Matrizen in Neuronalen Netzwerken

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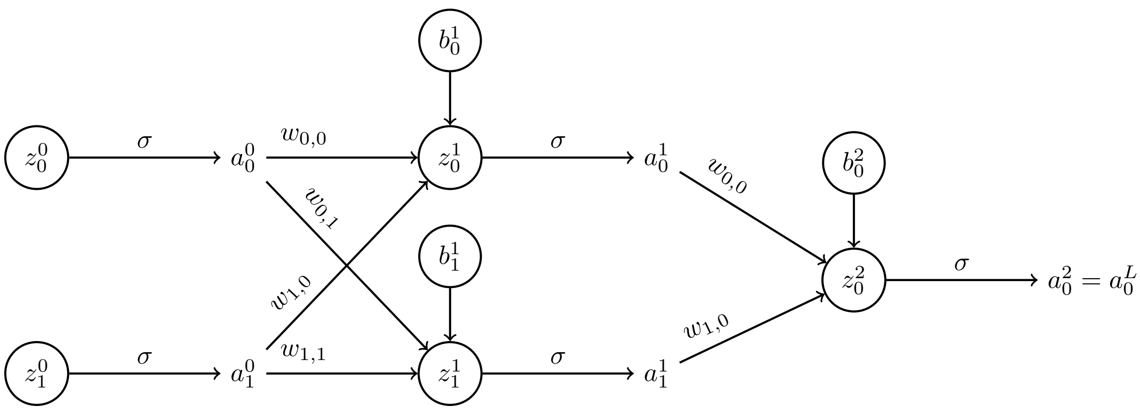
1 Feed-Forward

1.1 Grafik



1.2 Formeln

$$z_0^1 = a_0^0 \cdot w_{0,0} + a_1^0 \cdot w_{1,0} + b^1$$

allgemein:

$$z_0^1 = b^1 + \sum_{j=0}^{n} a_j^0 \cdot w_{j,0}$$

$$a_j^l = \sigma(z_j^l)$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

1.3 Matrizendarstellung

$$\vec{z}^l = \begin{pmatrix} z_0^l \\ z_1^l \\ ... \\ z_j^l \end{pmatrix} \qquad \vec{a}^l = \begin{pmatrix} a_0^l \\ a_1^l \\ ... \\ a_j^l \end{pmatrix} \qquad \vec{b}^l = \begin{pmatrix} b_0^l \\ b_1^l \\ ... \\ b_j^l \end{pmatrix} \qquad W^l = \begin{pmatrix} w_{0,0}^l & w_{1,0}^l & ... & w_{j,0}^l \\ w_{0,1}^l & w_{1,1}^l & ... & w_{j,1}^l \\ ... & ... & ... & ... \\ w_{0,i}^l & w_{1,i}^l & ... & w_{j,i}^l \end{pmatrix}$$

$$\vec{a}^l = \vec{z}^{l-1} \cdot W^l + \vec{b}^l$$

$$\begin{pmatrix} a_0^l \\ a_1^l \\ ... \\ a_j^l \end{pmatrix} = \begin{pmatrix} z_0^{l-1} \\ z_1^{l-1} \\ ... \\ z_j^{l-1} \end{pmatrix} \cdot \begin{pmatrix} w_{0,0}^l & w_{1,0}^l & ... & w_{j,0}^l \\ w_{0,1}^l & w_{1,1}^l & ... & w_{j,1}^l \\ ... & ... & ... & ... \\ w_{0,i}^l & w_{1,i}^l & ... & w_{j,i}^l \end{pmatrix} + \begin{pmatrix} b_0^l \\ b_1^l \\ ... \\ b_j^l \end{pmatrix}$$

$$\begin{pmatrix} a_0^l \\ a_1^l \\ ... \\ a_j^l \end{pmatrix} = \begin{pmatrix} w_{0,0}^l \cdot z_0^{l-1} + w_{1,0}^l \cdot z_1^{l-1} + ... + w_{j,0}^l \cdot z_j^{l-1} \\ w_{0,1}^l \cdot z_0^{l-1} + w_{1,1}^l \cdot z_1^{l-1} + ... + w_{j,1}^l \cdot z_j^{l-1} \\ ... \\ w_{0,i}^l \cdot z_0^{l-1} + w_{1,i}^l \cdot z_1^{l-1} + ... + w_{j,i}^l \cdot z_j^{l-1} \end{pmatrix} + \begin{pmatrix} b_0^l \\ b_1^l \\ ... \\ b_j^l \end{pmatrix}$$

2 Backpropagation

2.1 Kosten Funktion

$$C_j = (a_j^L - y_j)^2$$

$$C = \sum_{j=0}^{n} (a_j^L - y_j)^2$$

2.2 Ableitung der Kosten Funktion

$$\frac{dC_j}{dw_{j,i}^l} = 2 \cdot (a_j^L - y_j) \cdot \frac{da_j^L}{dw_{j,i}^l}$$

$$\frac{dC_j}{db_j^l} = 2 \cdot (a_j^L - y_j) \cdot \frac{da_j^L}{db_j^l}$$

2.3 Ableitung der sigmoid Funktion

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$\frac{d}{dx}\sigma(x) = \frac{0 \cdot 1 + e^{-x} - 1 \cdot e^{-x} \cdot -1}{(1 + e^{-x})^2} = \frac{e^{-x}}{(1 + e^{-x})^2} = \frac{1 + e^{-x} - 1}{(1 + e^{-x})^2}$$

$$= \frac{1 + e^{-x}}{(1 + e^{-x})^2} - \frac{1}{(1 + e^{-x})^2} = \frac{1}{1 + e^{-x}} - \frac{1}{(1 + e^{-x})^2}$$

$$= \frac{1}{1 + e^{-x}} \cdot (1 - \frac{1}{1 + e^{-x}}) = \sigma(x) \cdot (1 - \sigma(x))$$