Finite Residual Rule of Sum (FRSR):
From Quantum Entanglement to Cosmological Curvature

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Abstract

The Finite Residual Rule of Sum (FRSR) unifies quantum entanglement, vacuum energy, and spacetime curvature into a single coherent framework. Developed from the principle that finite energy residuals within entangled bands give rise to observable curvature, FRSR provides a cosmological interpretation where discrete negative-energy bands entangled with the continuous positive-energy sector generate the observed cosmological constant Λ. This approach bridges microscopic entanglement structure and macroscopic curvature, offering a falsifiable path to explain Λ without ultraviolet divergences or ad-hoc fine-tuning.

1 Introduction: Finite Residual Rule of Sum

(Originally developed under the working title "God Plays Dice / Λ Origin.")

We propose that the observed cosmological constant Λ is the tiny, finite residual of a symmetric vacuum composed of a *finite number of negative-energy bands* entangled with their positive-energy counterparts.

Each band contributes an energy density weighted by its entanglement strength, and the sum reproduces Λ without UV divergences or ad-hoc fine-tuning.

This yields a compact **finite-residual sum rule** that is testable in principle and compatible with Lorentz invariance and $w \simeq -1$.

The cosmological constant is the leftover curvature of broken time symmetry.

2 Framework: Existence Within Spacetime

Quantum entanglement — expressed by the weights $\mathcal{E}_i$ — encodes **non-spatial structure**, a network of pure correlations that exist beyond classical spacetime. The global coupling constant κ governs how this hidden informational structure *projects into* the geometric fabric of spacetime.

When discrete negative-energy ($-$E) bands exchange information with the continuous positive-energy (+E) field, their interaction translates informational geometry into **vacuum energy density**, which in turn produces measurable **spacetime curvature** Λ.

In essence, entanglement defines the invisible architecture of existence, while curvature is its visible trace in the universe.

Level	Domain	Symbol	Description / Units
1	Entanglement weight	$\mathcal{E}_i$	Dimensionless correlation factor
2	Coupling constant	κ	Dimensionless global coupling
3	Energy density	$\rho_{-,i}$	Negative-band energy density $[\mathrm{M\,L^{-3}}]$
4	Curvature	Λ	Spacetime curvature $[\mathrm{L^{-2}}]$

3 Master Equation (Cosmological Core)

Projecting the Finite Residual Rule of Sum (FRSR) into the spacetime domain yields a simple energy–curvature correspondence. Each discrete negative-energy band contributes a weighted share to the total curvature, producing the observed cosmological constant:

$$\Lambda_{\mathrm{obs}} = \lambda_{\mathrm{bare}} + \frac{8\pi G}{c^2}\,\kappa \sum_{i=1}^{N_{\mathrm{fin}}} \mathcal{E}_i\,\rho_{-,i}.$$

Here:

- G — Newton's constant.
- c — speed of light.
- Λ_{obs} — observed cosmological constant.
- λ_{bare} — bare curvature term.

The term λ_{bare} represents the geometric curvature of empty spacetime in the absence of any entangled contributions. It can be interpreted as the zero-point offset in the curvature–energy relationship and remains a constant reference background, not fitted by FRSR parameters.

- κ — global dimensionless coupling.
- N_{fin} — number of discrete negative-energy (−E) bands.
- $\rho_{-,i}$ — energy density of band i.
- $\mathcal{E}_i$ — entanglement weight coupling each −E band to the continuous +E field.

This compact form represents the **Einstein–FRSR bridge**, where finite residuals in the −E domain translate directly into measurable curvature in the +E universe.

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3.1 Derivation Outline (Summary)

1. **Einstein's Energy–Curvature Identity** General Relativity connects curvature to vacuum energy density as
$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}.$$

2. **FRSR Finite-Band Structure** The finite set of discrete −E bands, each with entanglement weight $\mathcal{E}_i$ and energy density $\rho_{-,i}$, defines a structured residual energy field. The coupling κ transfers this entangled energy into curvature.

3. **Combination into the FRSR Equation** Substituting the FRSR structure into Einstein's identity gives the full expression
$$\Lambda_{\text{obs}} = \lambda_{\text{bare}} + \frac{8\pi G}{c^2}\,\kappa \sum_{i=1}^{N_{\text{fin}}} \mathcal{E}_i\,\rho_{-,i}.$$
The observed curvature is thus the sum of a geometric baseline (λ_{bare}) and a finite-residual correction arising from quantum entanglement.

Note: FRSR does *not* quantize spacetime itself. Instead, it quantizes the *vacuum energy source* that drives curvature, leaving the Einstein geometry continuous and classical.

Narrow-band estimate (origin): Starting from the zero-point energy density integral,
$$\rho_{\text{vac}} = \frac{\hbar c}{2}\int_0^{k_{\max}} \frac{4\pi k^2\,dk}{(2\pi)^3}\,k = \frac{\hbar c}{4\pi^2}\int_0^{k_{\max}} k^3\,dk,$$
the FRSR replacement of the continuous spectrum by finite −E bands treats each band as a narrow slice of width Δk_i centered at k_i, yielding
$$\rho_{-,i} \approx \frac{\hbar c}{4\pi^2}\,k_i^3\,\Delta k_i.$$

Projection from Momentum to Spacetime. Each finite −E band exists in Fourier (momentum) space and contributes a bounded spectral energy density. The parameter Δk_i defines the finite momentum width of the band — effectively the coherence interval of that mode. When projected into spacetime, these spectral densities appear as real-space vacuum energy contributions $\rho_{-,i}$ that source curvature via the coupling term $\kappa\,\mathcal{E}_i$. Thus, the transition from spectral structure to curvature arises through entanglement-mediated energy transfer between the −E (spectral) and +E (geometric) domains. The small asymmetry between these sectors—imperfect cancellation across finite bandwidths Δk_i—produces the finite residual Λ_{FRSR}.

4 Motivation

The central empirical fact is that the Universe exhibits a tiny but non-zero cosmological constant, Λ_{obs}, with an equation of state $w \simeq -1$ that appears stable over cosmic time. Standard quantum-field estimates of vacuum energy overshoot by $\sim 10^{122}$ in Planck units and rely on delicate cancellations. FRSR addresses this tension by proposing that the vacuum is not featureless: a **finite set of negative-energy (−E) bands** remains entangled with the **continuous positive-energy (+E) sector**. A small structural asymmetry between these sectors leaves a **finite residual** that manifests as Λ.

Objectives of FRSR

- Provide a GR-compatible map from entanglement weights $\{\mathcal{E}_i\}$ and band densities $\{\rho_{-,i}\}$ **directly to curvature** via a single finite-sum rule.
- Remove UV divergence by assuming $N_{\text{fin}} < \infty$ and isotropy, while **preserving** $w = -1$ (Lorentz-invariant vacuum stress).
- Make the framework **falsifiable** through measurable constraints on the entanglement-weighted band sum $\sum_i \mathcal{E}_i \rho_{-,i}$, fixed by the observed Λ_{obs}, and through the possibility of **extremely small, scale-dependent residuals** $\delta\Lambda$ consistent with cosmological observations.
- Remain **agnostic about microdynamics**: a possible stochastic substrate interpretation may be developed in future work, but it is **not required** for the cosmological results presented here.

Although FRSR can be interpreted statistically—each finite entangled band acting as a weighted transition between information and curvature—the present formulation remains geometric, expressed through the four fundamental translations: information → coupling → energy → curvature.

> In the FRSR framework, the observable universe (the +E sea) represents the visible projection of a deeper entangled duality. The −E sea, unobservable and non-temporal, acts as the source of finite informational impulses — the domain where "God plays dice." Each such impulse induces a curvature response in the +E continuum, producing the small but measurable residual identified as Λ. Forward time emerges because the +E domain continuously reacts to earlier −E correlations; these stored correlations form our causal memory. The −E domain itself does not evolve in time — it establishes the very conditions that make temporal evolution possible.

GR permits boundary conditions or background fields that define geometry before time evolution; in this sense the −E sector functions as a fixed entanglement substrate relative to the evolving +E domain.

At the macroscopic scale, FRSR is fully deterministic, describing how entanglement-mediated curvature arises from finite residual structures in energy space. However, at microscopic scales, the same finite-band architecture could admit a stochastic interpretation: each discrete −E band may represent an ensemble-averaged fluctuation within the entanglement field. This perspective does not alter the deterministic geometry but provides an open pathway toward modeling microscopic variability and linking FRSR to measurable quantum phenomena.

5 Physical Interpretation

FRSR interprets the vacuum as a continuous +E field entangled with a finite set of discrete −E bands; the resulting finite residual projects into spacetime as vacuum energy (curvature).

In this framework, the −E sector resides naturally in the spectral (momentum) domain and manifests geometrically only through its entanglement-weighted energy densities projected into spacetime curvature.

Key physical insights include:

- Residuals represent intrinsic finite spectral features of the vacuum with measurable strength distributions.
- The framework allows finite-memory effects via the discrete −E band structure $\{k_i, \Delta k_i, \mathcal{E}_i\}$.
- Entanglement-weighted couplings $(\kappa\,\mathcal{E}_i)$ connect finite-band structure to curvature, enabling parameter inference from cosmological data.

6 Falsifiability

The FRSR framework is falsifiable through its explicit predictions on the distribution and statistics of finite residuals:

- Constraints on the entanglement-weighted band sum $\sum_i \mathcal{E}_i \rho_{-,i}$ can validate or refute the assumed finite-band structure.
- Deviations between observed and predicted values of Λ or its local-scale fluctuations $\delta\Lambda$ provide a direct test of the FRSR framework.
- Spatiotemporal analyses of cosmological data (e.g., CMB anisotropies, BAO, Type-Ia supernovae, weak lensing) can test for tiny, scale-dependent deviations $\delta\Lambda$ predicted by finite-band structure.

7 Relation to Existing Theories

FRSR aligns with foundational approaches linking quantum entanglement and spacetime geometry — such as ER=EPR, Ryu–Takayanagi, and Jacobson's thermodynamic gravity. However, unlike these geometric–entropic models, FRSR introduces a finite, entanglement-weighted energy formulation: curvature arises not from entropy or wormhole topology but from quantized vacuum energy coupling between −E and +E sectors.

8 Conclusion

The Finite Residual Rule of Sum (FRSR) presents a unified bridge between quantum entanglement, vacuum energy, and spacetime curvature. By interpreting the cosmological constant as the finite residual of a balanced but asymmetric entangled vacuum, FRSR provides a falsifiable framework that connects microscopic correlations to macroscopic geometry.

Future work will focus on extending the model toward multi-band simulations, investigating potential links with quantum field renormalization, and testing whether measurable curvature fluctuations $(\delta\Lambda)$ can reveal the fingerprints of finite residual dynamics.

References

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