

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019****Subject Code: 3110015****Date: 01/01/2020****Subject Name: Mathematics –2****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

	Marks
Q.1 (a) Find the length of curve of the portion of the circular helix $\vec{r}(t) = \cos t \hat{i} + \sin t \hat{j} + t \hat{k}$ from $t = 0$ to $t = \pi$	03
(b) $\int_{(1,2)}^{(3,4)} (xy^2 + y^3)dx + (x^2y + 3xy^2)dy$ is independent of path joining the points $(1, 2)$ and $(3, 4)$. Hence, evaluate the integral.	04
(c) Verify tangential form of Green's theorem for $\vec{F} = (x - \sin y)\hat{i} + (\cos y)\hat{j}$, where C is the boundary of the region bounded by the lines $y = 0, x = \pi/2$ and $y = x$.	07
Q.2 (a) Find the Laplace transform of $f(t)$ defined as	03
$f(t) = \begin{cases} \frac{t}{k} & 0 < t < k \\ 1 & t > k \end{cases}$	
(b) Find the inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$	04
(c) (i) Calculate the curl of the vector $xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$ (ii) The temperature at any point in space is given by $T = xy + yz + zx$. Determine the derivative of T in the direction of the vector $3\hat{i} - 4\hat{k}$ at the point $(1, 1, 1)$.	07
OR	
(c) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = \vec{r} $, and \vec{a} is a constant vector. Find the value of $\text{div} \left(\frac{\vec{a} \times \vec{r}}{r^n} \right)$	07
Q.3 (a) Find constants a, b and c such that $\vec{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.	03
(b) Using Fourier cosine integral representation show that $\int_0^\infty \frac{\cos \omega x}{k^2 + \omega^2} d\omega = \frac{\pi e^{-kx}}{2k}$	04
(c) Solve the following differential equations:	07
(i) $\cos(x + y) dy = dx$	
(ii) $\sec^2 y \frac{dy}{dx} + x \tan y = x^3$	

OR

- Q.3 (a)** Find the Laplace transform of (i) $\int_0^t \frac{\sin t}{t} dt$ (ii) $t^2 u(t-3)$ **03**
- (b)** Using Convolution theorem obtain $L^{-1}\left(\frac{1}{s(s^2 + a^2)}\right)$ **04**
- (c)** Find the power series solution of $\frac{d^2 y}{dx^2} + xy = 0$ **07**
- Q.4 (a)** Find the Laplace transform of the waveform **03**
 $f(t) = \left(\frac{2t}{3}\right), 0 \leq t \leq 3$
- (b)** Using the Laplace transforms, find the solution of the initial value problem **04**
 $y'' + 25y = 10 \cos 5t \quad y(0) = 2, y'(0) = 0$
- (c)** Using variation of parameter method solve $(D^2 + 1)y = x \sin x$ **07**
- OR**
- Q.4 (a)** Solve $y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$ **03**
- (b)** Solve $y''' - 3y'' + 3y' - y = 4e^t$ **04**
- (c)** Solve $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ using method of undetermined **07**
coefficients.
- Q.5 (a)** Classify the singular points of the equation $x^3(x-2)y'' + x^3y' + 6y = 0$ **03**
- (b)** Solve $(D^2 + 4)y = \cos 2x$ **04**
- (c)** Solve (i) $ye^x dx + (2y + e^x)dy = 0$ (ii) $\frac{dy}{dx} + 2y \tan x = \sin x$ **07**
- OR**
- Q.5 (a)** Solve $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$ **03**
- (b)** If $y_1 = x$ is one of solution of $x^2 y'' + xy' - y = 0$ find the second solution. **04**
- (c)** Using Frobenius method solve $x^2 y'' + 4xy' + (x^2 + 2)y = 0$ **07**

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-1/2 EXAMINATION – WINTER 2021****Subject Code:3110015****Date:21/03/2022****Subject Name:Mathematics - 2****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

		Marks
Q.1	(a) Find $L\{t^3 e^{-4t}\}$.	03
	(b) Find $L^{-1}\left\{\frac{6e^{-2s}}{s^2 + 4}\right\}$.	04
	(c) Verify Green's theorem for the function $\bar{F} = (x + y)i + 2xyj$ and C is the rectangle in the xy-plane bounded by $x = 0, y = 0, x = a, y = b$.	07
Q.2	(a) Find $L\{te^{4t} \cos 2t\}$.	03
	(b) Find the Fourier cosine integral of $f(x) = \frac{\pi}{2} e^{-x}, x \geq 0$.	04
	(c) (i) Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point (2,1,3) in the direction of $\bar{a} = (1,0,-2)$.	03
	(ii) If $\bar{F} = (2y + 3)i + xzj + (yz - x)k$, evaluate $\int_C \bar{F} \cdot d\bar{r}$ along the path C: $x = 2t^2, y = t, z = t^3$ from $t=0$ to $t=1$.	04
	OR	
	(c) Solve in series $3xy'' + 2y' + y = 0$ using Frobenius method.	07
Q.3	(a) Find the arc length of the curve (semi-circular) $x(t) = \cos t, y(t) = \sin t, z(t) = 0; 0 \leq t \leq \pi$.	03
	(b) A vector field is given by $\bar{F} = (x^2 + xy^2)i + (y^2 + x^2y)j$. Show that \bar{F} is irrotational and find its scalar potential.	04
	(c) Use divergence theorem for $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ over the surface of rectangular parallelepiped, $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$ to evaluate $\iiint_S \bar{F} \cdot \hat{n} ds$.	07
	OR	
Q.3	(a) Solve $\frac{dy}{dx} - y \cot x = 2x \sin x$.	03
	(b) Solve $y'' + y' - 12y = e^{6x}$.	04
	(c) Solve $\frac{dy}{dt} - 4y = 2e^{2t} + e^{4t}$ by Laplace transformation.	07

- Q.4** (a) Solve $\frac{dy}{dx} + \frac{y}{x} = y^3$. **03**
- (b) Solve $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$. **04**
- (c) Solve $y'' + 9y' = 2x^2$ using the method of undetermined coefficients. **07**
- OR**
- Q.4** (a) Solve $4xp^2 = (3x - a)^2$. **03**
- (b) Solve $x^2 y'' + xy' - 4y = x^2$. **04**
- (c) (i) Express $2 - 3x + 4x^2$ in terms of Legendre's polynomial. **03**
- (ii) Find ordinary and singular points of $2x^2 y'' + 6xy' + (x + 3)y = 0$. **04**
- Q.5** (a) Solve $(y - px)(p - 1) = p$. **03**
- (b) Solve $(D^3 + D)y = \cos x$. **04**
- (c) Solve $y'' + 4y = \sec 2x$ by using the method of variation of parameters. **07**
- OR**
- Q.5** (a) Solve $(D^3 - 6D^2 + 11D - 6)y = 0$. **03**
- (b) Solve $(2x + 3)^2 y'' - 2(2x + 3)y' - 12y = 6x$. **04**
- (c) Find the series solution of $(1 + x^2)y'' + xy' - 9y = 0$ near the ordinary point $x=0$. **07**

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-I & II (NEW) EXAMINATION – SUMMER-2019****Subject Code: 3110015****Date: 01/06/2019****Subject Name: Mathematics –2****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

		Marks
Q.1	(a) Find the Fourier integral representation of $f(x) = \begin{cases} x & ; x \in (0, a) \\ 0 & ; x \in (a, \infty) \end{cases}$	03
	(b) Define: Unit step function. Use it to find the Laplace transform of $f(t) = \begin{cases} (t-1)^2 & ; t \in (0, 1] \\ 1 & ; t \in (1, \infty) \end{cases}$	04
	(c) Use the method of undetermined coefficients to solve the differential equation $y'' - 2y' + y = x^2 e^x$.	07
Q.2	(a) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$; where $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ and C is the curve given by the parametric equation $\vec{C} : r(t) = t^2\hat{i} + t\hat{j}; 0 \leq t \leq 2$.	03
	(b) Apply Green's theorem to find the outward flux of a vector field $\vec{F} = \frac{1}{xy}(x\hat{i} + y\hat{j})$ across the curve bounded by $y = \sqrt{x}$, $2y = 1$ and $x = 1$.	04
	(c) Integrate $f(x, y, z) = x - yz^2$ over the curve $C = C_1 + C_2$, where C_1 is the line segment joining (0,0,1) to (1,1,0) and C_2 is the curve $y=x^2$ joining (1,1,0) to (2,4,0).	07
OR		
	(c) Check whether the vector field $\vec{F} = e^{y+2z}\hat{i} + x e^{y+2z}\hat{j} + 2x e^{y+2z}\hat{k}$ is conservative or not. If yes, find the scalar potential function $\phi(x, y, z)$ such that $\vec{F} = \text{grad } \phi$.	07
Q.3	(a) Write a necessary and sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact differential equation. Hence check whether the differential equation $[(x+1)e^x - e^y]dx - xe^y dy = 0$ is exact or not.	03
	(b) Solve the differential equation $(1 + y^2)dx = (e^{-\tan^{-1}y} - x)dy$	04
	(c) By using Laplace transform solve a system of differential equations $\frac{dx}{dt} = 1 - y$, $\frac{dy}{dt} = -x$, where $x(0) = 1, y(0) = 0$.	07
OR		
Q.3	(a) Solve the differential equation $(2x^3 + 4y)dx - xdy = 0$.	03

- (b) Solve: $(x + 1) \frac{dy}{dx} - y = e^{3x}(x + 1)^2$. 04
- (c) By using Laplace transform solve a differential equation $\frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 6y = e^{-t}$, where $y(0) = 0$, $y'(0) = -1$. 07
- Q.4** (a) Find the general solution of the differential equation 03
 $e^{-y} \frac{dy}{dx} + \frac{e^{-y}}{x} = \frac{1}{x^2}$
- (b) Solve : $\frac{d^3y}{dx^3} - 7 \frac{dy}{dx} + 6y = e^x$ 04
- (c) Find a power series solution of the differential equation $y'' - xy = 0$ near an ordinary point $x=0$. 07
- OR**
- Q.4** (a) Find the general solution of the differential equation 03
 $\frac{dy}{dx} + \frac{y}{x} - \sqrt{y} = 0$.
- (b) Solve : $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = x$ 04
- (c) Find a Frobenius series solution of the differential equation $2x^2y'' + xy' - (x + 1)y = 0$ near a regular-singular point $x=0$. 07
- Q.5** (a) Write Legendre's polynomial $P_n(x)$ of degree- n and hence obtain $P_1(x)$ and $P_2(x)$ in powers of x . 03
- (b) Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $y'' + xy' = 0$. 04
- (c) Solve the differential equation 07
 $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3 \cos x$
 by using the method of variation of parameters.
- OR**
- Q.5** (a) Write Bessel's function $J_p(x)$ of the first kind of order- p and hence show 03
 that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
- (b) Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $xy'' + y' = 0$. 04
- (c) Solve the differential equation $y'' + 25y = \sec 5x$ 07
 by using the method of variation of parameters.

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY**BE – SEMESTER 1&2 EXAMINATION – SUMMER 2020****Subject Code: 3110015****Date: 09/11/2020****Subject Name: Mathematics II****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

		Marks
Q.1	(a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the parabola $y^2 = x$ between the points (0, 0) and (1, 1) where $\vec{F} = x^2\hat{i} + xy\hat{j}$	03
	(b) Find the work done in moving particle from A (1, 0, 1) to B (2, 1, 2) along the straight-line AB in the force field $\vec{F} = x^2\hat{i} + (x - y)\hat{j} + (y + z)\hat{k}$	04
	(c) Verify green's theorem for $\oint_C (2xydx - y^2dy)$ where C is the boundary of the region bounded by the ellipse $3x^2 + 4y^2 = 12$	07
Q.2	(a) Find the Laplace transform of $te^{-4t} \sin 3t$.	03
	(b) Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$.	04
	(c) Show that the vector field $\vec{F} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}$ is conservative and find the corresponding scalar potential.	07
	OR	
	(c) Show that $\vec{F} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + x^2y\hat{k}$ is irrotational and find a scalar function ϕ such that $\vec{F} = \text{grad}\phi$.	07
Q.3	(a) Find the directional derivative of $f(x, y) = xy + xe^y + \cos(xy)$ at the point $P(1, 0)$ in the direction of $\vec{u} = 3\hat{i} - 4\hat{j}$.	03
	(b) Find the inverse Laplace transform of $\log\left(1 + \frac{1}{s^2}\right)$.	04
	(c) Find the singular solution and general solution of $y + px = x^4 p^2$	07
	OR	
Q.3	(a) Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.	03
	(b) Show that $\int_0^\infty \frac{\omega^3 \sin \omega x}{\omega^4 + 4} d\omega = \frac{\pi}{2} e^{-x} \cos x; x > 0$.	04
	(c) Find the power series solution of $y' - 2xy = 0; y(0) = 1$ near $x = 0$.	07

- Q.4** (a) Find the Laplace transform of $e^{-t} \{1 - u(t-2)\}$. **03**
- (b) Solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2, \frac{dx}{dt} = -1$ at $t = 0$. **04**
- (c) Solve $(D^2 - 1)y = xe^x \sin x$ **07**
- OR**
- Q.4** (a) Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ **03**
- (b) Using method of variation of parameter, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. **04**
- (c) Using method of undetermined coefficients solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^x$. **07**
- Q.5** (a) Classify the singular points of $x^2 y'' + xy' - 2y = 0$. **03**
- (b) Solve $\frac{d^2y}{dx^2} + 9y = \sin 2x \sin x$. **04**
- (c) Solve (i) $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$. **07**
- (ii) $\frac{dy}{dx} + y \cot x = 2 \cos x$.
- OR**
- Q.5** (a) Solve $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$. **03**
- (b) Solve $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + y = \cos(\ln x)$. **04**
- (c) Using Frobenius method solve $2x^2 y'' + xy' - (x+1)y = 0$. **07**

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER-I & II(NEW)EXAMINATION – SUMMER 2022

Subject Code:3110015

Date:22-08-2022

Subject Name:Mathematics - 2

Time:10:30 AM TO 01:30 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

	Marks
Q.1 (a) Find the Laplace transform of $t^2 e^{-3t}$.	03
(b) Define conservative vector field and potential function.	04
(c) Solve $y''' - 3y'' + 3y' - y = 4e^x$ using the method of undetermined coefficients.	07
Q.2 (a) Find the divergence of $\mathbf{F} = (x^2 - y)\mathbf{i} + (xy - y^2)\mathbf{j}$.	03
(b) Find Fourier cosine integral of $f(x) = e^{-kx} (x > 0, k > 0)$	04
(c) Integrate $f(x, y, z) = 3x^2 - 2y + z$ over the line segment C joining the origin to the point $(2, 2, 2)$.	07
OR	
(c) Write Green's theorem. Evaluate the integral $\oint_C \{xydy - y^2dx\}$ where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.	07
Q.3 (a) Obtain convolution of t and e^t .	03
(b) Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.	04
(c) Solve the initial value problem $y'' - y' - 2y = 0, y(0) = 1, y'(0) = 0$ using Laplace transform.	07
OR	
Q.3 (a) Find the inverse Laplace transform of $\frac{s-4}{s^2-4}$.	03
(b) State second shifting theorem and find the inverse Laplace transform of the function $\frac{se^{-\pi s}}{s^2+1}$.	04
(c) State convolution theorem and using it obtain the inverse Laplace transform of $\frac{1}{s(s^2+4)}$.	07
Q.4 (a) Solve $\frac{dy}{dx} - 2y = 4 - x$.	03
(b) Solve $p^2 + 2p \cot x = y^2$.	04
(c) Solve $y'' + 4y = 4 \tan 2x$ using the method of variation of parameters.	07
OR	
Q.4 (a) Find particular solution of $y'' - 2y' + y = \cos 3x$.	03
(b) Solve $x^2 y'' - 3xy' + 4y = 0$	04

- (c) Solve the initial value problem **07**
 $y''' + y' = 0,$
 $y(0) = 0, y'(0) = 1, y''(0) = 2$
- Q.5** (a) Write Legendre's and Bessel's differential equations. **03**
 (b) Solve the differential equation **04**
 $(y \cos x + 2xe^y) + (\sin x + x^2 e^y - 1)y' = 0$
 (c) Find the power series solution of the equation $(x^2 + 1)y'' + xy' -$ **07**
 $xy = 0$ in powers of x .
- OR**
- Q.5** (a) Write Legendre polynomials of degree one and two. **03**
 (b) Solve $y = 2px + p^2y$. **04**
 (c) Solve $x(x - 1)y'' + (3x - 1)y' + y = 0$ about $x = 0$ using Frobenius **07**
 method.
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