(c) Find the shortest and largest distance from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$. **07**

OR

Q.4 **(a)** Find the extreme values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ **03**

(b) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \left(x^2 + y^2\right) dydx$ by changing into polar coordinates. **04**

(c) (i) If $u = x^2 y + y^2 z + z^2 x$ then find out $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z}$ **07**

(ii) If $x^3 + y^3 = 6xy$ then find $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$

Q.5 **(a)** Evaluate $\iint\limits_R y \sin(xy)\, dA$, where R is the region bounded by $x = 1$, $x = 2$, $y = 0$ and $y = \dfrac{\pi}{2}$. **03**

(b) By changing the order of integration, evaluate $\int_0^3 \int_y^3 \dfrac{xdxdy}{x^2 + y^2}$ **04**

(c) Find the volume below the surface $z = x^2 + y^2$, above the plane $z = 0$, and inside the cylinder $x^2 + y^2 = 2y$. **07**

OR

Q.5 **(a)** Evaluate integral $\iint\limits_R \dfrac{rdrd\theta}{\sqrt{a^2 + r^2}}$ over the region R which is one loop of $r^2 = a^2 \cos 2\theta$ **03**

(b) Evaluate the integral $\int_1^e \int_1^{\log y} \int_1^{e^x} \left(x^2 + y^2\right) dzdxdy$. **04**

(c) Find the volume of the solid obtained by rotating the region R enclosed by the curves $y = x$ and $y = x^2$ about the line $y = 2$. **07**

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 3110014 **Date: 17/01/2020**

Subject Name: Mathematics – I

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Find the equations of the tenagent plane and normal line to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1)	**03**
	(b)	Evaluate $\lim_{x\to 0} \frac{xe^x - log(1+x)}{x^2}$	**04**
	(c)	Using Gauss Elimination method solve the following system -x+3y+4z =30 3x+2y-z =9 2x-y+2z =10	**07**
Q.2	**(a)**	Test the convergence of the series $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \cdots + \left(\frac{n}{2n+1}\right)^n + \cdots$	**03**
	(b)	Discuss the Maxima and Minima of the function $3x^2 - y^2 + x^3$	**04**
	(c)	Find the fourier series of $f(x) = \frac{(\pi - x)}{2}$ in the interval (0,2π)	**07**
		OR	
	(c)	Change the order of integration and evaluate $\int_0^1 \int_x^1 siny^2\, dy\, dx$	**07**
Q.3	**(a)**	Find the value of $\beta\left(\frac{7}{2}, \frac{5}{2}\right)$	**03**
	(b)	Obtain the fourier cosine series of the function f(x) = e^x in the range (0,l)	**04**
	(c)	Find the maximum and minimum distance fom the point (1,2,2) to the sphere $x^2 + y^2 + z^2 = 36$	**07**
		OR	
Q.3	**(a)**	Test the convergence of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ...$	**03**
	(b)	Evaluate $\iint (x^2 - y^2)\, dx\, dy$ over the triangle with the vertices (0,1), (1,1), (1,2)	**04**
	(c)	Find the volume of the solid generated by rotating the plane region bounded by $y = \frac{1}{x}$, x=1 and x=3 about the X axis.	**07**
Q.4	**(a)**	Evaluate $\int_0^{\pi} \int_0^{\sin\theta} r\, dr\, d\theta$	**03**
	(b)	Express f(x) = $2x^3 + 3x^2 - 8x + 7$ in terms of (x-2)	**04**

	(c)	Using Gauss-Jordan method find A^{-1} for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$	**07**

OR

Q.4	**(a)**	Using Cayley-Hamilton Theorem find A^{-1} for $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$	**03**
	(b)	Evaluate $\int_0^{\infty} \frac{dx}{x^2+1}$	**04**
	(c)	Test the convergence of the series $\frac{x}{1\cdot 2} + \frac{x^2}{3\cdot 4} + \frac{x^3}{5\cdot 6} + \frac{x^4}{7\cdot 8} + \cdots$	**07**
Q.5	**(a)**	Evaluate $\int_0^1 \int_1^2 xy\, dy\, dx$	**03**
	(b)	Find the eigen values and eigenvectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$	**04**
	(c)	If u = f (x-y, y-z, z-x) then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	**07**

OR

Q.5	**(a)**	Find the directional derivatives of f = $xy^2 + yz^2$ at the point (2,-1,1) ,in the direction of i+2j+2k.	**03**
	(b)	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$	**04**
	(c)	Evaluate $\iiint xyz\, dx\, dy\, dz$ over the positive octant of the sphere $x^2 + y^2 + z^2$ = 4	**07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II (NEW) EXAMINATION – WINTER 2020

Subject Code:3110014 **Date:16/03/2021**

Subject Name:Mathematics – I

Time:10:30 AM TO 12:30 PM **Total Marks:47**

Instructions:

1. **Attempt any THREE questions from Q1 to Q6.**
2. **Q7 is compulsory.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Expand $\sin x$ in powers of $(x-\pi/2)$.	**03**
	(b)	Evaluate $\lim_{x\to 0}\dfrac{\tan^2 x - x^2}{x^2\tan^2 x}$.	**04**
	(c)	**(i)** Check the convergence of $\int_4^\infty \dfrac{3x+5}{x^4+7}dx.$	**03**
		(ii) The region between the curve $y=\sqrt{x}$, $0\le x\le 4$ and the line $x = 4$ is revolved about the x – axis to generate a solid. Find its volume.	**04**
Q.2	**(a)**	If $u=\operatorname{cosec}^{-1}\left(\dfrac{x+y}{x^2+y^2}\right)$, show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u.$	**03**
	(b)	Check the convergence of the series $\sum_{n=1}^{\infty}\dfrac{2^n+5}{3^n}$.	**04**
	(c)	**(i)** Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right).$	**03**
		(ii) Test the convergence of $\sum_{n=1}^{\infty}\dfrac{2^n}{n^3+1}$.	**04**
Q.3	**(a)**	Solve the following equations by Gauss' elimination method: $x+y+z=6, x+2y+3z=14, 2x+4y+7z=30.$	**03**
	(b)	If $u=f(x-y,y-z,z-x)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0.$	**04**
	(c)	**(i)** Find the equation of the tangent plane and normal line to the surface $x^2+2y^2+3z^2=12$ at (1, 2, -1).	**03**
		(ii) For $f(x,y)=x^3+y^3-3xy$, find the maximum and minimum values.	**04**
Q.4	**(a)**	Find the rank of the matrix $\begin{bmatrix}8&0&0&16\\0&0&0&6\\0&9&9&9\end{bmatrix}$.	**03**
	(b)	If $u=f(x+at)+g(x-at)$, prove that $\dfrac{\partial^2 u}{\partial t^2}=a^2\dfrac{\partial^2 u}{\partial x^2}.$	**04**

(c) **(i)** Show that the function $f(x,y)=\begin{cases}\dfrac{2x^2y}{x^3+y^3}, (x,y)\neq(0,0)\\ 0, (x,y)=(0,0)\end{cases}$ is not continuous at the origin. **03**

(ii) Find the shortest distance from the point (1,2,2) to the sphere $x^2+y^2+z^2=16$. **04**

Q.5 **(a)** Use Gauss-Jordan method to find A^{-1}, if $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$. **03**

(b) Using Caley-Hamilton theorem find A^2, if $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$. Also find A^{-1}. **04**

(c) Find the Fourier cosine series for $f(x)=x^2, 0<x<\pi$. Hence show that $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=\frac{\pi^2}{12}$. **07**

Q.6 **(a)** Evaluate $\iint_R e^{2x+3y}dA$, where R is the triangle bounded by $x = 0$, $y = 0$, $x + y = 1$. **03**

(b) Find the eigen values and eigen vectors for the matrix $\begin{bmatrix}0 & 0 & -2\\ 1 & 2 & 1\\ 1 & 0 & 3\end{bmatrix}$. **04**

(c) Evaluate $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}dA$ by changing the order of integration. **07**

Q.7 Evaluate $\int_0^{\pi/2}\int_{a(1-\cos\theta)}^{a} r^2\,dr\,d\theta$. **05**

OR

Q.7 Find the area enclosed within the curves $y = 2-x$ and $y^2 = 2(2-x)$. **05**

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2021

Subject Code:3110014 Date:19/03/2022

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	If $u = \log(tanx + tany + tanz)$ then show that $$sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z} = 2$$	**03**
	(b)	Evaluate $$\lim_{x\to 0}\frac{e^x - e^{-x} - 2log(1+x)}{xsinx}$$	**04**
	(c)	Find the extreme values of the function $$f(x,y) = x^3 + y^3 - 3x - 12y + 20$$	**07**
Q.2	**(a)**	Use Ratio test to check the convergence of the series $$\sum_{n=1}^{\infty}\frac{2^n+1}{3^n+1}$$	**03**
	(b)	Find the Maclaurin's series of $cosx$ and use it to find the series of sin^2x.	**04**
	(c)	Find the Fourier series of $f(x) = x^2$ in the interval $(0,2\pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$	**07**
		OR	
	(c)	Find the Fourier series of $f(x) = 2x - x^2$ in the interval (0,3).	**07**
Q.3	**(a)**	Find the directional derivative of $f(x,y,z) = xyz$ at the point $P(-1,1,3)$ in the direction of the vector $\bar{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.	**03**
	(b)	Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to row echelon form.	**04**
	(c)	Find the eigenvalues and corresponding eigenvectors of the matrix $$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$	**07**
		OR	
Q.3	**(a)**	If $u = f(x-y, y-z, z-x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	**03**
	(b)	Find the inverse of the following matrix by Gauss-Jordan method:	**04**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

(c) Verify Cayley-Hamilton theorem for the following matrix and use it to find A^{-1} **07**

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Q.4 **(a)** If 1 is an eigenvalue of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ then find its corresponding eigen vector. **03**

(b) Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x - 2)$ **04**

(c) Solve following system by using Gauss Jordan method **07**

$$x + 2y + z - w = -2$$
$$2x + 3y - z + 2w = 7$$
$$x + y + 3z - 2w = -6$$
$$x + y + z + w = 2$$

OR

Q.4 **(a)** Use integral test to show that the following infinite series is convergent **03**

$$\sum_{n=1}^{\infty} \frac{1}{n(1 + \log^2 n)}$$

(b) For the odd periodic function defined below, find the Fourier series **04**

$$f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$$

(c) Determine the radius and interval of convergence of the following infinite series **07**

$$x - \frac{x^2}{4} + \frac{x^3}{9} - \frac{x^4}{16} + \cdots$$

Q.5 **(a)** Show the following limit does not exist using different path approach **03**

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y^2}{x^4 + y^4}$$

(b) Evaluate the following integral along the region R **04**

$$\iint_R (x + y)\,dy\,dx$$

where R is the region bounded by $x = 0, x = 2, y = x, y = x + 2$. Also, sketch the region.

(c) Change the order of integration and hence evaluate the same. Do sketch the region. **07**

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\,dy\,dx$$

OR

Q.5 **(a)** The following integral is an improper integral of which type? Evaluate **03**

$$\int_0^{\infty} \frac{dx}{x^2 + 1}$$

(b) If $x = rsin\theta cos\varphi, y = rsin\theta sin\varphi, z = rcos\theta$, then find the jacobian **04**

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)}$$

(c) Find the volume of the solid generated by rotating the region bounded by $y = x^2 - 2x$ and $y = x$ about the line $y = 4$. **07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019

Subject Code: 3110014 **Date: 06/06/2019**

Subject Name: Mathematics – I

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Use L'Hospital's rule to find the limit of $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{lnx}\right)$.	**03**
	(b)	Define Gamma function and evaluate $\int_0^\infty e^{x^2}dx$.	**04**
	(c)	Evaluate $\int_0^3\int_{\frac{\sqrt{x}}{3}}^1 e^{y^2}dydx$.	**07**
Q.2	**(a)**	Define the convergence of a sequence (a_n) and verify whether the sequence whose n^{th} term is $a_n=\left(\frac{n+1}{n-1}\right)^n$ converges or not.	**03**
	(b)	Sketch the region of integration and evaluate the integral $\iint_R (y-2x^2)dA$ where R is the region inside the square $\lvert x\rvert+\lvert y\rvert=1$.	**04**
	(c)	**(i)** Find the sum of the series $\sum_{n\geq 2}\frac{1}{4^n}$ and $\sum_{n\geq 1}\frac{4}{(4n-3)(4n+1)}$. **(ii)** Use Taylor's series to estimate $sin38°$.	**07**
		OR	
	(c)	Evaluate the integrals $\int_0^\infty\int_0^\infty\frac{1}{(1+x^2+y^2)^2}dxdy$ and $\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{ln3}\frac{\pi e^{2x}sin\pi y^2}{y^2}dxdydz$.	**07**
Q.3	**(a)**	If an electrostatic field E acts on a liquid or a gaseous polar dielectric, the net dipode moment P per unit volume is $P(E)=\frac{e^E+e^{-E}}{e^E-e^{-E}}-\frac{1}{E}$. Show that $\lim_{E\to 0^+}P(E)=0$.	**03**
	(b)	For what values of the constant k does the second derivative test guarantee that $f(x,y)=x^2+kxy+y^2$ will have a saddle point at (0,0)? A local minimum at (0,0)?	**04**
	(c)	Find the series radius and interval of convergence for $\sum_{n=0}^\infty\frac{(3x-2)^n}{n}$. For what values of x does the series converge absolutely?	**07**
		OR	
Q.3	**(a)**	Determine whether the integral $\int_0^3\frac{dx}{x-1}$ converges or diverges.	**03**
	(b)	Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=1, x=4$ about the line $y=1$.	**04**

	(c)	Check the convergence of the series $\sum_{n=1}^{\infty}\frac{(lnn)^3}{n^3}$ and $\sum_{n=0}^{\infty}(-1)^n(\sqrt{n+\sqrt{n}}-\sqrt{n})$.	07
Q.4	(a)	Show that the function $f(x,y)=\frac{2x^2y}{x^4+y^2}$ has no limit as (x,y) approaches to (0,0).	03
	(b)	Suppose f is a differentiable function of x and y and $g(u,v)=f(e^u+sinv,e^u+cosv)$. Use the following table to calculate $g_u(0,0),g_v(0,0),g_u(1,2)$ and $g_v(1,2)$.	04

	f	g	f_x	f_y
(0,0)	3	6	4	8
(1,2)	6	3	2	5

	(c)	Find the Fourier series of 2π −periodic function $f(x)=x^2, 0<x<2\pi$ and hence deduce that $\frac{\pi^2}{6}=\sum_{n=0}^{\infty}\frac{1}{n^2}$.	07

OR

Q.4	(a)	Verify that the function $u=e^{-\alpha^2k^2t}\cdot sinkx$ is a solution f the heat conduction euation $u_t=\alpha^2u_{xx}$.	03
	(b)	Find the half-range cosine series of the function $f(x)=\begin{cases}2, & -2<x<0\\ 0, & 0<x<2\end{cases}$.	04
	(c)	Find the points on the sphere $x^2+y^2+z^2=4$ that are closest to and farthest from the point $(3,1,-1)$.	07
Q.5	(a)	Find the directional derivative $D_uf(x,y)$ if $f(x,y)=x^3-3xy+4y^2$and u is the unit vector given by angle $\theta=\frac{\pi}{6}$. What is $D_uf(1,2)$?	03
	(b)	Find the area of the region bounded y the curves $y=sinx, y=cosx$ and the lines $x=0$ and $x=\frac{\pi}{4}$.	04
	(c)	Prove that $A=\begin{bmatrix}0&0&-2\\1&2&1\\1&0&3\end{bmatrix}$ is diagonalizable and use it to find A^{13}.	07

OR

Q.5	(a)	Define the rank of a matrix and find the rank of the matrix $A=\begin{bmatrix}2&-1&0\\4&5&-3\\1&-4&7\end{bmatrix}$.	03
	(b)	Use Gauss-Jordan algorithm to solve the system of linear equations $2x_1+2x_2-x_3+x_5=0$ $-x_1-x_2+2x_3-3x_4+x_5=0$ $x_1+x_2-2x_3-x_5=0$ $x_3+x_4+x_5=0$	04
	(c)	State Cayley-Hamilton theorem and verify if for the matrix $A=\begin{bmatrix}4&0&1\\-2&1&0\\-2&0&1\end{bmatrix}$.	07

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II(NEW)EXAMINATION – SUMMER 2022

Subject Code:3110014 **Date:02-08-2022**

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

				Marks
Q.1	**(a)**		Is $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent for $p>1$? Justify your answer.	**03**
	(b)	(1)	Find $\lim_{x \to a} \frac{\sin x - \sin a}{(x-a)^2}$	**02**
		(2)	Is $\int_0^{\infty} \frac{\sin^2 x}{x^2}$ convergent? Justify your answer.	**02**
	(c)	(1)	Find the length of curve $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \le x \le 4$.	**04**
		(2)	Prove that $Gamma\,(n) = (n-1)\,Gamma\,(n-1)$.	**03**
Q.2	**(a)**		Investigate the convergence of $\sum_{1}^{\infty} \frac{n^2}{7^n}$.	**03**
	(b)		Investigate the convergence of $\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$	**04**
	(c)		Find Fourier series of $f(x) = x^2$, $-\pi < x < \pi$.	**07**
			OR	
	(c)		Find Fourier series of $f(x) = \begin{cases} x, & -1 < x < 0 \\ 2, & 0 < x < 1 \end{cases}$.	**07**
Q.3	**(a)**		Find the derivative of $f(x,y) = x^2 + xy + y^2$ in the direction $\hat{i} + \hat{j}$ at $P(1,1)$.	**03**
	(b)		Find the tangent plane of $z = e^x \cos y$ at $P(0,0,0)$.	**04**
	(c)		Find local extreme values of $f(x,y) = xy - x^2 - y^2 - x$.	**07**
			OR	
Q.3	**(a)**		Explain second derivative test for local extreme values.	**03**
	(b)		Let $f = \ln r$, where $\bar{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ and $r = \lvert \bar{r} \rvert$. Find $grad\ f$.	**04**
	(c)		Determine the minimum value of $x^2 y z^2$ subject to the condition $x + y + 2z = 5$ using method of Lagrange multipliers.	**07**
Q.4	**(a)**		Evaluate $\int_{y=0}^{1} \int_{x=0}^{2} \frac{1}{\sqrt{4-x^2}\sqrt{1-y^2}}\,dxdy$.	**03**

(b) Evaluate the integral $\int_0^2 \int_{x/2}^1 \frac{1}{3} e^{y^2} dy\,dx$ by change of order. **04**

(c) (1) Find the area of the region covered by $x=1,\ x=4,\ y=0$ and $y=\sqrt{x}$. **04**

(2) Evaluate $\int_{x=0}^{1} \int_{y=0}^{x^{1/4}} \int_{z=0}^{y^2} \sqrt{z}\ dz\ dy\ dx$ **03**

OR

Q.4 **(a)** Evaluate $\iint_R xy\ dA$ where R is the region bounded by x axis, the ordinate $x=2a$ and the curve $x^2=4ay$. **03**

(b) Evaluate the integral $\int_{y=0}^{1} \int_{x=0}^{\cos^{-1} y} e^{\sin x}\ dx\,dy$ by change of order. **04**

(c) (1) Change in to polar coordinates then solve $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-\left(x^2+y^2\right)} dy\ dx$. **04**

(2) Let $x+y=u$ and $y=uv$ are given transformations. Find Jacobian for change of variables. **03**

Q.5 **(a)** Find characteristic equation of $A=\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$. **03**

(b) Find Maclaurin's series for $f(x)=e^{2x}\sinh x$ and show at least up to x^4 term. **04**

(c) Solve **07**

$x+y+w=1,\ 2x+z+w=3,\ 2y+z+2w=2$.

OR

Q.5 **(a)** Show that give matrix $A=\begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$ satisfies its Characteristic equation. **03**

(b) Show that $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges. **04**

(c) Show that $A=\begin{bmatrix} 0 & 1 & 0 \\ -2 & -1 & 2 \\ -4 & -8 & 7 \end{bmatrix}$ is diagonalizable. Find the matrix of eigen vectors and diagonal matrix. **07**

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