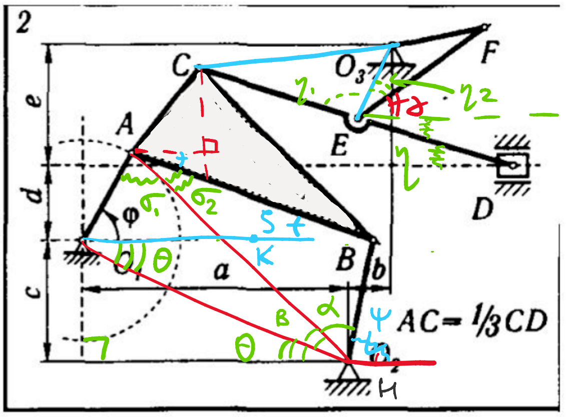
TM Homework Anton Buguev B19-RO-01

Task 1



1) Point A:

$$\boxed{\begin{aligned} x_A &= O_1A\cos(\varphi+\omega t) \\ y_A &= O_1A\sin(\varphi+\omega t) \end{aligned}}$$

2) Point B:

1. $\theta = \arccos\left(\frac{a}{O_1O_2}\right)$, where

$$O_1O_2 = \sqrt{(x_{O_1}-x_{O_2})^2+(y_{O_1}-y_{O_2})^2}$$

2. $\frac{\sin\beta}{O_1A} = \frac{\sin(\varphi+\omega t+\theta)}{O_2A}$,

$\angle KO_1O_2 = \theta$ because they are alternate interior

$$O_2A = \sqrt{(x_{O_2}-x_A)^2+(y_{O_2}-y_A)^2}$$

$$\beta = \arcsin\left(\frac{O_1A}{O_2A}\cdot\sin(\varphi+\omega t+\theta)\right)$$

3. $AB^2 = O_2B^2+O_2A^2-2O_2B\cdot O_2A\cos(\alpha) \Rightarrow$

$$\Rightarrow \alpha = \arccos\left(\frac{O_2B^2+O_2A^2-AB^2}{2O_2B\cdot O_2A}\right)$$

4. $\psi = \pi-(\theta+\beta+\alpha) \Rightarrow$ $\boxed{\begin{aligned} x_B &= x_{O_2}+O_2B\cos(\psi) \\ y_B &= y_{O_2}+O_2B\cos(\psi) \end{aligned}}$

Important moment!

Point A cannot make the whole circle arond O_1.

Proof:

Let us use Grashof condition

$O_1O_2 = \sqrt{56^2+26^2} = 61,74 \Rightarrow AO_1$ - shortest link, O_1O_2 - longest $\Rightarrow$

$AO_1 + O_1O_2 = 82,74$; $AB + BO_2 = 79 \Rightarrow$ Grashof cond. does not hold, hence A cannot rotate fully around O_1

In simulation at some moment all points except A disapear, but later reaper when it is possible

3) Point C:

1. $\sigma_1 = \pi - (\varphi + \omega t + \theta + \beta)$

2. $\frac{\sin(\sigma_2)}{O_2B} = \frac{\sin(\alpha)}{AB} \Rightarrow \sigma_2 = \arcsin\left(\frac{O_2B}{AB} \cdot \sin\alpha\right)$

3. $\zeta = \pi - (\varphi + \omega t + \sigma_1 + \sigma_2)$

4. $BC^2 = AC^2 + AB^2 - 2AC \cdot BC\cos(\rho)$

$\rho = \arccos\left(\frac{AC^2 + AB^2 - BC^2}{2AC \cdot BC}\right)$

5. $\xi = \rho - \zeta \Rightarrow \boxed{\begin{array}{l} x_C = x_A + AC \cdot \cos(\xi) \\ y_C = y_A + AC \cdot \sin(\xi) \end{array}}$

4) Point D:

$\eta = \arcsin\left(\frac{y_C - 16}{CD}\right) \Rightarrow \boxed{\begin{array}{l} x_D = x_C + CD\cos(\eta) \\ y_D = 16 \end{array}}$

5) Point E:

$\boxed{\begin{array}{l} x_E = x_C + CE\cos(\eta) \\ y_E = y_C - CE\sin(\eta) \end{array}}$

6) Point F:

1. $\eta_1 = \arccos\left(\frac{CE^2 + EO_3^2 - CO_3^2}{2CE \cdot EO_3}\right)$, where

$EO_3 = \sqrt{(x_E - x_{O_3})^2 + (y_E - y_{O_3})^2}$, $CO_3 = \sqrt{(x_C - x_{O_3})^2 + (y_C - y_{O_3})^2}$

2. $\eta_2 = \arccos\left(\frac{EO_3^2 + EF^2 - O_3F^2}{2EO_3 \cdot EF}\right)$

3. $\gamma = \pi - (\eta_1 + \eta_2 + \eta) \Rightarrow \boxed{\begin{array}{l} x_F = x_E + EF\cos(\gamma) \\ y_F = y_E + EF\sin(\gamma) \end{array}}$

7) Velocity of each point is calculated using derivatives:

$$\boxed{\vec{v}_i = \begin{bmatrix} v_{xi} \\ v_{yi} \end{bmatrix} = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix}, \text{ where } i \in \{A, B, C, D, E, F\}}$$

Calculated using Python

8) Accelerations are derivatives of velocities:

$$\boxed{\vec{a}_i = \begin{bmatrix} a_{xi} \\ a_{yi} \end{bmatrix} = \begin{bmatrix} \dot{v}_{xi} \\ \dot{v}_{yi} \end{bmatrix} = \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix}, \text{ where } i \in \{A, B\}}$$

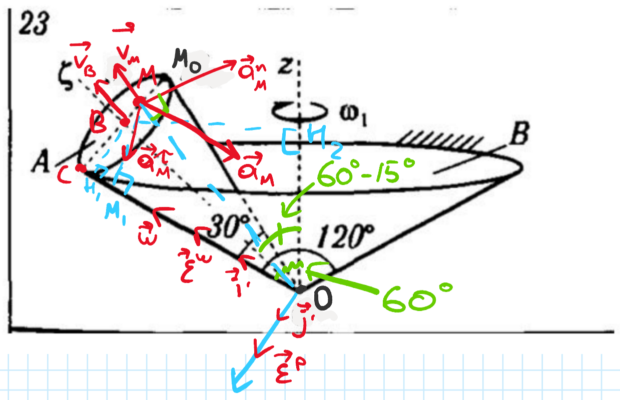
Calculated using Python

9) Angular velocity of the link is calculated using formula:

$$\boxed{|\vec{\omega}_{ij}| = \frac{|\vec{V}_{ij}|}{|ij|}, \text{ where } \vec{V}_{ij} = \vec{V}_i - \vec{V}_j;\ i, j \in \{A, B, C, D, E, F, O_1, O_2, O_3\}, i \neq j}$$

Calculated using Python

Task 2



1) The cone rotates around axis Z, IC of the cone is OC

2) B - center of base of cone

Since OC is IC $\Rightarrow \vec{V}_B = \cancel{\vec{V}_{H_1}}^{\,0} + \vec{V}_{BH_1} \Rightarrow V_B = V_{BH_1} = \omega BH_1 = \omega OB \sin(15°)$

Also we can write: $\vec{V}_B = \cancel{\vec{V}_{H_2}}^{\,0} + \vec{V}_{BH_2} \Rightarrow V_B = V_{BH_2} = \omega_1 BH_2 = \omega_1 OB \sin(60° - 15°)$

So we have:

$$\begin{cases} V_B = \omega OB \sin(15°), \\ V_B = \omega_1 OB \sin(45°). \end{cases} \Rightarrow \omega \sin(15°) = \omega_1 \sin(45°) \Rightarrow \omega = \frac{\sqrt{2}}{\sin(15°)} = 5{,}46 \frac{rad}{s}$$

$$\boxed{\omega = \omega_A = 5{,}46 \frac{rad}{s}}$$

3) $\vec{\varepsilon} = \dot{\vec{\omega}} = \underbrace{\frac{d\omega}{dt} \cdot \vec{i}'}_{\vec{\varepsilon}^{\,\omega}} + \underbrace{\frac{d\vec{i}'}{dt} \cdot \omega}_{\vec{\varepsilon}^{\,p}}$

$$\vec{\varepsilon}^{\,\omega} = \frac{d\omega}{dt} \cdot \vec{i}' = \frac{d\left(\omega_1 \cdot \frac{\sin(45^\circ)}{\sin(15^\circ)}\right)}{dt} \cdot \vec{i}' = \frac{\varepsilon_1 \sqrt{2}}{2\sin(15^\circ)} \cdot \vec{i}'$$

$$\vec{\varepsilon}^{\,p} = \frac{d\vec{i}'}{dt} \cdot \omega = (\vec{\omega}_1 \times \vec{i}') \cdot \omega = \omega\omega_1 \sin(60^\circ) \cdot \vec{j}' =$$

$$\vec{\varepsilon} = \frac{\varepsilon_1 \sqrt{2}}{2\sin(15^\circ)} \cdot \vec{i}' + \omega\omega_1 \sin(60^\circ)\, \vec{j}'$$

$$\varepsilon = \sqrt{\left(\frac{\varepsilon_1\sqrt{2}}{2\sin(15^\circ)}\right)^2 + \left(\omega\omega_1 \cdot \frac{\sqrt{3}}{2}\right)^2} = \sqrt{102{,}18\left(\frac{rad}{s^2}\right)^2 + 89{,}57\left(\frac{rad}{s^2}\right)^2} \Rightarrow$$

$$\Rightarrow \boxed{\varepsilon = \varepsilon_A = 13{,}85\ \frac{rad}{s^2}}$$

4) 1. $\angle CBO = 90^\circ \Rightarrow CB = OC \sin(15^\circ) = OM_0 \sin(15^\circ) = 10{,}35 \Rightarrow$

$\Rightarrow CM_0 = 2CB = 2OM_0 \sin(15^\circ) \Rightarrow CM = CM_0 - M_0M \Rightarrow$

$\Rightarrow CM = 2OM_0 \sin(15^\circ) - M_0M$

2. $\angle MCO = 90^\circ - 15^\circ = 75^\circ \Rightarrow MM_1 = CM \sin(75^\circ) \Rightarrow$

$\Rightarrow MM_1 = (2OM_0 \sin(15^\circ) - M_0M)\sin(75^\circ) = 15{,}17$

3. $\vec{V}_M = \cancel{\vec{V}_{M_1}}^{\,0} + \vec{V}_{MM_1} \Rightarrow V_M = \omega \cdot MM_1 = \underbrace{(2OM_0 \sin(15^\circ) - M_0M)\sin(75^\circ)}_{15{,}17}\omega$

$\boxed{V_M = 82{,}83\ m/s}$

5) $\vec{a}_M = \vec{a}_M^{\tau} + \vec{a}_M^{n}$

$a_M^{\tau} = \varepsilon \cdot OM = 13{,}85 \cdot 39 = 540{,}15$

$a_M^{n} = \omega^2 \cdot MM_1 = 452{,}24$

$OM = \sqrt{BM^2 + OB^2} = \sqrt{(\underbrace{CB - M_0M}_{5{,}35})^2 + (\underbrace{OC \cdot \cos(15^\circ)}_{38{,}64})^2} = 39$

$\alpha = \angle OMM_1 = \arccos\left(\frac{MM_1}{OM}\right) = 67^\circ$

$a_M = \sqrt{(a_M^{\tau})^2 + (a_M^{n})^2 \underbrace{- 2a_M^{\tau} \cdot a_M^{n} \cdot \cos(90^\circ + \alpha)}_{-449717{,}13}} \Rightarrow \boxed{a_M = 972{,}63\ \frac{m}{s^2}}$