6.0 Measures of Risk

若 $V_T = V_T(\bar{h})$: final wealth

那投资者希望找到 $\bar{h}$, s.t. $\max\limits_{\bar{h}} V_T(\bar{h})$ ← 很难发生

例如

$r=0,\ T=1$

$V_0=0$

$$\begin{array}{ccc} & & 12 \quad w_1 \\ 10 & \begin{array}{c}\nearrow \\ \searrow\end{array} & \\ & & 6 \quad w_2 \end{array}$$

$$t=0 \qquad \bar{h} = \begin{pmatrix} -h^0 \\ \frac{h^0}{10} \end{pmatrix}$$

从银行借

借的钱全拿去买股票

$$t=T=1 \quad V_1(\bar{h})(w) = -h^0 + \frac{h^0}{10} S_1(w) = \begin{cases} -h^0 + \frac{12}{10}h^0 = \frac{1}{5}\underline{h^0}, & \text{if } w=w_1 \\ -h^0 + \frac{6}{10}h^0 = -\frac{2}{5}\underline{h^0}, & \text{if } w=w_2 \end{cases}$$

很难找到 h^0, s.t. $V_1(\bar{h})(w_1)$ 和 $V_1(\bar{h})(w_2)$ 同时最大

故退而求其次，我们求 $\mathbb{E}[V_T(\bar{h})]$ 的最大值

Question 1: $\mathbb{E}[V_T(\bar{h})]$ 是对哪一个测度来做的？

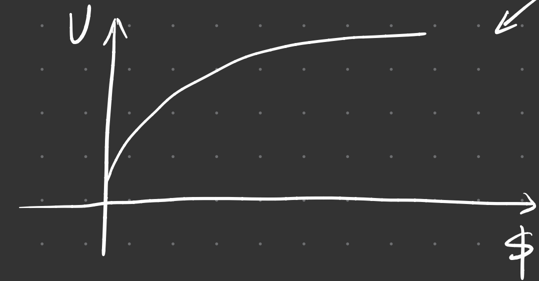
是用 Martingale measure 还是其它 P

或对所有Q来做，$\mathbb{E}_Q[V_T(\pi)]$ 再取 sup $\Rightarrow \sup_Q \mathbb{E}_Q[V_T(\pi)]$

Question 2: $\mathbb{E}[V_T(\pi)]$?

$\downarrow$

$\mathbb{E}[U(V_T(\pi))]$ U: utility function



整体递增

但随\$↑，增加的U越少

increasing and concav

最终：$\max_{\pi} \mathbb{E}[U(V_T(\pi))] \leftarrow$ expected utility optimisation

U = {

- risk-neutral linear $u(x)=x$
- risk-averse concave $-e^{-x}$, $\log x$
- risk- convex

对于相同 $\mathbb{E}[V_T(\pi)]$，要考虑风险

$V_T(\pi)\equiv 0$

$$V_T(\pi) = \begin{cases} 100 & \text{with prob. } 1/2 \\ -100 & \text{with prob. } 1/2 \end{cases}$$

$$V_T(\tilde{\pi}) = \begin{cases} 10^{10} & \text{with prob. } 1/2 \\ -10^{10} & \text{with prob. } 1/2 \end{cases}$$

Value at Risk $\longrightarrow$ 衡量风险
(VaR)

6.1 Monetary measure of risk

Consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$

Notation 1: A financial position $X: \Omega \mapsto \mathbb{R}$ is the discounted net worth at the end of the trading period.

本小节重点

Denote $\mathcal{X}$ a class of financial positions.

Definition 2: A mapping $\rho: \mathcal{X} \mapsto \mathbb{R}$ is called a monetary measure of risk if for any $X, Y \in \mathcal{X}$. ρ用来测量风险 ρ: 看成破产的风险

(i) (monotonity) If $X \leq Y$, then $\rho(X) \geq \rho(Y)$ Y这个点比X这个点赚的钱多,