Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (New) EXAMINATION – WINTER 2019

Subject Code: 3130005 **Date: 26/11/2019**

Subject Name: Complex Variables and Partial Differential Equations

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Find the real and imaginary parts of $f(z) = \frac{3i}{2+3i}$.	**03**
	(b)	State De-Movire's formula and hence evaluate $\left(1+i\sqrt{3}\right)^{100} + \left(1-i\sqrt{3}\right)^{100}$.	**04**
	(c)	Define harmonic function. Show that $u(x,y) = \sinh x \sin y$ is harmonic function, find its harmonic conjugate $v(x,y)$.	**07**
Q.2	**(a)**	Determine the Mobius transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ into $w_1 = -1,\ w_2 = -i,\ w_3 = 1$.	**03**
	(b)	Define $log z$, prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$.	**04**
	(c)	Expand $f(z) = \frac{1}{(z-1)(z+2)}$ valid for the region (i) $\lvert z\rvert < 1$ (ii) $1 < \lvert z\rvert < 2$ (iii) $\lvert z\rvert > 2$.	**07**
		OR	
	(c)	Find the image of the infinite strips (i) $\frac{1}{4} \le y \le \frac{1}{2}$ (ii) $0 < y < \frac{1}{2}$ under the transformation $= \frac{1}{z}$. Show the region graphically.	**07**
Q.3	**(a)**	Evaluate $\int_c (x - y + ix^2)dz$ where c is a straight line from $z = 0$ to $z = 1 + i$.	**03**
	(b)	Check whether the following functions are analytic or not at any point, (i) $f(z) = 3x + y + i(3y - x)$ (ii) $f(z) = z^{3/2}$.	**04**
	(c)	Using residue theorem, evaluate $\int_0^\infty \frac{dx}{(x^2+1)^2}$.	**07**
		OR	
Q.3	**(a)**	Expand Laurent series of $f(z) = \frac{1-e^z}{z}$ at $z = 0$ and identify the singularity.	**03**
	(b)	If $f(z) = u + iv$, is an analytic function, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\lvert Re f(z)\rvert^2 = 2\lvert f'(z)\rvert^2$.	**04**
	(c)	Evaluate the following: i. $\int_c \frac{z+3}{z-1} dz$ where c is the circle (a) $\lvert z\rvert = 2$ (b) $\lvert z\rvert = \frac{1}{2}$. ii. $\int_c \frac{\sin z}{\left(z-\frac{\pi}{4}\right)^3} dz$ where c is the circle $\lvert z\rvert = 1$.	**07**

Q.4	**(a)**	Evaluate $\int_0^{2+4i} Re(z)dz$ along the curve $z(t) = t + it^2$.	**03**
	(b)	Solve $x^2p + y^2q = (x + y)z$.	**04**
	(c)	Solve the equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ for the condition of heat along rod without radiation subject to the conditions (i) $\frac{\partial u}{\partial t} = 0$ for $x = 0$ and $x = l$; (ii) $u = lx - x^2$ at $t = 0$ for all x .	**07**

OR

Q.4	**(a)**	Solve $\frac{\partial^2 z}{\partial x^2} + 2\,\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{2x+3y}$.	**03**
	(b)	Solve $px + qy = pq$ using Charpit's method.	**04**
	(c)	Find the general solution of partial differential equation $u_{xx} = 9u_y$ using method of separation of variables.	**07**
Q.5	**(a)**	Using method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$.	**03**
	(b)	Solve $z(xp - yq) = y^2 - x^2$.	**04**
	(c)	A string of length $L = \pi$ has its ends fixed at $x = 0$ and $x = \pi$. At time $t = 0$, the string is given a shape defined by $f(x) = 50x(\pi - x)$, then it is released. Find the deflection of the string at any time t.	**07**

OR

Q.5	**(a)**	Solve $p^3 + q^3 = x + y$.	**03**
	(b)	Find the temperature in the thin metal rod of length l with both the ends insulated and initial temperature is $\sin {}^{\pi x}/_{l}$.	**04**
	(c)	Derive the one dimensional wave equation that governs small vibration of an elastic string . Also state physical assumptions that you make for the system.	**07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–III (NEW) EXAMINATION – WINTER 2020

Subject Code:3130005 Date:09/03/2021

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 12:30 PM Total Marks:56

Instructions:

1. **Attempt any FOUR questions out of EIGHT questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Show that the function $u = x^2 - y^2 + x$ is harmonic.	**03**
	(b)	Find the fourth roots of -1.	**04**
	(c)	(i) Find and sketch the image of the region $\lvert z\rvert > 1$ under the transformation $w = 4z.$	**03**
		(ii) Find the real and imaginary parts of $f(z) = z^2 + 3z.$	**04**
Q.2	**(a)**	Evaluate $\int_0^{2+i} z^2 dz$ along the line $y = x/2$.	**03**
	(b)	Determine the Mobius transformation that maps $z = 0,1,\infty$ onto $w = -1,-i,1$ respectively.	**04**
	(c)	(i) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$, where C is the circle $\lvert z\rvert = 1/2.$	**03**
		(ii) For which values of z does the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ convergent?	**04**
Q.3	**(a)**	Evaluate $\oint_C \frac{dz}{z^2}$ where C is a unit circle.	**03**
	(b)	Find the residue *Res(f(z),2i)* of the function $f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2 + 4)}$.	**04**
	(c)	Expand $f(z) = \frac{1}{(z+1)(z-2)}$ in Laurent's series in the region $(i)\lvert z\rvert < 1,$ $(ii) 1 < \lvert z\rvert < 2$, $(iii)\lvert z\rvert > 2.$	**07**
Q.4	**(a)**	Evaluate $\oint_C \frac{5z+7}{z^2 + 2z - 3} dz$, where $C : \lvert z-2\rvert = 2.$	**03**
	(b)	Evaluate by using Cauchy's Residue Theorem $\int_C \frac{5z-2}{z(z-1)} dz$, $C : \lvert z\rvert = 2$.	**04**
	(c)	Find Laurent's series that represent $f(z) = \frac{1}{z(z-1)}$ in the region $(i) 0 < \lvert z\rvert < 1$, $(ii) 0 < \lvert z-1\rvert < 1.$	**07**

Q.5	**(a)**	Solve $xp + yq = x - y$.	**03**
	(b)	Derive p.d.e. from $z = ax + by + ab$ by eliminating a and b.	**04**
	(c)	(i) Solve $(D^3 - 4D^2D' + 4DD'^2)z = 0$.	**03**
		(ii) Solve $pq = k$, where k is a constant.	**04**
Q.6	**(a)**	Solve $zp + yq = x$.	**03**
	(b)	Form the p.d.e. by eliminating ϕ from $x + y + z = \phi(x^2 + y^2 + z^2)$.	**04**
	(c)	(i) Solve $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.	**03**
		(ii) Solve $zpq = p + q$ by Charpit's method.	**04**
Q.7	**(a)**	Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x}$.	**03**
	(b)	Solve the p.d.e. $u_x = 4u_y$, $u(0, y) = 8e^{-3y}$.	**04**
	(c)	A string of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity kx for $0 \le x \le l/2$ and $k(l-x)$ for $l/2 \le x \le l$. Find the displacement $u(x,t)$.	**07**
Q.8	**(a)**	Solve $DD''(D - 2D' - 3)z = 0$.	**03**
	(b)	Solve the pde $u_{xx} = 16u_{xy}$.	**04**
	(c)	A bar of length 2 *m* is fully insulated along its sides. It is initially at a uniform temperature of $10^o C$ and at *t = 0* the ends are plunged into ice and maintained at a temperature of $0^o C$. Determine an expression for the temperature at a point P at a distance x from one end at any subsequent time t seconds after *t = 0*.	**07**

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GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – WINTER 2021

Subject Code:3130005 Date:17-02-2022

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 **(a)** Represent $z = 7i$ into polar form and find the argument of z and the principal value of the argument of z. **03**

(b) State De Moivre's theorem. Find and plot all roots of $(1+i)^{\frac{1}{3}}$ in the complex plane. **04**

(c) Using the method of separation of variables, solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ and $u = e^{-5y}$ *when* $x = 0$. **07**

Q.2 **(a)** Define an analytic function. Write the necessary and sufficient condition for function $f(z)$ to be analytic. Show that $f(z) = |z|^2$ is nowhere analytic. **03**

(b) Define the Mobious transformation. Determine the bilinear transformation which mapping the points $0, \infty, i$ *onto* $\infty, 2, 0$. **04**

(c) **Attempt the following.**

(i) Define the harmonic function. Show that $u = x^2 - y^2 + x$ is harmonic and find harmonic conjugate of u. **04**

(ii) Show that $f(z) = \begin{cases} \frac{\mathrm{Im}(z)}{|z|} & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$ is not continuous at $z = 0$ **03**

OR

(c) **Attempt the following.**

(i) Prove that $\cos^{-1} z = -i \ln(z + i\sqrt{1-z^2})$. **4**

(ii) Find the values of $\mathrm{Re}\, f(z)$ and $\mathrm{Im}\, f(z)$ at the point $7+2i$, where $f(z) = \frac{1}{1-z}$. **3**

Q.3 **(a)** Evaluate $\int_C \bar{z}\, dz$, where C is the right- half of the circle $|z| = 2$ and hence show that $\int_C \frac{dz}{z} = \pi i$. **03**

(b) Expand $f(z) = \sin z$ in a Taylor series about $z = \frac{\pi}{4}$ and write the Maclaurin series for e^{-z}. **04**

(c) Write the Cauchy integral theorem and Cauchy integral formula and hence evaluate: **07**

(i) $\oint_C \frac{e^z}{(z-1)(z-3)} dz; C: |z| = 2.$ (ii) $\oint_C e^z dz; C: |z| = 3$.

OR

Q.3 **(a)** Evaluate $\oint_C \frac{e^z}{z+i}dz$, where $c:|z-1|=1$. **03**

(b) Develop the following functions into Maclaurin series:(i) $\cos^2 z$ (ii) $e^z \cos z$. **04**

(c) Evaluate $\int_C \text{Re}(z^2)dz$, where C is the boundary of the square with vertices $0, i, 1+i, 1$ in the clockwise direction. **07**

Q.4 **(a)** Define the singular points of $f(z)$. Find the singularity of $f(z)$ and classify as pole, essential singularity or removable singularity. where $f(z)=\frac{1-e^z}{z}$. **03**

(b) State the Cauchy residue theorem. Find the residue at its poles of $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ and hence evaluate $\oint_C f(z)dz$, $C:|z|=3$. **04**

(c) Determine the Laurent series expansion of $f(z)=\frac{1}{(z+2)(z+4)}$ valid for regions **07**

(i) $|z|<2$

(ii) $2<|z|<4$

(iii) $|z|>4$

OR

Q.4 **(a)** Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=e^{x+2y}$. **03**

(b) Form a partial differential equation by eliminating the arbitrary functions form the equations $z=f(x+ay)+\phi(x-ay)$. **04**

(c) Show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}=\frac{\pi}{6}$. **07**

Q.5 **(a)** Form a partial differential equation by eliminating the arbitrary function form $u=f(\frac{x}{y})$. **03**

(b) State the Lagrange's linear partial differential equation of first order and hence solve $x(y-z)p+y(z-x)=z(x-y)$ **04**

(c) Using the method of separation of variables, solve $\frac{\partial u}{\partial x}=4\frac{\partial u}{\partial y}$ where $u(0,y)=8e^{-3y}$. **07**

OR

Q.5 **(a)** Obtain the general solution of $p+q^2=1$. **03**

(b) Solve by Charpit's method: $px+qy=pq$. **04**

(c) Find the solution of the wave equation $u_{tt}=c^2u_{xx}$, $0\le x\le L$ satisfying the condition $u(0,t)=u(L,t)=0$, $u_t(x,0)=0$, $u(x,0)=\frac{\pi x}{L}$, $0\le x\le L$. **07**

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GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III EXAMINATION – SUMMER 2020

Subject Code: 3130005 Date:27/10/2020

Subject Name: Complex Variables and Partial Differential Equations

Time: 02:30 PM TO 05:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	If $u = x^3 - 3xy$ is find the corresponding analytic function $f(z) = u + iv$.	**03**
	(b)	Find the roots of the equation $z^2 - (5+i)z + 8 + i = 0$.	**04**
	(c)	(i) Determine and sketch the image of $\lvert z\rvert = 1$ under the transformation $w = z + i$.	**03**
		(ii) Find the real and imaginary parts of $f(z) = z^2 + 3z$.	**04**
Q.2	**(a)**	Evaluate $\int_C \left(x^2 - iy^2\right)dz$ along the parabola $y = 2x^2$ from (1,2) to (2,8).	**03**
	(b)	Find the bilinear transformation that maps the points $z = \infty, i, 0$ into $w = 0, i, \infty$.	**04**
	(c)	(i) Evaluate $\oint_C \frac{e^{-z}dz}{z+1}$, where C is the circle $\lvert z\rvert = 1/2$.	**03**
		(ii) Find the radius of convergence of $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2} z^n$.	**04**
		OR	
	(c)	(i) Find the fourth roots of -1.	**03**
		(ii) Find the roots of $\log z = i\frac{\pi}{2}$.	**04**
Q.3	**(a)**	Find $\oint_C \frac{1}{z^2}dz$, where $C : \lvert z\rvert = 1$.	**03**
	(b)	For $f(z) = \frac{1}{(z-1)^2(z-3)}$, find Residue of *f(z)* at *z=1*.	**04**
	(c)	Expand $f(z) = \frac{1}{(z+2)(z+4)}$ in a Laurent series for the regions $(i)\lvert z\rvert < 2$, $(ii) 2 < \lvert z\rvert < 4$, $(iii)\lvert z\rvert > 4$.	**07**
		OR	
Q.3	**(a)**	Find $\oint_C \frac{z+4}{z^2+2z+5}dz$, where $C : \lvert z+1\rvert = 1$.	**03**
	(b)	Evaluate using Cauchy residue theorem $\int_C \frac{e^{2z}}{(z+1)^3}dz$; C: $4x^2 + 9y^2 = 16$.	**04**
	(c)	Expand $f(z) = \frac{1}{z(z-1)(z-2)}$ in Laurent's series for the regions $(i)\lvert z\rvert < 1$, $(ii) 1 < \lvert z\rvert < 2$, $(iii)\lvert z\rvert > 2$.	**07**

Q.4	**(a)**	Solve $xp + yq = x - y$.	**03**
	(b)	Derive partial differential equation by eliminating the arbitrary constants a and b from $z = ax + by + ab$.	**04**
	(c)	(i) Solve the p.d.e. $2r + 5s + 2t = 0$.	**03**
		(ii) Find the complete integral of $p^2 = qz$.	**04**

OR

Q.4	**(a)**	Find the solution of $x^2 p + y^2 q = z^2$.	**03**
	(b)	Form the partial differential equation by eliminating the arbitrary function ϕ from $z = \phi\left(\frac{y}{x}\right)$.	**04**
	(c)	(i) Solve the p.d.e. $\left(D^2 - D'^2 + D - D'\right)z = 0$.	**03**
		(ii) Solve by Charpit's method $yzp^2 - q = 0$.	**04**

Q.5	**(a)**	Solve $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$.	**03**
	(b)	Solve the p.d.e. $u_x + u_y = 2(x + y)u$ using the method of separation of variables.	**04**
	(c)	Find the solution of the wave equation $u_{tt} = c^2 u_{xx}$, $0 \le x \le \pi$ with the initial and boundary conditions $u(0,t) = u(\pi,t) = 0; t > 0$, $u(x,0) = k(\sin x - \sin 2x), u_t(x,0) = 0; 0 \le x \le \pi$. $(c^2 = 1)$	**07**

OR

Q.5	**(a)**	Solve the p.d.e. $r + s + q - z = 0$.	**03**
	(b)	Solve $2u_x = u_t + u$ given $u(x,0) = 4e^{-3x}$ using the method of separation of variables.	**04**
	(c)	Find the solution of $u_t = c^2 u_{xx}$ together with the initial and boundary conditions $u(0,t) = u(2,t) = 0; t \ge 0$ and $u(x,0) = 10; 0 \le x \le 2$.	**07**

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GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2021

Subject Code:3130005 Date:05/10/2021

Subject Name: Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Find the fourth roots of -1.	**03**
	(b)	Find the real and imaginary parts of $(-1-i)^7+(-1+i)^7$.	**04**
	(c)	(i) Find the analytic function whose real part is $u=x^2-y^2$.	**04**
		(ii) Show that $u=\dfrac{x}{x^2+y^2}$ is harmonic.	**03**
Q.2	**(a)**	Find the image in the w – plane of the circle $\lvert z-3\rvert=2$ in the z – plane under the inversion mapping $w=\dfrac{1}{z}$.	**03**
	(b)	Evaluate $\int_C (x^2-iy^2)dz$ along the parabola $y=2x^2$ from (1 , 2) to (2 , 8).	**04**
	(c)	State Cauchy's Integral Formula. Evaluate $\oint_C \dfrac{e^{2z}}{(z+1)^4}dz$, where C is the circle $\lvert z\rvert=2$.	**07**
		OR	
	(c)	(i) Evaluate $\oint_C \dfrac{e^{-z}}{z+1}dz$, where C is the circle $\lvert z\rvert=\dfrac{1}{2}$.	**04**
		(ii) Determine and sketch the image of $\lvert z\rvert=1$ under the transformation $w=z+i$.	**03**
Q.3	**(a)**	Find the radius of convergence of $\sum_{n=1}^{\infty}(3+4i)^n z^n$.	**03**
	(b)	Find the residues of the function $f(z)=\dfrac{1}{(z-1)^2(z-3)}$ at each of its poles in the finite z – plane.	**04**
	(c)	Expand $f(z)=\dfrac{1}{(z+1)(z+3)}$ in a Laurent series for the regions (i) $\lvert z\rvert<1$, (ii) $1<\lvert z\rvert<3$, (iii) $\lvert z\rvert>3$.	**07**
		OR	
Q.3	**(a)**	Find the bilinear transformation which transforms $z=2,1,0$ into $w=1,0,i$.	**03**
	(b)	Evaluate $\int_C \dfrac{e^{2z}}{(z+1)^3}dz$ by using Cauchy Residue Theorem, where C is the ellipse $4x^2+9y^2=16$.	**04**

	(c)	Find the Laurent series for the function $f(z)=\dfrac{1}{z(1-z)}$ in the region (i) $\lvert z+1\rvert<1$, (ii) $1<\lvert z+1\rvert<2$, (iii) $\lvert z+1\rvert>2$.	**07**
Q.4	**(a)**	Find the real and imaginary parts of $f(z)=z^3+3z$.	**03**
	(b)	Derive partial differential equation by eliminating arbitrary constants *a* and *b* from $z=(x^2+a)(y^2+b)$.	**04**
	(c)	(i) Solve $x^2p+y^2q=z^2$.	**03**
		(ii) Solve $q=3p^2$ by Charpit's method.	**04**
		OR	
Q.4	**(a)**	Show that the function $f(z)=xy+iy$ is continuous everywhere but not analytic.	**03**
	(b)	Solve $yq-xp=z$.	**04**
	(c)	(i) Find the complete integral of $q=pq+p^2$.	**03**
		(ii) Solve $px+qy=pq$ by Charpit's method.	**04**
Q.5	**(a)**	Solve $r-2s+t=\sin(2x+3y)$.	**03**
	(b)	Solve $(D^2-D'^2+D-D')z=0$.	**04**
	(c)	Solve $u_x=4y_y$, $u(0,y)=8e^{-3y}$ by the method of separation of variables.	**07**
		OR	
Q.5	**(a)**	Solve $(DD'+D-D'-1)z=xy$.	**03**
	(b)	Solve $(D^2+3DD'+2D'^2)z=x+y$.	**04**
	(c)	A tightly stretched string with fixed end points *x = 0*, *x = l* is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$, find the displacement *u(x,t)*.	**07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (NEW) EXAMINATION – SUMMER 2022

Subject Code:3130005 **Date:11-07-2022**

Subject Name:Complex Variables and Partial Differential Equations

Time:02:30 PM TO 05:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Express the complex number $-\sqrt{3}-i$ in polar form.	**03**
	(b)	Use De Moiver's theorem and find $\sqrt[3]{64i}$.	**04**
	(c)	Verify that $u = 2x - x^3 + 3xy^2$ is harmonic in the whole complex plane and finds it's harmonic conjugate function $v(x, y)$.	**07**
Q.2	**(a)**	Discuss Continuity of the function $f(z)$ at the origin: $f(z) = \begin{cases} \frac{Im(z)}{z}, & z \neq 0 \\ 0 & z = 0 \end{cases}$	**03**
	(b)	1) Define $Log(x + iy)$ 2) Determine $Log(-1 + i\ \ 2)$ 3) Determine all values of $log(1 + i)$	**04**
	(c)	Find the image of the circle $\lvert z + i\rvert = 2$ under the transformation $w = \frac{1}{z}$. Also, show the regions graphically.	**07**
		OR	
	(c)	Check whether the function $f(z) = \sin z$ is analytic or not. If so, find its derivative.	**07**
Q.3	**(a)**	Evaluate $\oint_C \frac{\sin z}{(z-\pi)^2} dz$, where C is the circle $\lvert z\rvert = 4$	**03**
	(b)	Find the Laurent's series that represent $f(z) = \frac{1}{(z-2)(z-3)}$ in the region $2 < \lvert z\rvert < 3$.	**04**
	(c)	Find the residues of the function $\mathrm{f(z)} = \frac{\mathrm{z}}{(\mathrm{z}+1)^2(\mathrm{z}^2-4)}$ at its poles.	**07**
		OR	
Q.3	**(a)**	Evaluate $\int_0^{2+i} z^2 dz$ along the line $y = x$	**03**
	(b)	Evaluate $\oint_C \frac{3z+4}{z^2+2z-3} dz$, where C is $\lvert z\rvert = 2$	**04**
	(c)	Using Residue theorem, evaluate the following Integral: $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$	**07**
Q.4	**(a)**	Expand $f(z) = \frac{\sin z}{z^4}$ in Laurent's series about $z = 0$ and identify the singularity.	**03**
	(b)	Solve: $\frac{\partial^2 z}{\partial x^2} + 10\frac{\partial^2 z}{\partial x \partial y} + 25\frac{\partial^2 z}{\partial y^2} = e^{3x+2y}$	**04**
	(c)	Solve $x^2 p + y^2 q = (x + y)z$	**07**

OR

Q.4 **(a)** Find the fixed points of the transformation, $w = \frac{z-1}{z+1}$ **03**

(b) Solve: $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \cos x$ **04**

(c) Solve: $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2\sin y$ when $x=0$ and $z=0$ when y is an odd multiple of $\frac{\pi}{2}$ **07**

Q.5 **(a)** Solve $xp + yq = 3z$ **03**

(b) Solve by the method of separation of variables $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, where $u(0, y) = 8e^{-3y}$ **04**

(c) A tightly stretched string with fixed end points at $x = 0$ and $x = 10$ is initially given by the deflection $f(x) = kx(10 - x)$. If it is released from this position, then find the deflection of the string. **07**

OR

Q.5 **(a)** Find complete and singular solution of $z = px + qy + pq$ **03**

(b) Using Charpit's method, solve $q = 3p^2$. **04**

(c) A rod of 30 cm long has its ends A and B are kept at 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to 0°C and kept so. Find the resulting temperature $u(x, t)$ from the end A. **07**