

For the solution let us use the following algorithm:

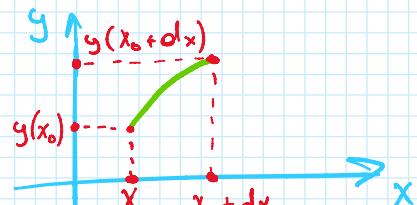
1) Divide the plot into a lot of sectors with small step along x axis, let us make it  $\boxed{dx=0,0001}$

2) So we will have the following graph:

Since  $dx$  is very small, we can

calculate the length of this graph not using formula

$$l = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$
 but using Euclidean distance



because we can assume that this is a straight line:

$$\boxed{l = \sqrt{(x_0 - (x_0 + dx))^2 + (y(x_0) - y(x_0 + dx))^2}}$$

3) Knowing the length of that piece, let us use formula  $v = \sqrt{2as + v_0^2}$ . Using this formula we can find velocity and acceleration for each segment. Since we want our body to move as fast as possible, we set  $a = a_{\max}^t$  and  $v_0$  is calculated for previous segment, so initially it is 0.

If calculated  $v \leq v_{\max}$ , we need to check that  $a^n \leq a_{\max}^n$

$$4) a^n = \frac{v^2}{P} \quad \text{where} \quad P = \frac{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{\frac{d^2y}{dx^2}}$$

If both conditions hold we save calculated  $v$  as velocity for this sector. But if at least one of the conditions fail, we try to decrease  $a^t$  by small value and calculate everything again until we find appropriate values for  $v, a^t$  and  $a^n$ .

However, it may be that it is impossible to find appropriate values. In this case we need to go to the previous segment decrease values of  $v$  and  $a^t$  and recalculate everything again.

5) Since we have a condition that in the end body must stop, we can consider last segments

as motion back, for example, from  $x=4$  to  $x=3,5$ .

Thus we divided our model into 2 parts:

1 - Motion on  $x \in [0; 3,5]$  with  $v_0 = 0$  for  
the smallest time

2 - Motion on  $4 \geq x \geq 3,5$  with  $v_0 = 0$  for  
the smallest time

## 6) Pseudocode:

1. create lists for required data

2.  $dx = 0,0001$

$da = 0,1$

3.  $i = 0$

while ( $i < 3,5 \cdot dx$ ):

| find  $\ell$  // Euclidean dist.

|  $v_0 = v\_data[i]$

|  $v = \sqrt{2\ell a^r + v_0^2}$

| if ( $v \leq v_{max}$  and  $a^r \leq a_{max}^r$ ):

| | if ( $a^r < -a_{max}^r$ ):

| | |  $i--$

| | else:

| | |  $a^r -= da$

| | continue

| else:

| | save  $v, a^r, a^r$

| .

| save  $v, \hat{a^r}, a^n$

| i++

4.  $j = 4 / dx$

while ( $j \geq 3, 5 \cdot dx$ ):

| find  $\ell$  // Euclidean dist.

|  $V_0 = v\_data[j]$

|  $V = \sqrt{2\ell a^r + V_0^2}$

| if ( $V \leq v_{max}$  and  $a^n \leq a_{max}^n$ ):

| | if ( $\hat{a^r} < -a_{max}^r$ ):

| | | j++

| | else:

| | |  $\hat{a^r} -= da$

| | continue

| | else:

| | save  $v, \hat{a^r}, a^n$

| | j--

5.  $t = 0$

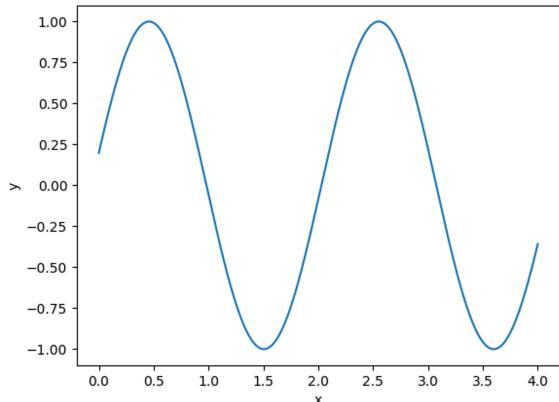
for i in len(dist\_data):

$t += dist\_data[i] / vel\_data[i]$

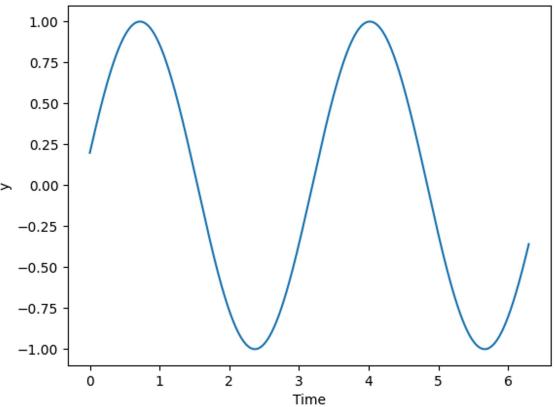
7) The best time is approx.  $t = 6,3 \text{ s}$

## 8) Plots

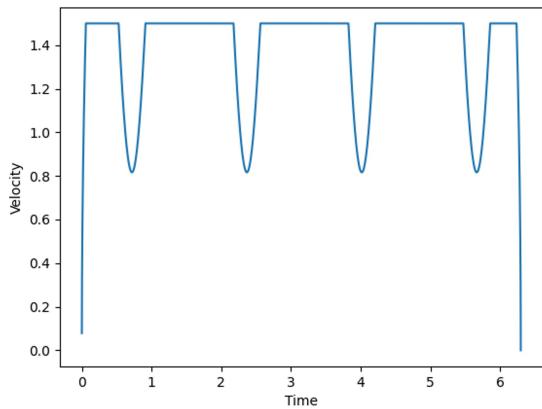
$y(x)$ :



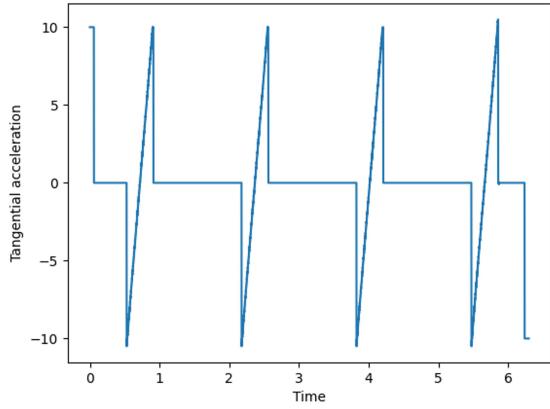
$y(t)$ :



$v(t)$ :



$a^t(t)$ :



$a^n(t)$ :

