Asssignment 2

Physics 451

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Contents

Question 1

Scaling Behavior (4 Points)

Question 1.a

Near critical points, the self-similarity under rescaling leads to characteristic power-law singularities. These dependences may be disguised, however, by less-singular corrections to scaling. An experiment measures the susceptibility $\chi(T)$ in a magnet for temperatures T slightly above the ferromagnetic transition temperature Tc. They find their data is fit well by the form

$$\chi(T) = A(T - T_c)^{-1.25} + B + C(T - T_c) + D(T - T_c)^{1.77} \tag{1.1}$$

Assuming this is the correct dependence near Tc, what is the critical exponent γ

Question 1.b

The pair correlation function $C(r, T) = \langle \sigma_i \sigma_{i+r} \rangle$ is measured in another, three-dimensional system just above T_c. It is found to be spherically symmetric, and of the form

$$C(r, T) = r^{-1.026} G(r(T - T_c)^{0.59}) \tag{1.2}$$

Where the function $G(x)$ is found to be roughly $exp(-x)$. What is the critical exponent ν?

Question 2

The recursion relations for nonzero magnetic field

Question 2.a

or nonzero magnetic field show that the function $A(K, h)$ satisfies the relation:

$$2e^{h(s_1+s_3)/2} \cosh[K(s_1 + s_3) + h] = A(K, h)e^{K' s_1 s_3 + h'(s_1+s_3)/2} \tag{2.1}$$

Question 2.b

Show that the recursion relations for nonzero magnetic field are

$$K' \frac{1}{4} \ln \frac{\cosh(2K+h)\cosh(2K-h)}{\cosh^2 h} \tag{2.2}$$

$$h' = h + \frac{1}{2} \ln \left[\frac{\cosh(2K+h)}{2K-h} \right] \tag{2.3}$$

and

$$\ln A(k,h) = \frac{1}{4} \ln \left[16 \cosh(2K+h)\cosh(2K-h)\cosh^2 h \right] \tag{2.4}$$

Question 2.c

Show that the recursion relations have a line of trivial fixed points satisfying $K^* = 0$ and arbitrary h^*, corresponding to the paramagnetic phase, and an unstable ferromagnetic fixed point at $K^* = \infty, h^* = 0$.

Question 2.d

Justify:

$$Z(K,h,N)A(K,h)^{N/2} Z(K',h',N/2) \tag{2.5}$$

Question 3

The partition function for the N-spin Ising chain can be written as the trace of the Nth power of hte transfer matrix T. Another way to reduce the number of degrees of freedom is to describe the system in terms of two-spin cells. We write Z as:

$$Z = \mathrm{Tr}\mathbf{T}^N = \mathrm{Tr}(\mathbf{T}^2)^{N/2} = \mathrm{Tr}\mathbf{T}'^{N/2} \tag{3.1}$$

The transfer matrix for two-spin cells. $\mathbf{T}^2$, can be written as

$$\mathbf{T}^2 = \mathbf{T}\mathbf{T} = \begin{matrix} e^{2K+2h} + e^{-2K} & e^{h} + e^{-h} \\ e^{-h} + e^{h} & e^{2K-2h} + e^{-2K} \end{matrix} \tag{3.2}$$

We require that $\mathbf{T}'$ have the same form as $\mathbf{T}$:

$$\mathbf{T}' = c\begin{matrix} e^{K'+h'} & e^{-K'} \\ e^{-K'} & e^{K'-h'} \end{matrix} \tag{3.3}$$

A parameter C must be introduced because matching the two equations requires three matrix elements, which is impossible with only two variables K' and h'.

Question 3.a

Show that the three unknowns satisfy the three conditions:

$$Ce^{K'}e^{h'} = e^{2K+2h} + e^{-2K} \tag{3.4}$$

$$Ce^{-K'} = e^{h} + e^{-h} \tag{3.5}$$

$$Ce^{K'}e^{-h'} = e^{2K-2h} + e^{-2K} \tag{3.6}$$

Question 3.b

Show that the solutions of the previous can be written as

$$e^{-2h'} = \frac{e^{2K-2h} + e^{-2K}}{e^{2K+2h} + e^{-2K}} \tag{3.7}$$

$$e^{4K'} = \frac{e^{4K} + e^{-2h} + e^{2h} + e^{-4K}}{\left(e^{h} + e^{-h}\right)^2} \tag{3.8}$$

$$C^4 = \left[e^{4K} + e^{-2h} + e^{2h} + e^{-4K}\right]\left[e^{h} + e^{-h}\right]^2 \tag{3.9}$$

Question 3.c

Show that the recursion relations in the previous reduce to

$$K' = R(K) = \frac{1}{2}\ln[\cosh(2K)] \tag{3.10}$$

for $h = 0$. For $h \neq 0$ start from some intial state, K_0, h_0 and calculate a typical renormalization group trajectory. To what phase (paramagnetic or ferromagnetic) does the fixed point correspond?

References