Asssignment 1

Physics 451

DATE February 10, 2026

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Contents

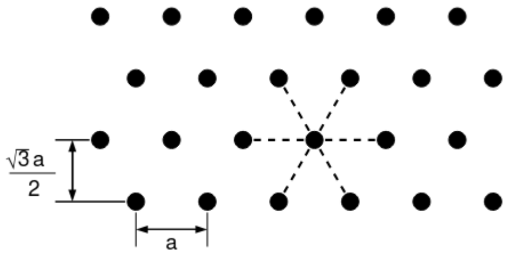

Figure 5.11: Each spin has six nearest neighbors on a hexagonal lattice. This lattice structure is sometimes called a triangular lattice.

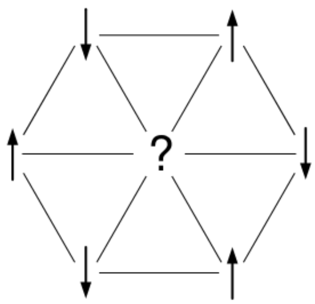

Figure 5.12: The six nearest neighbors of the central spin on a hexagonal lattice are successively antiparallel, corresponding to the lowest energy of interaction for an Ising antiferromagnet. The central spin cannot be antiparallel to all its neighbors and is said to be *frustrated*.

Figure 1: Caption

Question 1

So far we have considered the ferromagnetic Ising model for which the energy of interaction between two nearest neighbor spins is $J > 0$. Hence, all spins are parallel in the ground state of the ferromagnetic Ising model. In contrast, if $J < 0$, nearest neighbor spins must be antiparallel to minimize their energy of interaction.

Question 1.a

Sketch the ground state of the one-dimensional antiferromagnetic Ising model. Then do the same for the antiferromagnetic Ising model on a square lattice. What is the value of M for the ground state of an Ising antiferromagnet?

Question 1.b

Use Program `IsingAnitferromagnetSquareLattice` to simulate the antiferromagnetic Ising model on a square lattice at various temperatures and describe its qualitative behavior. Does the system have a phase transition at T ¿ 0? Does the value of M show evidence of a phase transition?

Question 1.c

In addition to the usual thermodynamic quantities the program calculates the staggered magnetization and the staggered susceptibility. The staggered magnetization is calculated by considering the square lattice as a checkerboard with black and red sites so that each black site has four red sites as nearest neighbors and vice versa. The staggered magnetization is calculated from $\sum c_i s_i$ where $c_i = +1$ for a black site and $c_i = -1$ for a red site. Describe the behavior of these quantities and compare them to the behavior of M and χ for the ferromagnetic Ising model.

Question 1.d

Consider the Ising antiferromagnetic model on a hexagonal lattice (see Figure 5.11), for which each spin has six nearest neighbors. The ground state in this case is not unique because of frustration (see Figure 5.12). Convince yourself that there are multiple ground states. Is the entropy zero or nonzero at T = 0? Use Program `IsingAntiferromagnetHexagonalLattice` to simulate the antiferromagnetic Ising model on a hexagonal lattice at various temperatures and describe its qualitative behavior. This system does not have a phase transition for $T > 0$. Are your results consistent with this behavior?

Question 2

Minima of free energy

Question 2.a

To understand the meaning of the various solutions of (5.108), expand the free energy in (5.126) about $m = 0$ with $H = 0$ and show that the form of $f(m)$ near the critical point (small m) is given by

$$f(m) = a + b(1 - \beta qJ)m^2 + cm^4 \tag{2.1}$$

Determine a, b, and c

Question 2.b

If H is nonzero but small, show that there is an additional term $-mH$ in 2.1

Question 2.c

Show that the minimum free energy for $T > T_c$ and $H = 0$ is at $m = 0$, and that $m = \pm m_0$ corresponds to a lower free energy for $T < T_c$.

Question 2.d

Use program `IsingMeanField` to plot $f(m)$ as a function of m for $T > T_c$ and $H = 0$. For what value of m does $f(m)$ have a minimum?

Question 2.e

Plot $f(m)$ FOR $T = 1$ and $H = 0$. Where are the minima of $f(m)$? Do they have the same depth? If so, what is the meaning of this result?

Question 2.f

Choose $H = 0.5$ and $T = 1$. Do the two minima have the same depth? The global minimum corresponds to the equilibrium or stable phase. If we quickly "flip" the field and let $H \to -0.5$, the minimum at $m \approx 1$ will become a local minimum. The system will remain in this local minimum for some time before it switches to the global minimum.