

GUJARAT TECHNOLOGICAL UNIVERSITY
BE –SEMESTER 1&2(NEW SYLLABUS)EXAMINATION- WINTER 2018

Subject Code: 3110014**Date: 07-01-2019****Subject Name: Mathematics - I****Time: 10:30 am to 01:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

	Marks
Q.1 (a) State Cayley– Hamilton theorem. Find eigen values of A and A^{-1} , where $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$	03
(b) State L' Hospital's Rule. Use it to evaluate $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x} \right]$	04
(c) Investigate convergence of the following integrals:	07
(i) $\int_5^{\infty} \frac{5x}{(1+x^2)^3} dx$	
(ii) $\int_0^{\infty} \frac{x^{10}(1+x^5)}{(1+x)^{27}} dx$	
Q.2 (a) Test the convergence of series $\sum_{n=1}^{\infty} \frac{4^n + 5^n}{6^n}$	03
(b) State the p-series test. Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{n+1}{n^3 - 3n + 2}$	04
(c) State D'Alembert's ratio test and Cauchy's root test. Discuss the convergence of the following series:	07
(i) $\sum_{n=1}^{\infty} \frac{4^n (n+1)!}{n^{n+1}}$	
(ii) $\sum_{n=2}^{\infty} \frac{n}{(\log n)^n}$	
OR	
(c) Test the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{x}{4 \cdot 5 \cdot 6} + \frac{x^2}{7 \cdot 8 \cdot 9} + \frac{x^3}{10 \cdot 11 \cdot 12} + \dots;$ $x \geq 0$	07
Q.3 (a) Reduce matrix $A = \begin{bmatrix} 1 & 5 & 3 & -2 \\ 2 & 0 & 4 & 1 \\ 4 & 8 & 9 & -1 \end{bmatrix}$ to row echelon form and find its rank.	03
(b) Derive half range sine series of $f(x) = \pi - x, 0 \leq x \leq \pi$	04
(c) Find the eigen values and corresponding eigen vectors for the matrix A where $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$	07

OR

- Q.3** (a) Expand $e^{x \sin(x)}$ in power of x up to the terms containing x^6 . 03
(b) Solve system of linear equation by Gauss Elimination method, if solution exists. 04
 $x + y + 2z = 9$; $2x + 4y - 3z = 1$; $3x + 6y - 5z = 0$
(c) Find Fourier series of $f(x) = \begin{cases} x, & -1 < x < 0 \\ 2, & 0 < x < 1 \end{cases}$ 07
- Q.4** (a) Discuss the continuity of the function f defined as 03
$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

(b) Define gradient of a function. Use it to find directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P(1, 1, 0)$ in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. 04
(c) Find the shortest and largest distance from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$. 07

OR

- Q.4** (a) Find the extreme values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ 03
(b) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) dy dx$ by changing into polar coordinates. 04
(c) (i) If $u = x^2y + y^2z + z^2x$ then find out $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ 07
(ii) If $x^3 + y^3 = 6xy$ then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
- Q.5** (a) Evaluate $\iint_R y \sin(xy) dA$, where R is the region bounded by $x = 1$, $x = 2$, $y = 0$ and $y = \frac{\pi}{2}$. 03
(b) By changing the order of integration, evaluate $\int_0^3 \int_y^3 \frac{xdxdy}{x^2 + y^2}$ 04
(c) Find the volume below the surface $z = x^2 + y^2$, above the plane $z = 0$, and inside the cylinder $x^2 + y^2 = 2y$. 07

OR

- Q.5** (a) Evaluate integral $\iint_R \frac{rdrd\theta}{\sqrt{a^2 + r^2}}$ over the region R which is one loop of $r^2 = a^2 \cos 2\theta$ 03
(b) Evaluate the integral $\int_1^e \int_1^{\log y} \int_1^{e^x} (x^2 + y^2) dz dx dy$. 04
(c) Find the volume of the solid obtained by rotating the region R enclosed by the curves $y = x$ and $y = x^2$ about the line $y = 2$. 07

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER- I & II (NEW) EXAMINATION – WINTER 2019****Subject Code: 3110014****Date: 17/01/2020****Subject Name: Mathematics – I****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

		MARKS
Q.1	(a) Find the equations of the tangent plane and normal line to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1)	03
	(b) Evaluate $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$	04
	(c) Using Gauss Elimination method solve the following system $-x+3y+4z=30$ $3x+2y-z=9$ $2x-y+2z=10$	07
Q.2	(a) Test the convergence of the series $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \dots + \left(\frac{n}{2n+1}\right)^n + \dots$	03
	(b) Discuss the Maxima and Minima of the function $3x^2 - y^2 + x^3$	04
	(c) Find the fourier series of $f(x) = \frac{(\pi-x)}{2}$ in the interval (0,2 π)	07
	OR	
	(c) Change the order of integration and evaluate $\int_0^1 \int_x^1 \sin y^2 dy dx$	07
Q.3	(a) Find the value of $\beta \left(\frac{7}{2}, \frac{5}{2}\right)$	03
	(b) Obtain the fourier cosine series of the function $f(x) = e^x$ in the range (0,l)	04
	(c) Find the maximum and minimum distance from the point (1,2,2) to the sphere $x^2 + y^2 + z^2 = 36$	07
Q.3	OR	
	(a) Test the convergence of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$	03
	(b) Evaluate $\iint (x^2 - y^2) dx dy$ over the triangle with the vertices (0,1), (1,1), (1,2)	04
	(c) Find the volume of the solid generated by rotating the plane region bounded by $y = \frac{1}{x}$, $x=1$ and $x=3$ about the X axis.	07
Q.4	(a) Evaluate $\int_0^\pi \int_0^{\sin \theta} r dr d\theta$	03
	(b) Express $f(x) = 2x^3 + 3x^2 - 8x + 7$ in terms of (x-2)	04

- (c) Using Gauss-Jordan method find A^{-1} for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ **07**

OR

- Q.4** (a) Using Cayley-Hamilton Theorem find A^{-1} for $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ **03**

- (b) Evaluate $\int_0^{\infty} \frac{dx}{x^2+1}$ **04**

- (c) Test the convergence of the series $\frac{x}{1 \cdot 2} + \frac{x^2}{3 \cdot 4} + \frac{x^3}{5 \cdot 6} + \frac{x^4}{7 \cdot 8} + \dots$ **07**

- Q.5** (a) Evaluate $\int_0^1 \int_1^2 xy \, dy \, dx$ **03**

- (b) Find the eigen values and eigenvectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$ **04**

- (c) If $u = f(x-y, y-z, z-x)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **07**

OR

- Q.5** (a) Find the directional derivatives of $f = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of $i+2j+2k$. **03**

- (b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ **04**

- (c) Evaluate $\iiint xyz \, dx \, dy \, dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = 4$ **07**

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-I & II (NEW) EXAMINATION – WINTER 2020****Subject Code:3110014****Date:16/03/2021****Subject Name:Mathematics – I****Time:10:30 AM TO 12:30 PM****Total Marks:47****Instructions:**

1. Attempt any **THREE** questions from Q1 to Q6.
2. **Q7 is compulsory.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

	Marks
Q.1 (a) Expand $\sin x$ in powers of $(x - \pi/2)$.	03
(b) Evaluate $\lim_{x \rightarrow 0} \frac{\tan^2 x - x^2}{x^2 \tan^2 x}$.	04
(c) (i) Check the convergence of $\int_4^{\infty} \frac{3x+5}{x^4+7} dx$.	03
(ii) The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the line $x = 4$ is revolved about the x -axis to generate a solid. Find its volume.	04
Q.2 (a) If $u = \cos e^{-1} \left(\frac{x+y}{x^2+y^2} \right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$.	03
(b) Check the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n + 5}{3^n}$.	04
(c) (i) Test the convergence of the series $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$.	03
(ii) Test the convergence of $\sum_{n=1}^{\infty} \frac{2^n}{n^3 + 1}$.	04
Q.3 (a) Solve the following equations by Gauss' elimination method: $x + y + z = 6, x + 2y + 3z = 14, 2x + 4y + 7z = 30$.	03
(b) If $u = f(x - y, y - z, z - x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	04
(c) (i) Find the equation of the tangent plane and normal line to the surface $x^2 + 2y^2 + 3z^2 = 12$ at $(1, 2, -1)$.	03
(ii) For $f(x, y) = x^3 + y^3 - 3xy$, find the maximum and minimum values.	04
Q.4 (a) Find the rank of the matrix $\begin{bmatrix} 8 & 0 & 0 & 16 \\ 0 & 0 & 0 & 6 \\ 0 & 9 & 9 & 9 \end{bmatrix}$.	03
(b) If $u = f(x + at) + g(x - at)$, prove that $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.	04

- (c) (i) Show that the function $f(x, y) = \begin{cases} \frac{2x^2y}{x^3 + y^3}, (x, y) \neq (0, 0) \\ 0, (x, y) = (0, 0) \end{cases}$ is not continuous at the origin. 03

- (ii) Find the shortest distance from the point (1, 2, 2) to the sphere $x^2 + y^2 + z^2 = 16$. 04

- Q.5** (a) Use Gauss-Jordan method to find A^{-1} , if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. 03

- (b) Using Caley-Hamilton theorem find A^2 , if $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Also find A^{-1} . 04

- (c) Find the Fourier cosine series for $f(x) = x^2, 0 < x < \pi$. Hence show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$. 07

- Q.6** (a) Evaluate $\iint_R e^{2x+3y} dA$, where R is the triangle bounded by $x = 0, y = 0, x + y = 1$. 03

- (b) Find the eigen values and eigen vectors for the matrix $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. 04

- (c) Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dA$ by changing the order of integration. 07

- Q.7** Evaluate $\int_0^{\pi/2} \int_{a(1-\cos\theta)}^a r^2 dr d\theta$. 05

OR

- Q.7** Find the area enclosed within the curves $y = 2 - x$ and $y^2 = 2(2 - x)$. 05

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-1/2 EXAMINATION – WINTER 2021****Subject Code:3110014****Date:19/03/2022****Subject Name:Mathematics - 1****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

	MARKS
Q.1 (a) If $u = \log(\tan x + \tan y + \tan z)$ then show that	03
$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$	
(b) Evaluate	04
$\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$	
(c) Find the extreme values of the function	07
$f(x, y) = x^3 + y^3 - 3x - 12y + 20$	
Q.2 (a) Use Ratio test to check the convergence of the series	03
$\sum_{n=1}^{\infty} \frac{2^n + 1}{3^n + 1}$	
(b) Find the Maclaurin's series of $\cos x$ and use it to find the series of $\sin^2 x$.	04
(c) Find the Fourier series of $f(x) = x^2$ in the interval $(0, 2\pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots$	07
OR	
(c) Find the Fourier series of $f(x) = 2x - x^2$ in the interval $(0, 3)$.	07
Q.3 (a) Find the directional derivative of $f(x, y, z) = xyz$ at the point $P(-1, 1, 3)$ in the direction of the vector $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$.	03
(b) Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to row echelon form.	04
(c) Find the eigenvalues and corresponding eigenvectors of the matrix	07
$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$	
OR	
Q.3 (a) If $u = f(x - y, y - z, z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	03
(b) Find the inverse of the following matrix by Gauss-Jordan method:	04

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

- (c) Verify Cayley-Hamilton theorem for the following matrix and use it to find A^{-1} 07

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

- Q.4 (a)** If 1 is an eigenvalue of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ then find its corresponding eigen vector. 03

- (b) Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x - 2)$ 04
 (c) Solve following system by using Gauss Jordan method 07

$$\begin{aligned} x + 2y + z - w &= -2 \\ 2x + 3y - z + 2w &= 7 \\ x + y + 3z - 2w &= -6 \\ x + y + z + w &= 2 \end{aligned}$$

OR

- Q.4 (a)** Use integral test to show that the following infinite series is convergent 03

$$\sum_{n=1}^{\infty} \frac{1}{n(1 + \log^2 n)}$$

- (b) For the odd periodic function defined below, find the Fourier series 04

$$f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$$

- (c) Determine the radius and interval of convergence of the following infinite series 07

$$x - \frac{x^2}{4} + \frac{x^3}{9} - \frac{x^4}{16} + \dots$$

- Q.5 (a)** Show the following limit does not exist using different path approach 03

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2y^2}{x^4 + y^4}$$

- (b) Evaluate the following integral along the region R 04

$$\iint_R (x + y) dy dx$$

where R is the region bounded by $x = 0, x = 2, y = x, y = x + 2$. Also, sketch the region.

- (c) Change the order of integration and hence evaluate the same. Do sketch the region. 07

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$$

OR

- Q.5 (a)** The following integral is an improper integral of which type? Evaluate 03

$$\int_0^{\infty} \frac{dx}{x^2 + 1}$$

- (b) If $x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi, z = r\cos\theta$, then find the jacobian **04**

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)}$$

- (c) Find the volume of the solid generated by rotating the region bounded by $y = x^2 - 2x$ and $y = x$ about the line $y = 4$. **07**

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-I & II(NEW)EXAMINATION – SUMMER 2022****Subject Code:3110014****Date:02-08-2022****Subject Name:Mathematics - 1****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

	Marks
Q.1 (a) Is $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent for $p > 1$? Justify your answer.	03
(b) (1) Find $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{(x-a)^2}$	02
(2) Is $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$ convergent? Justify your answer.	02
(c) (1) Find the length of curve $f(x) = \frac{x^3}{12} + \frac{1}{x}, 1 \leq x \leq 4.$	04
(2) Prove that $\Gamma(n) = (n-1) \Gamma(n-1).$	03
Q.2 (a) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{n^2}{7^n}.$	03
(b) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$	04
(c) Find Fourier series of $f(x) = x^2, -\pi < x < \pi.$	07
OR	
(c) Find Fourier series of $f(x) = \begin{cases} x, & -1 < x < 0 \\ 2, & 0 < x < 1 \end{cases}$	07
Q.3 (a) Find the derivative of $f(x, y) = x^2 + xy + y^2$ in the direction $\hat{i} + \hat{j}$ at $P(1,1).$	03
(b) Find the tangent plane of $z = e^x \cos y$ at $P(0,0,0).$	04
(c) Find local extreme values of $f(x, y) = xy - x^2 - y^2 - x.$	07
OR	
Q.3 (a) Explain second derivative test for local extreme values.	03
(b) Let $f = \ln r$, where $\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$ and $r = \vec{r} $. Find $\text{grad } f$.	04
(c) Determine the minimum value of $x^2 y z^2$ subject to the condition $x + y + 2z = 5$ using method of Lagrange multipliers.	07
Q.4 (a) Evaluate $\int_{y=0}^1 \int_{x=0}^2 \frac{1}{\sqrt{4-x^2} \sqrt{1-y^2}} dx dy.$	03

- (b) Evaluate the integral $\int_0^2 \int_{x/2}^1 \frac{1}{3} e^{y^2} dy dx$ 04

by change of order.

- (c) (1) Find the area of the region covered by $x=1$, $x=4$, $y=0$ and $y=\sqrt{x}$. 04

- (2) Evaluate $\int_{x=0}^1 \int_{y=0}^{x^{1/4}} \int_{z=0}^{y^2} \sqrt{z} dz dy dx$ 03

OR

- Q.4** (a) Evaluate $\iint_R xy dA$ where R is the region 03

bounded by x axis, the ordinate $x=2a$ and the curve $x^2=4ay$.

- (b) Evaluate the integral $\int_{y=0}^1 \int_{x=0}^{\cos^{-1} y} e^{\sin x} dx dy$ by change of order. 04

- (c) (1) Change in to polar coordinates then solve $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-(x^2+y^2)} dy dx$. 04

- (2) Let $x+y=u$ and $y=uv$ are given transformations. Find Jacobian for change of variables. 03

- Q.5** (a) Find characteristic equation of $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$. 03

- (b) Find Maclaurin's series for $f(x) = e^{2x} \sinh x$ and show at least up to x^4 term. 04

- (c) Solve $x+y+w=1$, $2x+z+w=3$, $2y+z+2w=2$. 07

OR

- Q.5** (a) Show that give matrix $A = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$ satisfies its Characteristic equation. 03

- (b) Show that $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges. 04

- (c) Show that $A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -1 & 2 \\ -4 & -8 & 7 \end{bmatrix}$ is diagonalizable. Find the matrix of eigen vectors and diagonal matrix. 07
