# Mechanics and Machines
# HW-DYN1

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## 1 Angle limits

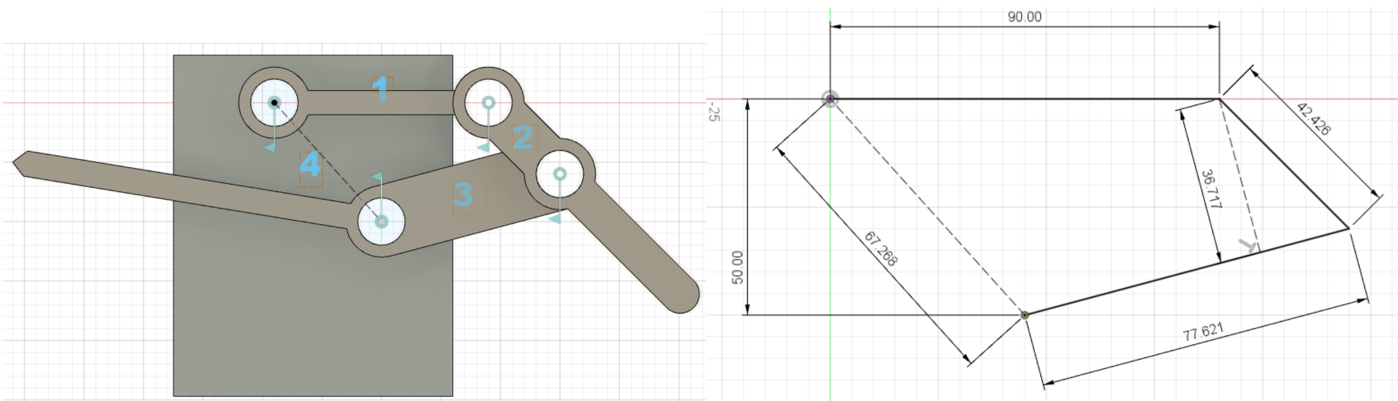


Figure 1: Model and its scheme

Since rotational link is the longest among all links, therefore the system cannot perform full rotation and it has limit angles that we need to find.

### 1.1 Code

1. First, let us find initial angle between links 1 and 4. Let us assume that joint 1 has coordinates $(0;0)$ which means joint between link 3 and base has coordinates $(45;50)$. And using $\arctan(50\backslash 45)$ we find that initial angle is $\alpha_c = 48.013°$.

2. Now let us find biggest positive angle (clockwise rotation). It reaches maximum when links 2 and 3 are aligned. It means that there will be triangle with sides 1, 4 and $2-3$. Using cosine theorem we can find angle between links 1 and 4 and subtracting initial angle we will find first limit angle:

$$(\mathit{2}+\mathit{3})^2 = \mathit{1}^2 + \mathit{4}^2 - 2 \cdot \mathit{1} \cdot \mathit{4} \cdot \cos(\beta_{1c}) \tag{1}$$

$$\beta_{1c} = \arccos(\frac{90^2 + 67.268^2 - (42.426 + 77.621)^2}{2 \cdot 90 \cdot 67.268}) = 98.484° \tag{2}$$

$$\theta_{1c} = \beta_{1c} - \alpha_c = 50.471° \tag{3}$$

3. Now let us find the smallest angle (counter-clockwise rotation). In this case we also have to consider collisions between objects. It can appear when link 1 touches link 3 and it will be the limit position. Since we know that radius of joint is 15 and width of link 3 is also 15, therefore if we draw orthogonal line from $2nd$ joint to link 3, then its length should be grater or equal than 30. It means that we can find angle between links 2 and 3 such that orthogonal line is 30, then we can find corresponding angle between links 1 and 4 and finally find $2nd$ limit angle:

$$\gamma_c = \arcsin\left(\frac{30}{42.426}\right) = 45^\circ \tag{4}$$

$$e_c^2 = \mathit{2}^2 + \mathit{3}^2 - 2 \cdot \mathit{2} \cdot \mathit{3} \cdot \cos(\gamma_c) = 3167.748mm^2 \tag{5}$$

where $e_c$ - diagonal between $2nd$ and $4th$ joints

$$e_c^2 = \mathit{1}^2 + \mathit{4}^2 - 2 \cdot \mathit{1} \cdot \mathit{4} \cdot \cos(\beta_{2c}) \tag{6}$$

$$\beta_{2c} = \arccos(\frac{\mathit{1}^2 + \mathit{4}^2 - e_c^2}{2 \cdot \mathit{1} \cdot \mathit{4}}) = 38.642^\circ \tag{7}$$

$$\theta_{2c} = \beta_{2c} - \alpha_c = 9.37^\circ \tag{8}$$

Concluding, $1st$ joint has range $\theta_{1c} + \theta_{2c} = 59.841^\circ$.

## 1.2 Fusion 360

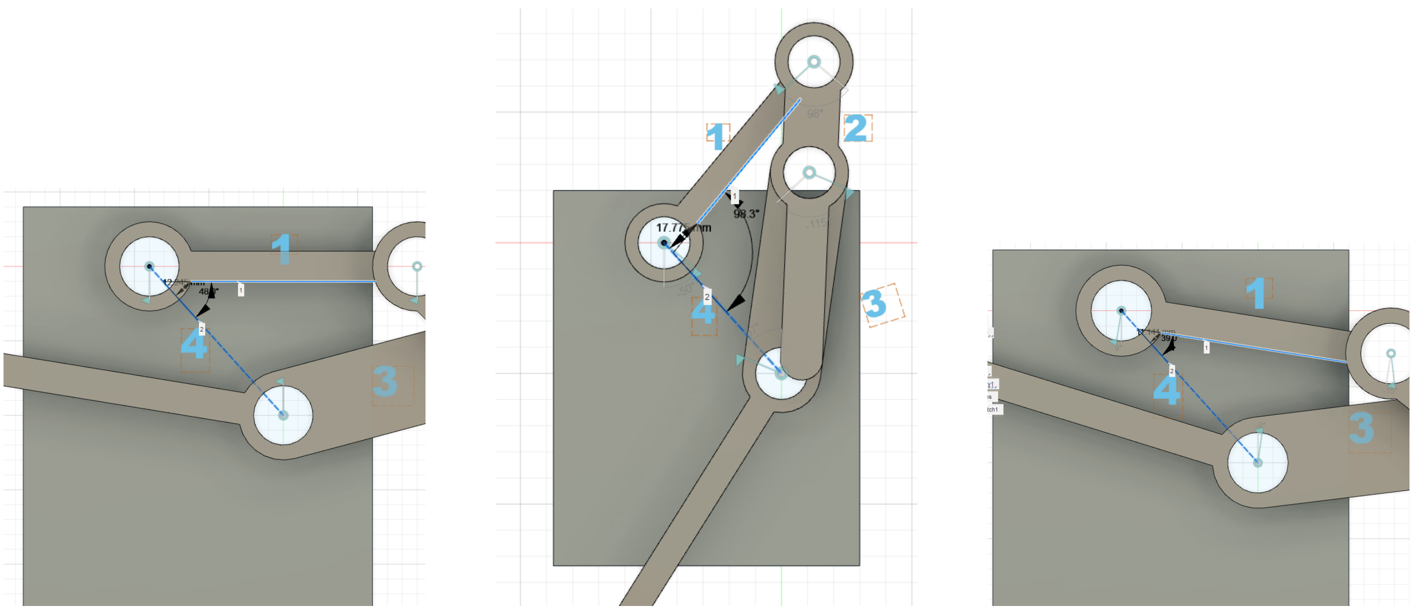


Figure 2: Model

In Fusion 360 we can find angles only using experimental approach:

1.
$$\alpha_f = 48^\circ,\ \beta_{1f} = 98.3^\circ \tag{9}$$

$$\theta_{1f} = \beta_{1f} - \alpha_f = 50.3^\circ \tag{10}$$

2.
$$\beta_{2f} = 39^\circ \tag{11}$$

$$\theta_{2f} = \beta_{2f} - \alpha_f = 9^\circ \tag{12}$$

Therefore, $1st$ joint has range $\theta_{1f} + \theta_{2f} = 59.3^\circ$

Concluding, Fusion 360 does not provide such precision, but results are still similar.

## 2 Torque

Required torque may be found using Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau$$

Unfortunately there is no solution of dynamics and simulation in ANSYS, because I did not understand how to apply Euler-Lagrange for the current system and did not find explanation how to measure torque in ANSYS.