Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–IV (NEW) EXAMINATION – WINTER 2020

Subject Code:3140708 Date:17/02/2021

Subject Name:Discrete Mathematics

Time:02:30 PM TO 04:30 PM Total Marks:56

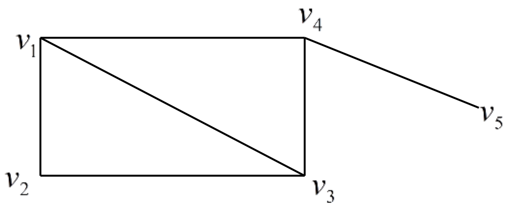
Instructions:

1. **Attempt any FOUR questions out of EIGHT questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Find the power sets of $(i)\{a\}$, $(ii)\{a,b,c\}$.	**03**
	(b)	If $f(x)=2x, g(x)=x^2, h(x)=x+1$ then find *(fog)oh* and *fo(goh).*	**04**
	(c)	(i) Let N be the set of natural numbers. Let R be a relation in N defined by xRy if and only if $x+3y=12$. Examine the relation for (i) reflexive (ii) symmetric (iii) transitive.	**03**
		(ii) Draw the Hasse diagram representing the partial ordering *{(a,b)/ a* divides *b*} on {1,2,3,4,6,8,12}.	**04**
Q.2	**(a)**	Let R be a relation defined in A={1,2,3,5,7,9} as R={(1,1), (1,3), (1,5), (1,7), (2,2), (3,1), (3,3), (3,5), (3,7), (5,1), (5,3), (5,5), (5,7), (7,1), (7,3), (7,5), (7,7), (9,9)}. Find the partitions of A based on the equivalence relation R.	**03**
	(b)	In a box there are 5 black pens, 3 white pens and 4 red pens. In how many ways can 2 black pens, 2 white pens and 2 red pens can be chosen?	**04**
	(c)	Solve the recurrence relation $a_n-4a_{n-1}+4a_{n-2}=n+3^n$ using undetermined coefficient method.	**07**
Q.3	**(a)**	Define self-loop, adjacent vertices and a pendant vertax.	**03**
	(b)	Define tree. Prove that if a graph *G* has one and only one path between every pair of vertices then *G* is a tree.	**04**
	(c)	(i) Find the number of edges in G if it has 5 vertices each of degree 2.	**03**
		(ii) Define complement of a subgraph by drawing the graphs.	**04**
Q.4	**(a)**	Show that the algebraic structure *(G,*)* is a group, where $G=\{(a,b)/a,b\in R, a\neq 0\}$ and * is a binary operation defined by $(a,b)*(c,d)=(ac,bc+d)$ for all $(a,b),(c,d)\in G$.	**03**
	(b)	Define path and circuit of a graph by drawing the graphs.	**04**
	(c)	(i) Show that the operation * defined by $x*y=x^y$ on the set N of natural numbers is neither commutative nor associative.	**03**
		(ii) Define ring. Show that the algebraic system $(Z_9,+_9,\bullet_9)$, where $Z_9=\{0,1,2,3,\ldots,8\}$ under the operations of addition and multiplication of congruence modulo 9, form a ring.	**04**

Q.5 **(a)** Define subgraph. Let H be a subgroup of *(Z,+)*, where H is the set of even integers and Z is the set of all integers and + is the operation of addition. Find all right cosets of H in Z. **03**

(b) Define adjacency matrix and find the same for **04**



(c) (i) Draw the composite table for the operation * defined by $x*y=x$, $\forall x, y \in S = \{a, b, c, d\}$. **03**

(ii) Show that an algebraic structure $(G, \bullet)$ is an abelian group, where $G=\{A,B,C,D\}$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\bullet$ is the binary operation of matrix multiplication. **04**

Q.6 **(a)** Define indegree and outdegree of a graph with example. **03**

(b) Prove that the inverse of an element is unique in a group *(G,*)*. **04**

(c) (i) Does a 3-regular graph with 5 vertices exist? **03**

(ii) Define centre of a graph and radius of a tree. **04**

Q.7 **(a)** Check the properties of commutative and associative for the operation * defined by $x*y=x+y-2$ on the set Z of integers. **03**

(b) Define group permutation. Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$. **04**

(c) (i) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. **03**

(ii) Obtain the d.n.f. of the form $(p \rightarrow q) \wedge (\neg p \wedge q)$. **04**

Q.8 **(a)** Find the domain of the function $f(x) = \sqrt{16 - x^2}$. **03**

(b) Define lattice. Determine whether POSET $\{\{1,2,3,4,5\};|\}$ is a lattice. **04**

(c) Show that the propositions $\neg(p \wedge q)$ and $\neg p \vee q$ are logically equivalent. **07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2021

Subject Code:3140708 Date:24/12/2021

Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Show that for any two sets A and B, $A-(A\cap B)=A-B$. **03**

(b) If $S=\{a,\ b,\ c\}$, find nonempty disjoint sets A_1 and A_2 such that $A_1\cup A_2=S$. Find the other solutions to this problem. **04**

(c) Using truth table state whether each of the following implication is tautology. **07**

a) $(p\wedge r)\rightarrow p$
b) $(p\wedge q)\rightarrow(p\rightarrow q)$
c) $p\rightarrow(p\vee q)$

Q-2 **(a)** Given $S=\{1,2,3,----,10\}$ and a relation R on S. Where, $R=\{\langle x,y\rangle|x+y=10\}$. What are the properties of relation R? **03**

(b) Let L denotes the relation "less than or equal to" and D denotes the relation "divides". Where xDy means "x divides y". Both L and D are defined on the set $\{1,2,3,\ 6\}$. Write L and D as sets, find $L\cap D$. **04**

(c) Let $X=\{1,2,3,4,5,6,7\}$ and $R=\{\langle x,y\rangle|x-y\ is\ divisible\ by\ 3\}$. Show that R is an equivalence relation on. Draw the graph of R. **07**

OR

(b) Define equivalence class generated by an element $x\in X$. Let Z be the set of integers and let R be the relation called "congruence modulo 3" defined by **07**

$$R=\{\langle x,y\rangle|x\in Z\wedge y\in Z\wedge(x-y)\ is\ divisible\ by\ 3\}$$

Determine the equivalences classes generated by the element of Z.

Q.3 **(a)** Let $f(x)$ be any real valued function. Show that $g(x)=\frac{f(x)+f(-x)}{2}$ is always an even function where as $h(x)=\frac{f(x)-f(-x)}{2}$ is always an odd function. **03**

(b) The Indian cricket team consist of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can cricket eleven be selected if we have select 1 wicket keeper and at least 4 bowlers? **04**

(c) Let A be the set of factors of particular positive integer m and $\leq$ be the relation divides, that is **07**

$\leq=\{\langle x,y\rangle|x\in A\wedge y\in A\wedge(x\ divides\ y)\}$ Draw the Hasse diagrams for

a) $m=45$
b) $m=210$.

OR

Q-3 **(a)** Find the composition of two functions $f(x)=e^x$ and $g(x)=x^3$, $(f\circ g)(x)$ and $(g\circ f)(x)$. Hence, show that $(f\circ g)(x)\neq(g\circ f)(x)$. **07**

(b) In a box, there are 5 black pens, 3 white pens and 4 red pens. In how many ways can 2 black pens, 2 white pens and 2 red pens can be chosen? **04**

(c) Let A be a given finite set and $\rho(A)$ its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Draw Hass diagram for $\langle\rho(A),\subseteq\rangle$ for **07**

a) $A=\{a,b,c\}$
b) $A=\{a,b,c,d\}$

Q.4	**(a)**	Let $\langle L, \leq \rangle$ be a lattice. Show that for $a, b, c \in L$, following inequalities holds. $$a \oplus (b \star c) = (a \oplus b) \star (a \oplus c)$$ $$a * (b \oplus c) = (a * b) \oplus (a * c)$$	**07**
	(b)	Let $G = \{(a, b) \mid a, b \in R\}$. Define binary operation $(*)$ on G as $(a, b), (c, d) \in G, (a, b) * (c, d) = (ac, bc + d)$. Show that an algebraic structure $(G,*)$ is a group.	**07**
		OR	
Q-4	**(a)**	Let G be the set of non-zero real numbers. Define binary operation $(*)$ on G as $a * b = \frac{ab}{2}$. Show that an algebraic structure $(G,*)$ is an abelian group.	**07**
	(b)	Explain the following terms with proper illustrations. a) Directed graphs b) Simple and elementary path c) Reachability of a vertex d) Connected graph.	**07**
Q-5	**(a)**	Show that sum of in-degrees of all the nodes of simple digraph is equal to the sum of out-degrees of all the nodes and this sum equal to the number of edges in it.	**07**
	(b)	Let $= \{1, 2, 3, 4\}$. For the relation R whose matrix is given, find the matrix of the transitive closure by using Warshall's algorithm. $$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$	**07**
		OR	
Q-5	**(a)**	Define tree and root. Also prove that tree with n vertices has $n - 1$ edges.	**07**
	(b)	Define in-degree and out-degree of a vertex and matrix of a relation. Let $A = \{a, b, c, d\}$ and let R be the relation on A that has the matrix $$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$	**07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV EXAMINATION – SUMMER 2020

Subject Code: 3140708 **Date:29/10/2020**

Subject Name: Discrete Mathematics

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

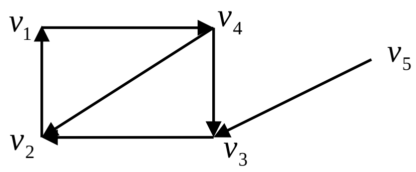
Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	If $A = \{a, b\}$ and $B = \{c, d\}$ and $C = \{e, f\}$ then find *(i)* $(A \times B) U (B \times C)$ $(ii) A \times (BUC)$.	**03**
	(b)	Define even and odd functions. Determine whether the function $f : I \to R^+$ defined by $f(x0 = 2x + 7$ is one-to-one or bijective.	**04**
	(c)	**(i)** Show that the relation $x \equiv y(\bmod m)$ defined on the set of integers I is an equivalence relation.	**03**
		(ii) Draw the Hasse diagram for the partial ordering $\{(A, B) / A \subseteq B\}$ on the power set $P(S)$, where $S = \{a, b, c\}$.	**04**
Q.2	**(a)**	Define equivalence class. Let R be the relation on the set of integers I defined by $(x - y)$ is an even integer, find the disjoint equivalence classes	**03**
	(b)	A committee of *5* persons is to be formed from *6* men and *4* women. In how many ways can this be done when (i) at least *2* women are included (ii) at most *2* women are included ?	**04**
	(c)	Solve the recurrence relation $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2$ using the method of undetermined coefficients.	**07**
		OR	
	(c)	Solve the recurrence relation using the method of generating function $a_n - 5a_{n-1} + 6a_{n-2} = 3^n, n \geq 2; a_0 = 0, a_1 = 2.$	**07**
Q.3	**(a)**	Define simple graph, degree of a vertex and complete graph.	**03**
	(b)	Define tree. Prove that there is one and only one path between every pair of vertices in a tree *T*.	**04**
	(c)	**(i)** A graph *G* has *15* edges, *3* vertices of degree *4* and other vertices of degree *3*. Find the number of vertices in *G*.	**03**
		(ii) Define vertex disjoint and edge disjoint subgraphs by drawing the relevant graphs.	**04**
		OR	
Q.3	**(a)**	Show that $(G, +_5)$ is a cyclic group, where *G={ 0, 1, 2, 3, 4 }.*	**03**
	(b)	Define the following by drawing graphs (i) weak component (ii) unilateral component (iii) strong component.	**04**
	(c)	**(i)** Construct the composite tables for *(i)* addition modulo *4* and *(ii)* multiplication modulo *4* for $Z_4 = \{0,1,2,3\}$. Check whether they have identity and inverse element.	**03**
		(ii) Define ring. Show that the set $M = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} / a, b \in R \right\}$ is not a ring under the operations of matrix addition and multiplication.	**04**

Q.4 **(a)** Define algebraic structure, semi group and monoid. Also give related examples. **03**

(b) Use Warshall's algorithm to obtain path matrix from the adjacency matrix of **04**



(c) **(i)** Is the algebraic system *(Q, *)* a group? Where Q is the set of rational numbers and * is a binary operation defined by $a*b = a+b-ab,\ \forall a,b \in Q.$ **03**

(ii) Let *(Z, +)* be a group, where Z is the set of integers and + is an operation of addition. Let H be a subgroup of Z consisting of elements multiple of 5. Find the left cosets of H in Z. **04**

OR

Q.4 **(a)** Prove that there are always an even number of vertices of odd degree in a graph. **03**

(b) Prove that every subgroup H of an abelian group is normal. **04**

(c) **(i)** Find the number of edges in a r – regular graph with n – vertices. **03**

(ii) A tree T has 4 vertices of degree 2, 4 vertices of degree 3, 2 vertices of degree 4. Find the number of pendant vertices in T. **04**

Q.5 **(a)** Show that the operation * defined by $x*y = x^y$ on the set N of natural numbers is neither commutative nor associative. **03**

(b) Prove that an algebraic structure *(G, *)* is an abelian group, where G is the set of non-zero real numbers and * is a binary operation defined by $a*b = \dfrac{ab}{2}$. **04**

(c) **(i)** Find out using truth table, whether $(p \wedge r) \rightarrow p$ is a tautology. **03**

(ii) Obtain the dnf of the form $\neg(p \rightarrow (q \wedge r))$. **04**

OR

Q.5 **(a)** If $f : x \rightarrow 2x$, $g : x \rightarrow x^2$ and $h : x \rightarrow (x+1)$, then show that $(fog)oh = fo(goh)$. **03**

(b) Define lattice. Determine whether the POSET $(\{1,2,3,4,5\};1)$ is lattice. **04**

(c) **(i)** If *p:* product is good, *q:* service is good, *r:* company is public limited, write the following in symbolic notations *(i)* either product is good or the service is good or both, *(ii)* either the product is good or service is good but not both, *(iii)* it is not the case that both product is good and company is public limited. **03**

(ii) For the universe of integers, let *p(x), q(x), r(x), s(x)* and *t(x)* be the following open statements: *p(x): x > 0; q(x): x* is even; *r(x): x* is a perfect square; *s(x): x* is divisible by *4*; *t(x): x* is divisible by *5*. Write the following statements in symbolic form: *(i)* At least one integer is even, *(ii)* There exists a positive integer that is even, *(iii)* If *x* is even then *x* is not divisible by *5*, *(iv)* No even integer is divisible by *5*, *(v)* There exists an even integer divisible by *5*. **04**

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2021

Subject Code:3140708 Date:07/09/2021

Subject Name:Discrete Mathematics

Time:02:30 PM TO 05:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Among 100 people at least how many of them were born in the same month?	**03**
	(b)	Prove that: $(A \cup B)' \equiv A' \cap B'$.	**04**
	(c)	Define the following: 1) Composition of functions 2) Monoid 3) Existential Quantifier 4) Partially Ordered Set 5) Boolean Algebra 6) Tree 7) Complete Graph	**07**
Q.2	**(a)**	Explain types of a Relation with a suitable example.	**03**
	(b)	Rewrite the following statements using quantifier variables and predicate symbols: 1) All birds can fly. 2) Some women are genius. 3) There is a student who likes Discrete Mathematics but not Probability and Statistics. 4) Each integer is either even or odd.	**04**
	(c)	Determine the validity of the argument given: If I study, then I will not fail in Discrete Mathematics. If I do not play cricket, then I will study. But I failed in Discrete Mathematics. -- Therefore I must have played cricket.	**07**
		OR	
	(c)	Find if the following is a tautology, contradiction or contingency. $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$	**07**
Q.3	**(a)**	Define: Bounded, Distributive and Complemented Lattices.	**03**
	(b)	Find the transitive closure of $R = \{(1,2), (3,4), (4,5), (4,1), (1,1)\}$. Where, $A = \{1,2,3,4,5\}$.	**04**
	(c)	Let A be a set of factors of positive integer m and relation is divisibility on A. For $m = 45$, show that POSET $(A, \leq)$ is a Lattice.	**07**
		OR	
Q.3	**(a)**	Draw the Hasse diagram of the set $\{1,3,9,18\}$ under partial order relation ‘divides’ and indicate those which are chains.	**03**
	(b)	Let $X = \{1,2,3, \dots ,7\}$ and $R = \{(x, y): x - y \; is \; divisble \; by \; 3\}$. Show that R is an equivalence relation. Draw the graph of R.	**04**

(c) Solve the recurrence relation: **07**

$$a_{n+2} - 5a_{n+1} + 6a_n = 2.$$

Q.4 **(a)** Define group with example. Give an example of a non-abelian group. **03**

(b) Let $H = \{[0],[3]\}$ in Z_6 under addition. Find left and right cosets in $< Z_6, +_6 >$. **04**

(c) Prove that $G = \{1,2,3,4,5,6\}$ is a finite abelian group of order 6 with respect to multiplication modulo 7. **07**

OR

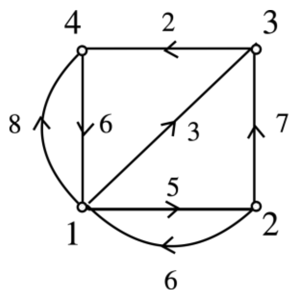
Q.4 **(a)** Define subgroup and group Homomorphism. **03**

(b) Is addition a binary operation on {-1,0,1}? Justify. **04**

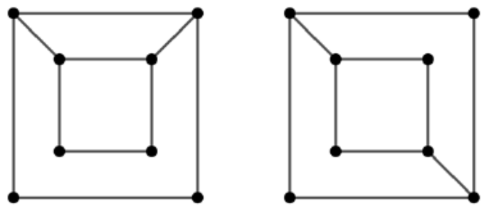
(c) Explain Cosets and Lagrange's theorem. **07**

Q.5 **(a)** How many nodes are necessary to construct a graph with exactly 8 edges in which each node is of degree 2. **03**

(b) Find the shortest path between each pair of vertices for a simple digraph using Warshall's Algorithm. **04**



(c) Define Isomorphic Graphs. Verify the following graphs are Isomorphic or not (Justify). **07**



OR

Q.5 **(a)** Define Cyclic graph, Null graph and Strongly connected graph. **03**

(b) Draw a graph which is regular but not bipartite. **04**

(c) For the following set of weights construct an optimal binary prefix code. For each weight in the set, give corresponding code word. **07**

5, 7, 8, 15, 35, 40

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2022

Subject Code:3140708 Date:02-07-2022

Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks: 70

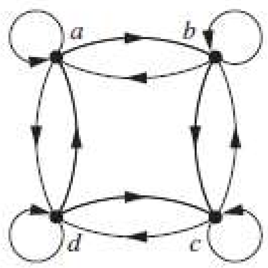
Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Determine whether each of these statements is true or false. 1) $0 \in \emptyset$ 2) $\emptyset \subset \{0\}$ 3) $\{0\} \in \{0\}$ 4) $\emptyset \in \{\emptyset\}$ 5) $\{\emptyset\} \in \{\{\emptyset\}\}$ 6) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$	**03**
	(b)	Determine whether f is a function from the set of all bit strings to the set of integers if 1) $f(s)$ is the position of a 0 bit in S. 2) $f(s)$ is the number of a 1 bits in S. Find the range of each of the following functions that assigns: 3) to a bit string the number of one bits in the string 4) to each bit string twice the number of zeros in that string.	**04**
	(c)	1) Find the bitwise OR, and bitwise XOR of the bit string 1111 0000, 1010 1010 2) Show that the function $f: R \to R^+ \cup \{0\}$ defined by $f(x) = \|x\|$ is not invertible. Modify the domain or codomain of f so that it becomes invertible. 3) Let S be subset of a universal set ∪. The characteristic function f_S : $\cup \to \{0,1\}$, $f_S(x) = 1, if\ x \in S\ and\ 0\ is\ x \notin S.$ Let A and B be sets. Then show that $f_{A \cap B}(x) = f_A(x).f_B(x)$	**07**
Q.2	**(a)**	Let P(x) be the statement *"$x = x^2$ "*. If the domain consists of the integers, what are the truth values of the following? 1) $\exists x\, P(x)$ 2) $\forall x\, \neg P(x)$ 3) $\exists x\, \neg P(x)$	**03**
	(b)	Identify the error or errors in this argument that supposedly shows that if ∃x P(x) ∧ ∃x Q(x) is true then ∃x (P(x) ∧ Q(x)) is true. 1. ∃x P(x) ∧ ∃x Q(x) Premise 2. ∃x P(x) Simplification from (1) 3. *P(c)* Existential instantiation from (2) 4. ∃x Q(x) Simplification from (1) 5. *Q(c)* Existential instantiation from (2) 6. $P(c) \wedge Q(c)$ Conjunction from (3) and (5) 7. $\exists x\, (P(x) \wedge Q(x))$ Existential generalization	**04**
	(c)	1) Use a truth table to verify the De Morgan's law $\neg(p \vee q) \equiv \neg p \wedge \neg q$ 2) Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.	**07**

OR

	(c)	1) Suppose that the domain of Q(x, y, z) consists of triples x, y, z, where x = 0, 1, or 2, y = 0 or 1, *and* z = 0 or 1. Write out following propositions using disjunctions and conjunctions. i) $\exists z \neg Q(0,0,z)$ ii). $\forall y Q(0,y,0)$ 2) i) Show that $\exists x\ P(x) \wedge \exists x\ Q(x)$ and $\exists x(P(x) \wedge Q(x))$ are not logically equivalent. ii) Show that $\exists x(P(x) \vee Q(x))$ and $\exists x\ P(x) \vee \exists x\ Q(x)$ are logically equivalent.	07
Q.3	(a)	For the relation {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)} on the set {1,2,3,4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive..(Justify your answer if the property is not satisfied)	03
	(b)	For the following relations on the set of real numbers, $R_1 = \{(a,b) \in R^2 \mid a \geq b\}$, $R_2 = \{(a,b) \in R^2 \mid a \leq b\}$ $R_3 = \{(a,b) \in R^2 \mid a \neq b\}$ find 1) $R_1 \oplus R_3$ 2) $R_2 o\ R_3$	04
	(c)	1) Draw the Hasse diagram for the poset ({2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72}, |). Hence find $glb(\{60,72\})$ and all maximal elements. 2) Determine whether the relation with the directed graph shown is an equivalence relation.	07

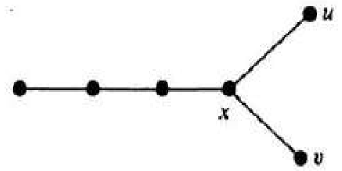


OR

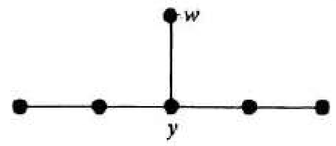
Q.3	(a)	Suppose that the relations R_1 and R_2 on a set A are represented by the matrices $M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ What are the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$?	03
	(b)	Use Warshall's algorithm to find the transitive closures of the relation $R = \{(1,4), (2,1), (2,3), (3,1), (3,4), (4,3)\}$ on $\{1, 2, 3, 4\}$	04
	(c)	1) Draw the Hasse diagram for the poset({2, 4, 5, 10, 12, 20, 25}, |). Hence, find the are maximal and minimal elements. 2) Which of these collections of subsets are partitions of {1, 2, 3, 4, 5, 6}? Justify your answer if it is not a partition. i) {1, 2}, {2, 3, 4}, {4, 5, 6} ii) {1, 4, 5}, {2, 6}	07
Q.4	(a)	Explain Path and Circuit of a graph.	03

(b) 1) Define Isomorphic Graphs **04**

2) Verify the following graphs are Isomorphic or not (Justify).



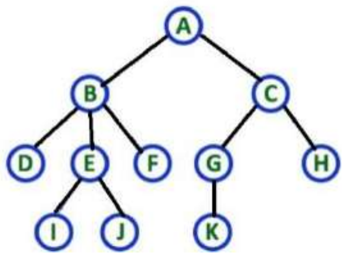
Graph -1



Graph-2

(c) 1) Define Subtree and Degree of a Node **07**

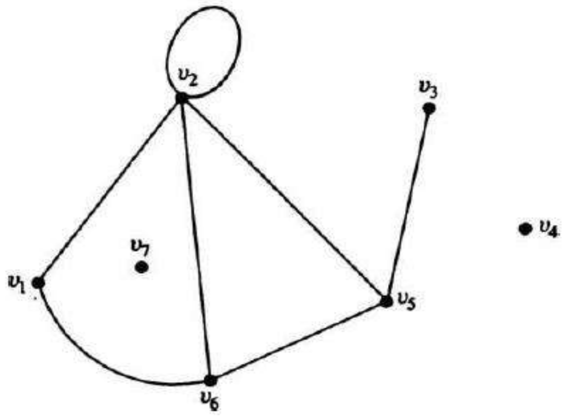
2) Determine degree of the each node for the following tree.



OR

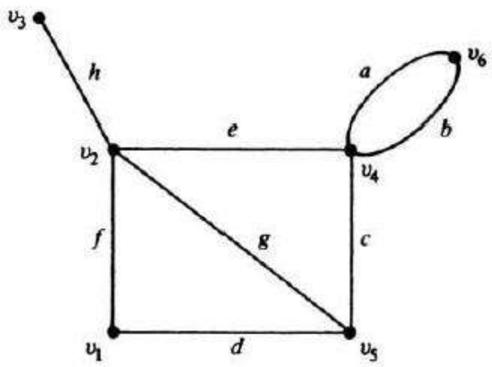
Q.4 (a) 1) Define: i) Isolated vertex ii) Null graph **03**

2) Identify Isolated vertex/vertices from the following graph



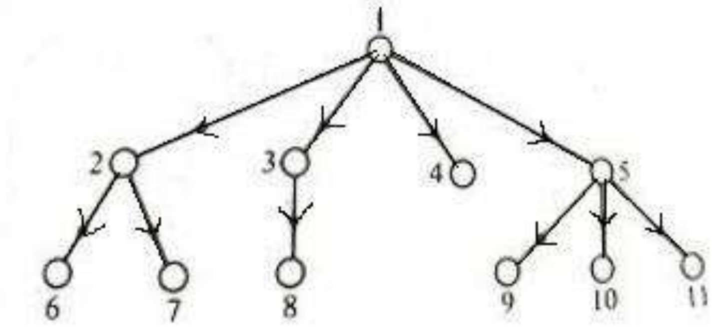
(b) 1) Define incidence Matrix of a Graph **04**

2) Find incidence matrix for the following graph



(c) 1) Define: M-ary Tree and Binary Tree. **07**

2) Represent the following directed tree as Binary tree



Q.5 **(a)** 1) Define: SemiGroups. **03**

2) Let S = {a, b, c, d}. The following multiplication table, define operations $\cdot$ on S. Is $\langle S, \cdot \rangle$ semigroup? Justify

$\cdot$	a	b	c	d
a	a	b	c	d
b	b	a	a	b
c	c	b	a	a
d	d	a	a	a

(b) Let $H = \{1, -1\}$ and $G = \{1, -1, i, -i\}$. $(H, \times)$ is a sub-group $(G, \times)$. Then find all left cosets and right cosets of H in G. **04**

(c) 1) Define:Ring. **07**

2) Write elements of the ring $\langle z_{10}, +_{10}, \times_{10} \rangle$. And find $-3, -8, 3^{-1}, 4^{-1}$

OR

Q.5 **(a)** Consider the set Q of a rational numbers. Let $*$ be the operation on Q defined by $a * b = a + b - ab$. Find **03**

1) $3 * 4$ 2) the identity element for $*$.

(b) Write the equivalence classes for congruence modulo 4 i.e.. z_4 **04**

Let the subset H={[0],[2]} is a subgroup of $G = \langle z_4, +_4 \rangle$. Then determine all left cosets of H in G.

(c) We are given the ring $\langle \{a, b, c, d\}, +, \cdot \rangle$ the operations $+$ and $\cdot$ on R are as shown in the following table. **07**

+	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	c	a	c
c	a	a	a	a
d	a	c	a	a

1) Does it have an identity?

2) What is the zero of this ring?

3) Find the additive inverse of each of its elements

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