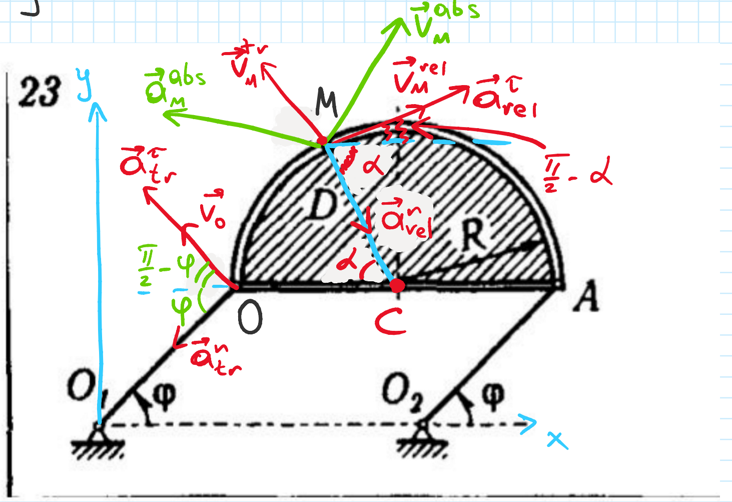
TM Homework 3 Anton Buguev B19-RO1

Task 1



1) $\varphi(t) = \pi \frac{t^3}{6}$

$$\boxed{\begin{aligned} X_0 &= O_1O\cos(\varphi(t)) \\ y_0 &= O_1O\sin(\varphi(t)) \end{aligned}}$$

$$\boxed{\begin{aligned} X_A &= X_0 + O_2A\cos(\varphi(t)) \\ y_A &= O_2A\sin(\varphi(t)) \end{aligned}}$$

2) $\alpha = \frac{OM}{R} = \frac{s_r(t)}{R} = \frac{6\pi t^2}{18} \Rightarrow \alpha = \frac{\pi}{3}t^2 \Rightarrow$

$$\boxed{\begin{aligned} X_M &= X_0 + R(1-\cos\alpha) \\ y_M &= y_0 + R\sin\alpha \end{aligned}}$$

when $\alpha \in (\pi + 2\pi n; 2\pi + 2\pi n]$ $n \in \mathbb{Z}$ values of α have to be inversed so M moves along the arc

3) $\vec{V}_0 = \vec{\omega}_0 \cdot O_1O = \dot{\vec{\varphi}}(t) \cdot O_1O = \frac{\pi t^2}{2} \cdot 20 = \boxed{10\pi t^2 = \vec{V}_M^{tr} \Rightarrow \vec{V}_0 \upuparrows \vec{V}_M^{tr}}$

4) $\boxed{V_M^{rel} = \dot{s}_r(t) = 12\pi t, \quad V_M^{rel} \perp MC}$

5) $\vec{V}_M^{abs} = \vec{V}_M^{rel} + \vec{V}_M^{tr}$

$$\begin{cases} V_x = V_x^{rel} + V_x^{tr}, \\ V_y = V_y^{rel} + V_y^{tr}; \end{cases} \quad \boxed{\begin{cases} V_x = 12\pi t \cdot \cos(\frac{\pi}{2} - \alpha) - 10\pi t^2 \cos(\frac{\pi}{2} - \varphi), \\ V_y = 12\pi t \sin(\frac{\pi}{2} - \alpha) + 10\pi t^2 \cdot \sin(\frac{\pi}{2} - \varphi). \end{cases}}$$

Components of V_M^{rel} have to be inversed when $\alpha < 0$ for correct direct direction of vector (implemented in Python)

$$\boxed{|\vec{V}_M^{abs}| = \sqrt{V_x^2 + V_y^2}}$$

6) $\vec{a}_M^{abs} = \vec{a}_{tr}^{\tau} + \vec{a}_{tr}^{n} + \vec{a}_{rel}^{\tau} + \vec{a}_{rel}^{n}$

1. $a^{\tau}_{tr} = O_1O \cdot \varepsilon = O_1O \cdot \dot{\omega} = \underline{20 \cdot \pi t}$

2. $a^{n}_{tr} = O_1O \cdot \omega^2 = 20 \cdot \frac{(\pi t^2)^2}{4} = \underline{5(\pi t^2)^2}$

3. $a^{\tau}_{rel} = (V^{rel}_{M})' = \underline{12\pi}$

4. $a^{n}_{rel} = \frac{(V^{rel}_{M})^2}{R} = \underline{\frac{(12\pi t)^2}{18}}$

7) $\begin{cases} a_x = a^{tr}_x + a^{rel}_x, \\ a_y = a^{tr}_y + a^{rel}_y; \end{cases}$ $\begin{cases} a_x = (a^{tr}_x)^{\tau} + (a^{tr}_x)^{n} + (a^{rel}_x)^{\tau} + (a^{rel}_x)^{n}, \\ a_y = (a^{tr}_y)^{\tau} + (a^{tr}_y)^{n} + (a^{rel}_y)^{\tau} + (a^{rel}_y)^{n}; \end{cases}$

$$\boxed{\begin{aligned} a_x &= -a^{\tau}_{tr} \cdot \cos(\tfrac{\pi}{2} - \varphi) - a^{n}_{tr} \cos(\varphi) + \underline{a^{\tau}_{rel} \cos(\tfrac{\pi}{2} - \alpha)} + a^{n}_{rel} \cos(\alpha) \\ a_y &= a^{\tau}_{tr} \sin(\tfrac{\pi}{2} - \varphi) - a^{n}_{tr} \sin(\varphi) + \underline{a^{\tau}_{rel} \sin(\tfrac{\pi}{2} - \alpha)} - a^{n}_{rel} \sin(\alpha) \end{aligned}}$$

When $\alpha < 0$ underlined components have to be inversed to have correct direction of vector (implemented in Python)

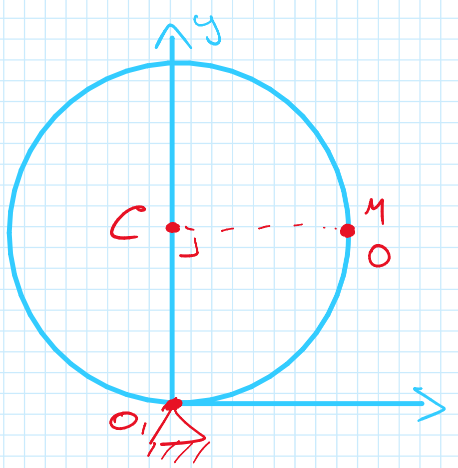
$$\boxed{|\vec{V}^{abs}_{M}| = \sqrt{a_x^2 + a_y^2}}$$

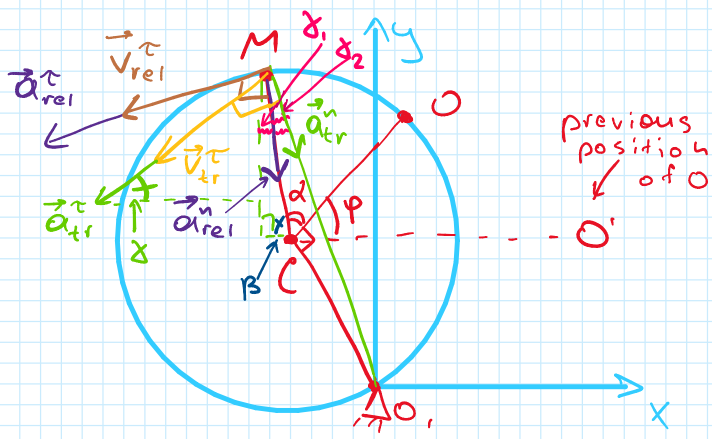
8) Point M reaches O every time when $\alpha = 2\pi k, k \in \mathbb{Z}$

$\alpha = \frac{OM}{R} = 2\pi k \Rightarrow \frac{6\pi t^2}{R} = 2\pi k \Rightarrow t = \sqrt{6k} \quad k \in \mathbb{Z}$

It means that M reaches O at $\boxed{t = \sqrt{6k} \quad k \in \mathbb{Z}}$

Task 2





1) Let us derive position of center C, then we will be able to find position of p. O and p. M

1. p. C makes full circle arond O_1 =>

$$\boxed{\begin{aligned} x_c &= -R\sin(\varphi(t)) \\ y_c &= R\cos(\varphi(t)) \end{aligned}}$$

2. $$\boxed{\begin{aligned} x_o &= x_c + R\cos(\varphi(t)) \\ y_o &= y_c + R\sin(\varphi(t)) \end{aligned}}$$

3. $\underline{\alpha} = \frac{\smile OM}{R} \Rightarrow \alpha = \frac{75\pi(0{,}1t + 0{,}3t^2)}{30} = \underline{2{,}5\pi(0{,}1t + 0{,}3t^2)}$

$\beta = \pi - (\alpha + \varphi)$

$$\boxed{\begin{aligned} x_M &= x_c - R\cos(\beta) \\ y_M &= y_M + R\sin(\beta) \end{aligned}}$$

2) $\vec{V}_M^{abs} = \vec{V}_M^{tr} + \vec{V}_M^{rel}$

by cosine theorem for ΔO_1CM

1. $V_M^{tr} = \omega \cdot O_1M = \dot{\varphi} \cdot R\sqrt{2(1 - \cos(\frac{\pi}{2} + \alpha))} \Rightarrow$

$\Rightarrow \underline{V_M^{tr} = (2 - 0{,}6t) \cdot 30\sqrt{2(1 - \cos(\frac{\pi}{2} + \alpha))}}$

2. $V_M^{rel} = \dot{s} \Rightarrow V_M^{rel} = \underline{75\pi(0{,}1 + 0{,}6t)}$

3. $$\boxed{\begin{aligned} V_x^{abs} &= -V_M^{rel}\cos(\beta) - \underline{V_M^{tr}\cos(\gamma)} \\ V_y^{abs} &= -V_M^{rel}\sin(\beta) - \underline{V_M^{tr}\sin(\gamma)} \end{aligned}}$$

When M reaches O_1 underlined components have to be inversed for correct direction of vector

$$\boxed{|\vec{V}_M^{abs}| = \sqrt{(V_x^{abs})^2 + (V_y^{abs})^2}}$$

3) $\vec{a}_M^{abs} = \vec{a}_M^{rel\_n} + \vec{a}_M^{rel\_\tau} + \vec{a}_M^{tr\_n} + \vec{a}_M^{tr\_\tau}$

1. $a_M^{rel\_n} = \frac{(\dot{s})^2}{R} = \frac{(75\pi(0{,}1+0{,}6t))^2}{30} \Rightarrow$

$\Rightarrow a_M^{rel\_n} = 187{,}5\pi^2(0{,}1+0{,}6t)^2$

2. $a_M^{rel\_\tau} = \ddot{s} = 75\pi \cdot 0{,}6 \Rightarrow a_M^{rel\_\tau} = 45\pi$

3. $$\boxed{\begin{aligned} a_x^{rel} &= a_M^{rel\_n} \cdot \cos(\beta) - a_M^{rel\_\tau} \cdot \sin(\beta) \\ a_y^{rel} &= -a_M^{rel\_n} \sin(\beta) - a_M^{rel\_\tau} \cos(\beta) \end{aligned}}$$

$$\boxed{|\vec{a}_M^{rel}| = \sqrt{(a_x^{rel})^2 + (a_y^{rel})^2}}$$

4. $a_M^{tr\_n} = \omega^2 \cdot O_1M \Rightarrow$

$\Rightarrow a_M^{tr\_n} = (2-0{,}6t)^2 \cdot 30\sqrt{2(1-\cos(\frac{\pi}{2}+\alpha))}$

5. $a_M^{tr\_\tau} = \varepsilon \cdot O_1M = \dot{\omega}\, O_1M = -0{,}6 \cdot 30\sqrt{2(1-\cos(\frac{\pi}{2}+\alpha))} \Rightarrow$

$\Rightarrow a_M^{tr\_\tau} = -18\sqrt{2(1-\cos(\frac{\pi}{2}+\alpha))}$

6. $\gamma_1 = \frac{\pi}{2} - (\pi - \alpha - \varphi)$

$\gamma_2 = \frac{\pi}{4} - \frac{\alpha}{2}$ ← from $\triangle O_1CM$

$\gamma = \gamma_1 + \gamma_2$

$$\boxed{\begin{aligned} a_x^{tr} &= -a_M^{tr\_n} \cdot \sin(\gamma) + a_M^{tr\_\tau} \cdot \cos(\gamma) \\ a_y^{tr} &= a_M^{tr\_n} \cdot \cos(\gamma) + a_M^{tr\_\tau} \cdot \sin(\gamma) \end{aligned}}$$

When p. M reaches O_1 value of each component has to be inversed for correct direction of vector (implemented in Python)

$$\boxed{|\vec{a}_M^{tr}| = \sqrt{(a_x^{tr})^2 + (a_y^{tr})^2}}$$

4) Point M reaches p.O for the first time when

$\alpha = 2\pi \Rightarrow 2{,}5\pi(0{,}1t + 0{,}3t^2) = 2\pi$

$0{,}75t^2 + 0{,}25t - 2 = 0$

$75t^2 + 25t - 200 = 0$

$D = 625 + 60000 = 60625$

$t_1 = \dfrac{-25 + 246{,}22}{150} = 1{,}47\,s$

$t_2 = \dfrac{-25 - 246{,}22}{150} = -1{,}8\,s$ — wrong answer because $t \geq 0$ $\Rightarrow$

$\Rightarrow \boxed{t = 1{,}47\,s}$