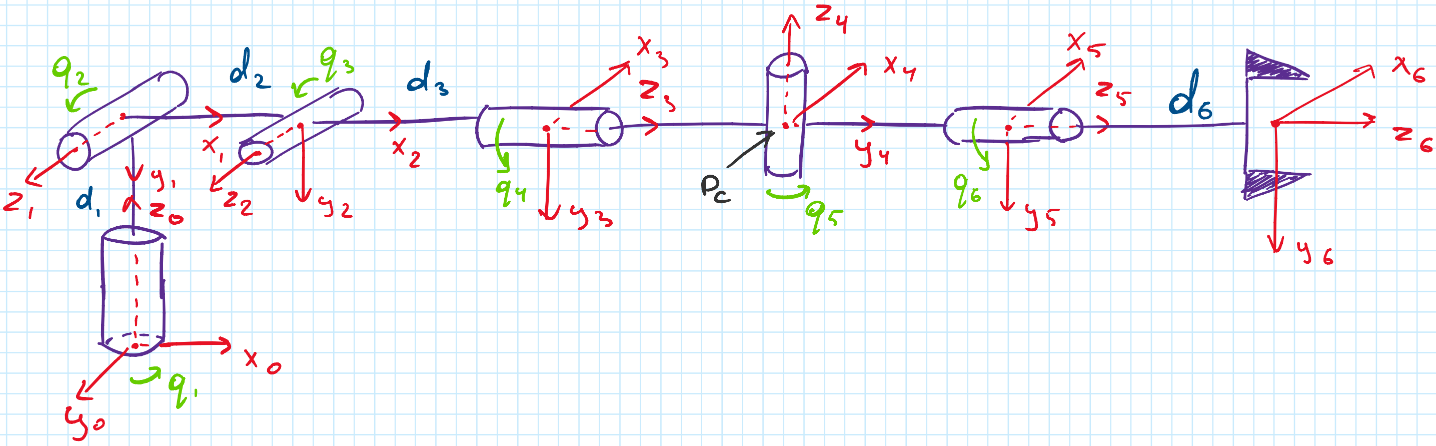
FoR HA1 Anton Buguev B19-RO-01

Anthropomorphic model with ZXZ wrist configuration

Zero configuration:



Forward Kinematics:

$$T = T_{base} \cdot \underbrace{R_z(q_1)\, T_z(d_1)\, R_x\left(\tfrac{\pi}{2}\right)}_{{}^0T_1} \cdot \underbrace{R_z(q_2)\, T_x(d_2)}_{{}^1T_2} \cdot \underbrace{R_z(q_3)\, T_x(d_3)\, R_y\left(-\tfrac{\pi}{2}\right)}_{{}^2T_3} \cdot$$

$$\cdot \underbrace{R_z(q_4)\, R_x\left(-\tfrac{\pi}{2}\right)}_{{}^3T_4} \cdot \underbrace{R_z(q_5)\, R_x\left(\tfrac{\pi}{2}\right)}_{{}^4T_5} \cdot \underbrace{R_z(q_6)\, T_z(d_6)}_{{}^5T_6} \cdot T_{tool}$$

Inverse Kinematics (Pieper's method):

So we have $\boxed{{}^0T_6 = {}^0T_1\, {}^1T_2\, {}^2T_3\, {}^3T_4\, {}^4T_5\, {}^5T_6}$.

We can write $\boxed{{}^0T_6 = {}^0T_3 \cdot {}^3T_6}$

Also we need location of wrist center P_c that is

$\boxed{P_c = {}^0T_6 \cdot T_z^{-1}(d_6)}$ take last column

In fact P_c equals to the values of last column ${}^0T_3$:

$$
{}^0T_3 = \underbrace{\begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{{}^0T_1} \cdot \underbrace{\begin{bmatrix} c_2 & -s_2 & 0 & c_2 d_2 \\ s_2 & c_2 & 0 & s_2 d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{{}^1T_2} \cdot \underbrace{\begin{bmatrix} 0 & -s_3 & c_3 & c_3 d_3 \\ 0 & c_3 & -s_3 & s_3 d_3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{{}^2T_3} =
$$

$$
= \underbrace{\begin{bmatrix} s_1 & -c_1 c_2 s_3 - c_1 s_2 c_3 & -c_1 c_2 c_3 + c_1 s_2 s_3 & c_1 c_2 c_3 d_3 - c_1 s_2 s_3 d_3 + c_1 c_2 d_2 \\ -c_1 & -s_1 c_2 s_3 - s_1 s_2 c_3 & -s_1 c_2 c_3 + s_1 s_2 s_3 & s_1 c_2 c_3 d_3 - s_1 s_2 s_3 d_3 + s_1 c_2 d_2 \\ 0 & -s_2 s_3 + c_2 c_3 & -s_2 c_3 - c_2 s_3 & s_2 c_3 d_3 + c_2 s_3 d_3 + s_2 d_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{{}^0T_3}
$$

So we have;

$$
\begin{cases} x_c = c_1 c_2 c_3 d_3 - c_1 s_2 s_3 d_3 + c_1 c_2 d_2 \, , \\ y_c = s_1 c_2 c_3 d_3 - s_1 s_2 s_3 d_3 + s_1 c_2 d_2 \, , \\ z_c = s_2 c_3 d_3 + c_2 s_3 d_3 + s_2 d_2 + d_1 \, . \end{cases}
$$

$$
\frac{y_c}{x_c} = \frac{s_1 (c_2 c_3 d_3 - s_2 s_3 d_3 + c_2 d_2)}{c_1 (c_2 c_3 d_3 - s_2 s_3 d_3 + c_2 d_2)}
$$

$$
\frac{y_c}{x_c} = \tan_1 \Rightarrow \boxed{q_1 = \operatorname{atan2}(y_c, x_c)}
$$

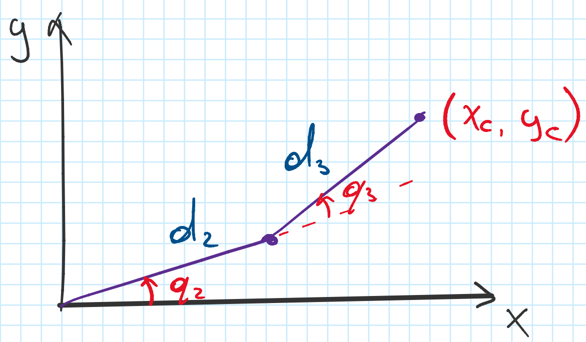
if $c_2 c_3 d_3 - s_2 s_3 d_3 + c_2 d_2 \neq 0$
otherwise q_1 is any

$$
y_c^2 + x_c^2 = d_2^2 + d_3^2 - 2 d_2 d_3 \cos(\pi - q_3)
$$

$$
c_3 = \frac{y_c^2 + x_c^2 - (d_2^2 + d_3^2)}{2 d_2 d_3} \Rightarrow
$$

$$
\Rightarrow s_3 = \sqrt{1 - c_3^2} \Rightarrow
$$

$$
\Rightarrow \boxed{q_3 = \operatorname{atan2}(s_3, c_3)}
$$



$$
s_2 = \frac{y_c (d_2 + d_3 c_3) - x_c d_3 s_3}{d_2^2 + d_3^2 + 2 d_2 d_3 c_3} \, , \quad c_2 = \frac{x_c (d_2 + d_3 c_3) + y_c d_3 s_3}{d_2^2 + d_3^2 + 2 d_2 d_3 c_3} \Rightarrow
$$

$$\boxed{q_2 = \operatorname{atan2}(s_2, c_2)}$$

After we find q_1, q_2, q_3 we can calculate ${}^0R_3$ and using it we can find ${}^3R_6$ as $\boxed{{}^3R_6 = {}^0R_3^{-1} \cdot {}^0R_6}$

$$ {}^3R_6 = \underbrace{\begin{bmatrix} c_4 & -s_4 & 0 \\ s_4 & c_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}}_{{}^3R_4} \cdot \underbrace{\begin{bmatrix} c_5 & -s_5 & 0 \\ s_5 & c_5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}}_{{}^4R_5} \cdot $$

$$ \cdot \underbrace{\begin{bmatrix} c_6 & -s_6 & 0 \\ s_6 & c_6 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}^5R} = \underbrace{\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}}_{{}^3R_6} $$

$$ {}^3R_6 = \begin{bmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{bmatrix} \Rightarrow \frac{k_2}{k_1} = \tan_4 \Rightarrow \boxed{q_4 = \operatorname{atan2}(k_2, k_1)}, $$

$$ \boxed{q_5 = \arccos(k_3)}, \quad \frac{-b_3}{a_3} = \tan_6 \Rightarrow \boxed{q_6 = \operatorname{atan2}(-b_3, a_3)} $$