$\Longrightarrow \quad M_t = \mathbb{E}[M_T | \mathcal{F}_t] \quad \forall t \le T$

$\Longrightarrow \quad M$ is a martingale

Corollary 10: Let U be an adapted process with $\mathbb{E}|U| < \infty$, $\forall t$

Then U is a supermartingale

$\Longleftrightarrow$ For any stopping time τ the stopped process U^τ is a supermartingale

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5.2 American claims

Definition 11. An American contingent claim is a security that can be exercised at time t between 0 and its expiration time T.

Definition 12: An exercise strategy for an American contingent claim C is $\mathcal{F}_T$-measurable random variable τ taking values in $\{0, 1, 2, \cdots, T\}$.

注意：这里的 τ 不一定是 stopping time

The payoff obtained by using τ is equal to $C_{\tau(w)} = C_{\tau(w)}(w)$ for $w \in \Omega$.

与欧式相比，美式的 τ 是不确定的，因为不确定走哪条 w_i

Example 13: (1) An American put option on the ith asset and with strike $k>0$ pays the amount $C_t^{put} = (k - S_t^i)^+$ if it is exercised at time t.

(2) The payoff at time t of the corresponding American call option is given by $C_t^{call} = (S_t^i - k)^+$

Remark 14: The concept of American contingent claim can be regarded as a generalization of European contingent claim.

If C^E is a European contingent claim, then we can define a corresponding American contingent claim C^A by

$$C_t^A = \begin{cases} 0 & \text{if } t < T \\ C^E & \text{if } t = T \end{cases}$$

Definition 15: The $H_t = \frac{C_t}{B_t}$ of discounted payoff of C will be called discounted American claim associated with C.