Fundamentals of Robotics
Homework 4

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1 Dynamic model derivation

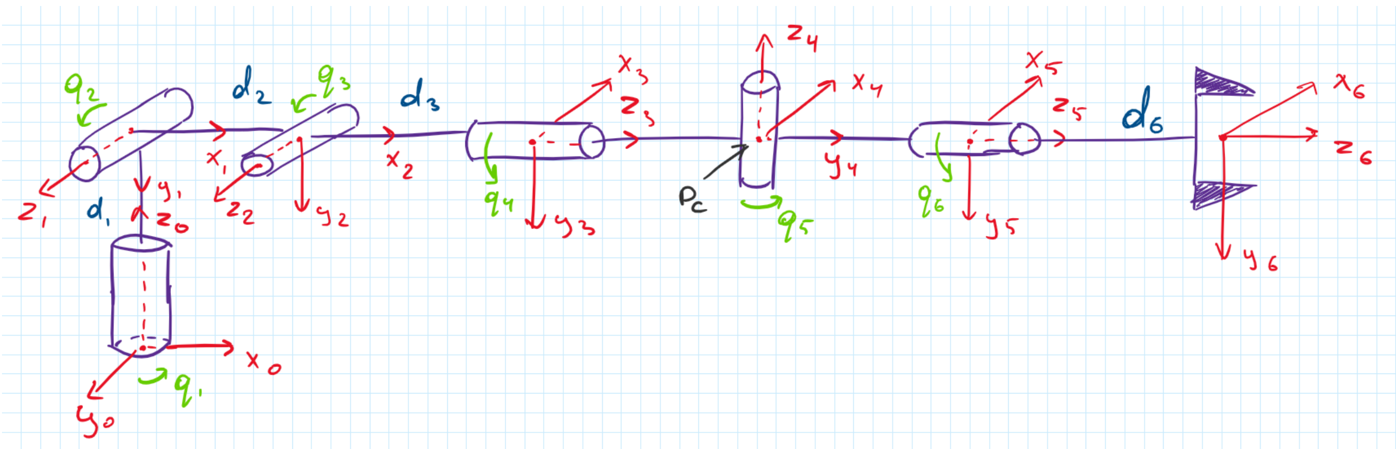

Figure 1: Anthropomorphic 'xyx' model

Let us use Euler-Lagrange approach for the model on Figure 1:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$$

where

$$\begin{cases} M(q) = \sum_{i=1}^{n} m_i J_{v_i}^T J_{v_i} + J_{\omega_i}^T R_i I R_i^T J_{\omega_i}, \\ C(q, \dot{q}) = \sum_{k=1}^{n} c_{ijk} \dot{q}_k, \\ c_{ijk} = \frac{1}{2}(\frac{\partial m_{ij}}{\partial q_k} + \frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{jk}}{\partial q_i}), \\ g(q) = \sum_{k=1}^{n} (J_{v_i}^k)^T m_k g_0. \end{cases} \tag{1}$$

2 Inertia Matrix

1. First, let us find matrix J_v. To do so we need to find transformation matrices for each center of mass:

$$CoM_1 = Rz(q_1) \cdot Tz(d_{c_1}) \cdot Rx(\frac{\pi}{2})$$

$$CoM_2 = Rz(q_1) \cdot Tz(d_1) \cdot Rx(\frac{\pi}{2}) \cdot Rz(q2) \cdot Tx(d_{c_2})$$

$$CoM_3 = Rz(q_1) \cdot Tz(d_1) \cdot Rx(\frac{\pi}{2}) \cdot Rz(q_2) \cdot Tx(d_2) \cdot Rz(q_3) \cdot Tx(d_{c_3}) \cdot Ry(-\frac{\pi}{2})$$

$$CoM_4 = Rz(q_1) \cdot Tz(d_1) \cdot Rx(\frac{\pi}{2}) \cdot Rz(q_2) \cdot Tx(d_2) \cdot Rz(q_3) \cdot Tx(d_3) \cdot Ry(-\frac{\pi}{2}) \cdot Rz(q_4) \cdot Tz(d_{c_4}) \cdot Rz(-\frac{pi}{2})$$

$$CoM_5 = Rz(q_1) \cdot Tz(d_1) \cdot Rx(\frac{\pi}{2}) \cdot Rz(q_2) \cdot Tx(d_2) \cdot Rz(q_3) \cdot Tx(d_3) \cdot Ry(-\frac{\pi}{2}) \cdot Rz(q_4) \cdot Tz(d_4) \cdot Rz(-\frac{\pi}{2}) \cdot Rz(q_5) \cdot Ty(d_{c_5}) \cdot Rx(\frac{\pi}{2})$$

$$CoM_6 = Rz(q_1) \cdot Tz(d_1) \cdot Rx(\frac{\pi}{2}) \cdot Rz(q_2) \cdot Tx(d_2) \cdot Rz(q_3) \cdot Tx(d_3) \cdot Ry(-\frac{\pi}{2}) \cdot$$
$$\cdot Rz(q_4) \cdot Tz(d_4) \cdot Rz(-\frac{\pi}{2}) \cdot Rz(q_5) \cdot Ty(d_5) \cdot Rx(\frac{\pi}{2}) \cdot Rz(q_6) \cdot Tz(d_{c_5})$$

   And elements of J_v^i will be derivatives of last column of CoM$_i$ with respect to q_j.

2. Elements of J_ω^i are elements of 3^{rd} column of CoM$_i$.

3. Let us assume that all links have cubic form therefore matrix I will look this way:

$$I = \begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 & I_6 \end{bmatrix},$$

   where

$$I_i = \begin{bmatrix} \frac{m_i}{12}(2 \cdot d_i^2) & 0 & 0 \\ 0 & \frac{m_i}{12}(2 \cdot d_i^2) & 0 \\ 0 & 0 & \frac{m_i}{12}(2 \cdot d_i^2) \end{bmatrix}$$

4. Matrix R_i is matrix that is composed of first 3 rows and columns of matrix CoM$_i$.

Now we are ready to compute matrix $M(q)_{6\times 6}$.

3 Coriolis Force

Coriolis force can be calculated using formulas above for $C(q, \dot{q})$ and c_{ijk}.

4 Gravity Force

Gravity force can also be calculated using formulas above.