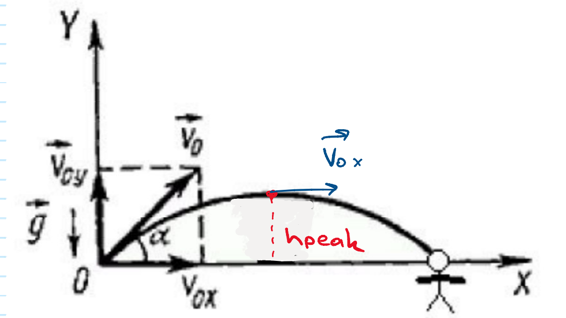
TM Homework 5 Anton Buguev B19-RO-01

Task 1



R.O.: bullet (particle, since it is very small comparing to the distance of shot) planar motion

Conditions (for 1 and 2 subtasks):

initial	peak	final
$x=0$	$x=\frac{1}{2}L$	$x=L$
$y=0$	y - ?	$y=0$
$V=V_0=870\,m/s$	$V=V_{0x}$	$V=V_0$
$V_x=V_0\cos\alpha$	$V_x=V_0\cos\alpha$	$V_x=V_0\cos\alpha$
$V_y=V_0\sin\alpha$	$V_y=0$	$V_y=-V_0\sin\alpha$
$t=0$	$t=t_{up}$ - ?	$t=t_f=2t_{up}$ - ?
α - ?		α

Force analysis (for 1 and 2 subtasks):

$G=mg$

Solution:

1) $\cancel{V_{y\,peak}}^{0} = V_{0y} - g t_{up} \Rightarrow t_{up} = \frac{V_0 \sin\alpha}{g} \Rightarrow t_f = \frac{2V_0\sin\alpha}{g}$

2) $L = \cancel{x_0}^{0} + V_{0x} t_f + \frac{1}{2}\cancel{a_x}^{0} t_f^2 \Rightarrow t_f = \frac{L}{V_0\cos\alpha}$

$\Rightarrow$

$$\frac{2V_0\sin\alpha}{g} = \frac{L}{V_0\cos\alpha} \Rightarrow V_0^2\sin(2\alpha) = Lg \Rightarrow$$

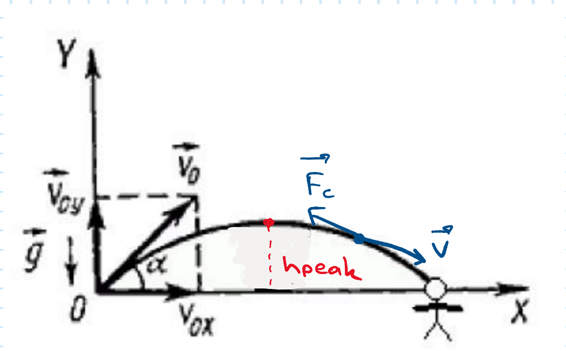
$$\Rightarrow \alpha = \frac{1}{2}\arcsin\left(\frac{Lg}{V_0^2}\right) = \frac{1}{2}\arcsin\left(\frac{1500\cdot 9{,}8}{870}\right) \Rightarrow \boxed{\alpha = 0{,}556^\circ}$$

3) $y_{peak} = \cancel{y_0}^{0} + V_{0y}t_{up} - \frac{1}{2}g t_{up}^2 = \frac{(V_0\sin\alpha)^2}{2g} = \frac{(870\cdot\sin(0{,}556))^2}{2\cdot 9{,}8} \Rightarrow$

$$\Rightarrow \boxed{y_{peak} = h_{max} = 3{,}64\,m}$$

3rd subtask:

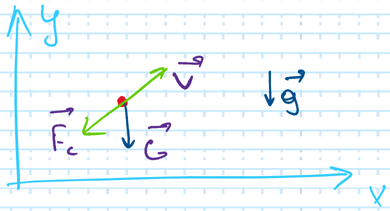
Conditions:



initial:	final:
$x=0$	$x \approx L$
$y=0$	$y=0$
$v: V_0 = 870 m/s$	v - ?
$v_x = V_0 \cos\alpha$	v_x - ?
$v_y = V_0 \sin\alpha$	v_y - ?
$t=0$	t - ?
α - ?	

Force analysis:

$\vec{G} = m\vec{g}, \quad \vec{F}_c = -kv\vec{v}$



Solution:

1) $x: -F_{cx} = ma_x \Rightarrow a_x = \frac{-F_{cx}}{m} \Rightarrow \underline{a_x = -\frac{k}{m} v v_x}$

$y: -F_{cy} - G = ma_y \Rightarrow a_y = -g - \frac{F_{cy}}{m} \Rightarrow \underline{a_y = -g - \frac{k}{m} v v_y}$

2) Let us solve this task numerically (in Python).

Knowing velocity at initial moment we can find initial acceleration:

$a_{x0} = -\frac{k}{m} V_c V_{x0}^2, \quad a_{y0} = -g - \frac{k}{m} V_c V_{y0}^2,$ where $V_c = \sqrt{V_{x0}^2 + V_{y0}^2}$

Then using small value of time (Δt) we can find v when time is $\cancel{t_0}\,0 + \Delta t$ using formulas: $\underline{V_{x1} = V_{x0} + a_{x0} \Delta t}$ and $\underline{V_{y1} = V_{y0} + a_{y0} \Delta t}$.

The same for coordinates x and y:

$x_1 = x_0 + V_{x0} \Delta t + \frac{1}{2} a_{x0} \Delta t^2, \quad y_1 = y_0 + V_{y0} \Delta t + \frac{1}{2} a_{y0} \Delta t^2$

And accelerations: $a_{x1} = -\frac{k}{m} V_c V_{x1}, \quad a_{y1} = -g - \frac{k}{m} V_c V_{y1}$

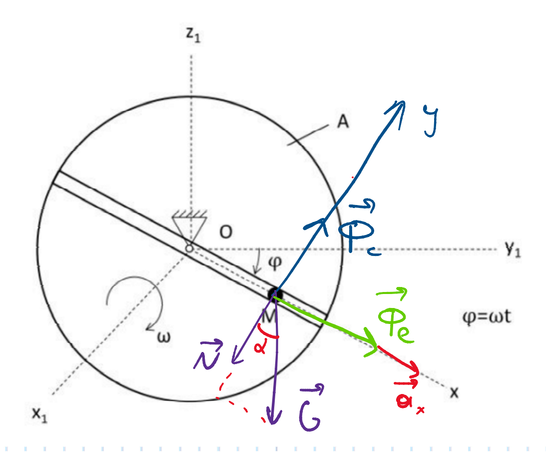
where $V_c = \sqrt{V_{x_1}^2 + V_{y_1}^2}$

So using big amount of steps we can find coordinates, velocity and acceleration at each moment of time.

For our task let us take several values of α and then choose the one that mostly satisfies conditions.

Task 2

R.O.: particle M (planar mot.)
disk (rot. mot.)



Conditions:

initial	final
$t=0$	$t-?$
$x=0$	$x=r$
$\dot{x}=0{,}4$	$\dot{x}-?$
$\varphi=0$	$\varphi=\omega t$

Force analysis:

$N,\ G=mg,\ \Phi_c=2m\omega\dot{x},\ \Phi_e=m\omega^2 x$

Solution:

$$\vec{G}+\vec{N}+\vec{\Phi}_c+\vec{\Phi}_e=m\vec{a}_x$$

1) $x:\ G\sin(\varphi)+\Phi_e=m\ddot{x}$

$$\cancel{m}g\sin(\varphi)+\cancel{m}\omega^2 x=\cancel{m}\ddot{x}$$

$$\ddot{x}-\omega^2 x=g\sin(\varphi)$$

$$x=-\frac{g\sin(\varphi)}{\omega^2}+C_1e^{-\omega t}+C_2e^{\omega t}$$

$$\dot{x}=-\omega\left(C_1e^{-\omega t}-C_1e^{\omega t}\right)$$

2) Substitute initial values:

1. $x(0)=0$:

$$0=-\frac{g\sin(\varphi)}{\omega^2}+C_1\underbrace{\cancel{e^{-\omega t}}}_{1}+C_2\underbrace{\cancel{e^{\omega t}}}_{1}\Rightarrow C_2=\frac{g\sin(\varphi)}{\omega^2}-C_1$$

2. $\dot{x}(0)=0{,}4$

$$0{,}4 = -\omega\left(C_1 \cancelto{1}{e^{-\omega t}} - C_1 \cancelto{1}{e^{\omega t}}\right) \Rightarrow 0{,}4 = -\omega\left(2C_1 - \frac{g\sin(\varphi)}{\omega^2}\right) \Rightarrow$$

$$2C_1 - \frac{g\sin(\varphi)}{\omega^2} = \frac{-2}{5\omega} \Rightarrow \boxed{C_1 = \frac{-2\omega + 5g\sin(\varphi)}{10\omega^2}} \Rightarrow$$

$$\Rightarrow C_2 = \frac{g\sin(\varphi)}{\omega^2} + \frac{2\omega - 5g\sin(\varphi)}{10\omega^2} \Rightarrow \boxed{C_2 = \frac{2\omega + 5g\sin(\varphi)}{10\omega^2}}$$

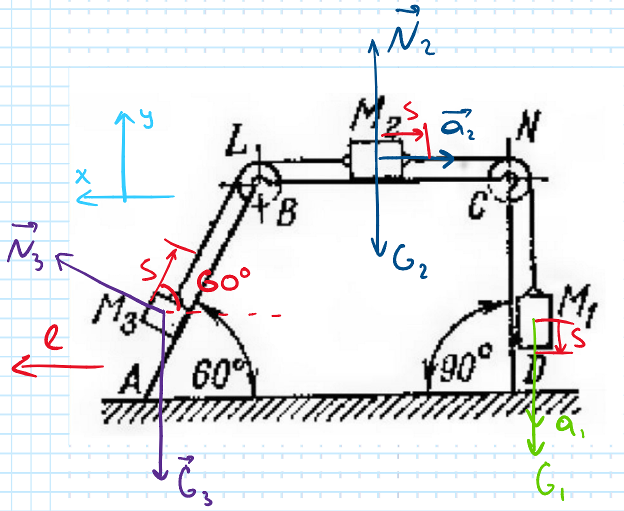
$$\boxed{x(t) = \frac{-g\sin(\varphi)}{\omega} - \frac{2\omega - 5g\sin(\varphi)}{10\omega^2}e^{-\omega t} + \frac{2\omega + 5g\sin(\varphi)}{10\omega^2}e^{\omega t}}$$

$$\boxed{v(t) = \dot{x}(t) = \frac{2\omega - 5g\sin(\varphi)}{\omega}e^{-\omega t} + \frac{2\omega + 5g\sin(\varphi)}{\omega}e^{\omega t}}$$

3) $y:\ N + G\cos\varphi = \Phi_c$

$$\boxed{N = 2m\omega\dot{x} - mg\cos\varphi}$$

Task 3



R.O.: bricks M_1, M_2, M_3,
transl. motion

Conditions:

initial	final
$t=0$	t - ?
$x=0$	x - ?
$\dot{x}=0$	$\dot{x}$ - ?

Force analysis:

$G_1 = M_1 g$, $G_2 = N_2 = M_2 g$, $G_3 = M_3 g$, $N_3 = G_3\cos(60°)$

Solution:

$$m\vec{a} = \vec{G}_1 + \vec{G}_2 + \vec{G}_3 + \vec{N}_2 + \vec{N}_3$$

$$x:\ m\ddot{x} = N_3\cos(90° - 60°)$$

$m\ddot{x} = N_3 \cos(30°)$

integrate:

$m\dot{x} = N_3 t \cos(30°) + C_1$

Put initial values: $m\dot{x} = 0$

integrate:

$mx = C_2$

Put initial values: $mx = 0$

So center of mass didn't move. Hence we have:

$M x_{init} = M x_{fin}$

Conditions:

	init.	fin.
M_1:	x_1	$x_1 + \ell$
M_2:	x_2	$x_2 + \ell - s$
M_3:	x_3	$x_3 + \ell - s\cos(60°)$

$$\frac{M_1 x_1 + M_2 x_2 + M_3 x_3}{\sum_{i=1}^{3} M_i} = \frac{M_1(x_1+\ell) + M_2(x_2+\ell-s) + M_3(x_3+\ell-s\cos(60°))}{\sum_{i=1}^{3} M_i}$$

$$M_1\ell + M_2\ell - M_2 s + M_3\ell - M_3 s\cos(60°) = 0 \Rightarrow$$

$$\Rightarrow \ell = \frac{s(M_2 + M_3\cos(60°))}{M_1 + M_2 + M_3} = \frac{15 + 10 \cdot 0{,}5}{45} \Rightarrow \boxed{\ell = 0{,}44\ m}$$

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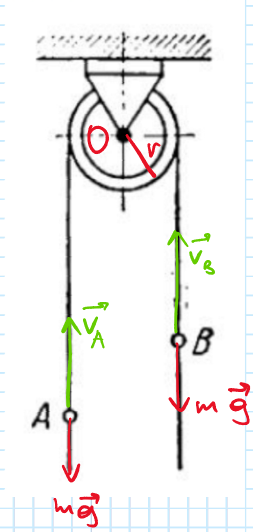
Task 4 on the next page

Task 4



R.O.: body A - transl. motion
load B - tranl. motion
pulley - rotation

Force analysis:

$G_A = G_B = mg$

Solution:

$\sum M_o(F_k^e) = 0$ because G_A and G_B are equal in magn. and different in direction

So we have:

$m v_A r + m v_B r + I\omega = 0$ where $I = \frac{m}{4} r^2$

$v_A = v_B - v$

$\cancel{m}(v_B - v)\cdot r + \cancel{m} v_B \cdot r + \frac{1}{4}\cancel{m} r^2 \omega = 0$

$(v_B - v)\cancel{r} + v_B \cancel{r} + \frac{1}{4}\cancel{r}\cdot v_B = 0$

$\frac{9}{4} v_B = v \Rightarrow \boxed{v_B = \frac{4}{9} v}$