ECON 613 Applied Econometrics Homework2

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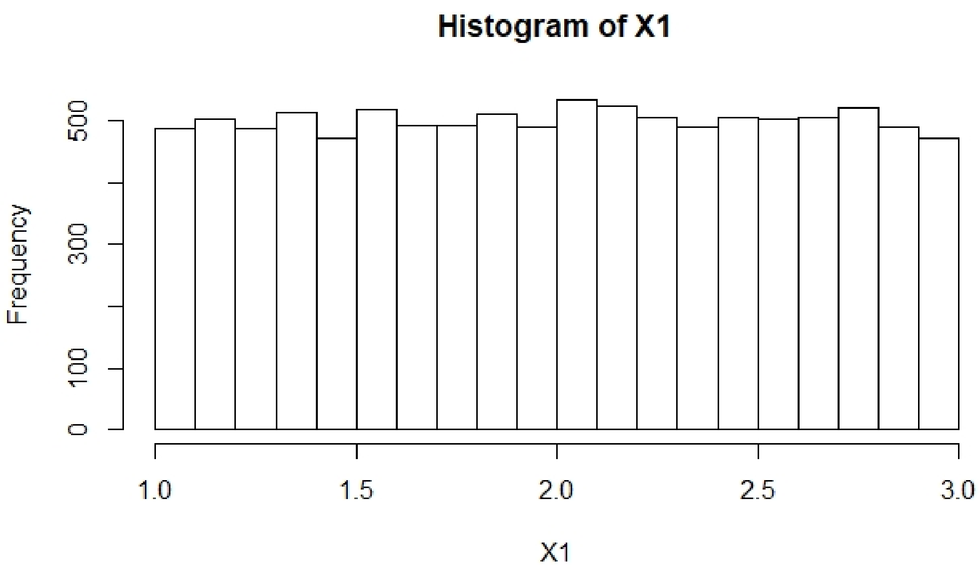
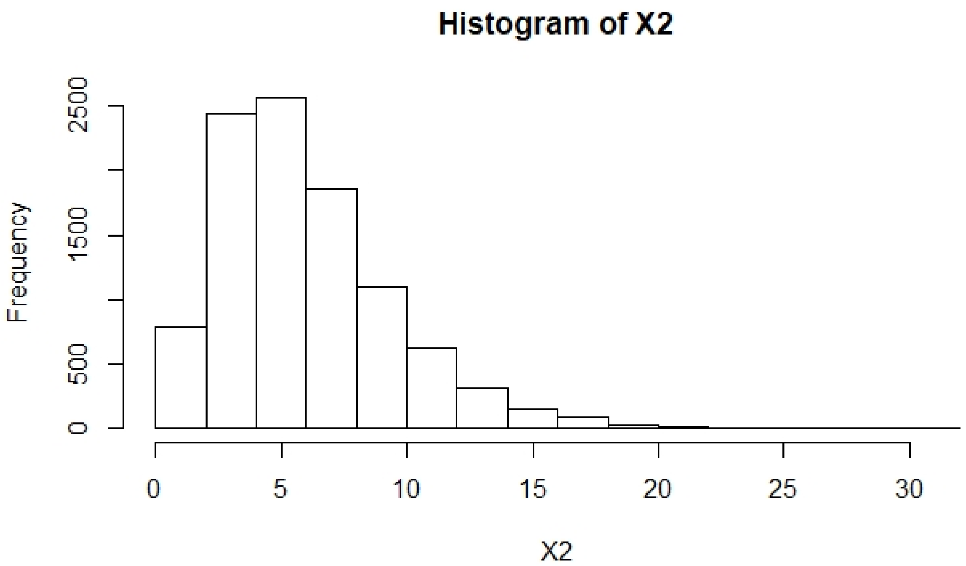
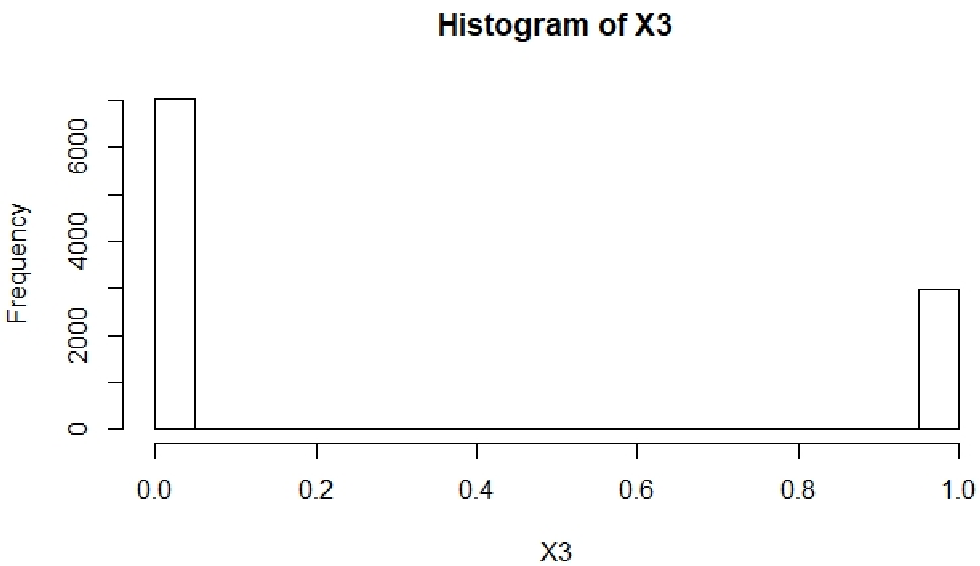
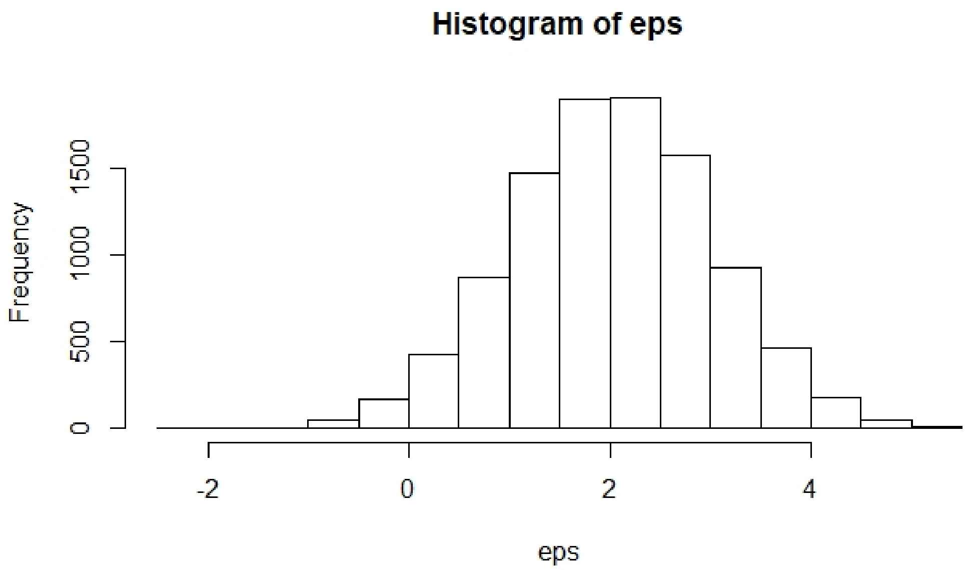
Exercise 1 Data Creation

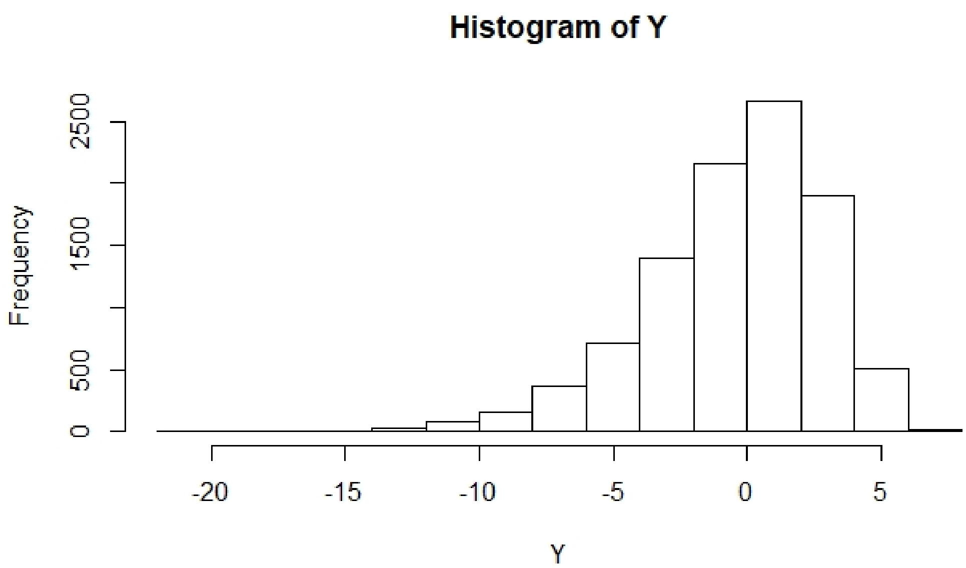
Histogram of the simulated variables will be reported here

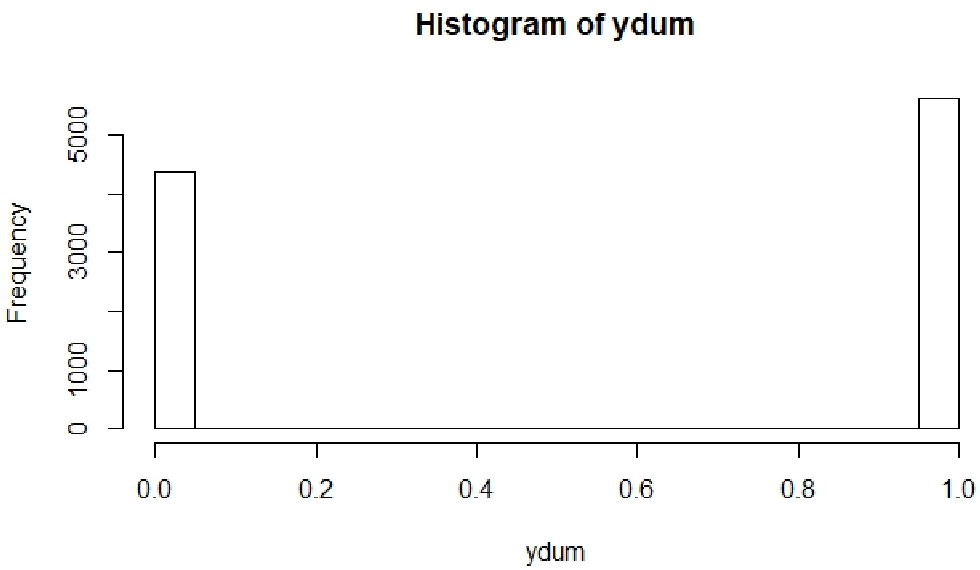



Histogram of X2
Frequency
0
500
1500
2500
0
5
10
15
20
25
30
X2


Histogram of X3
Frequency
0
2000
4000
6000
0.0
0.2
0.4
0.6
0.8
1.0
X3


Histogram of eps
Frequency
0
500
1000
1500
-2
0
2
4
eps


Histogram of Y
Frequency
0
500
1500
2500
-20
-15
-10
-5
0
5
Y


Histogram of ydum
Frequency
0
1000
3000
5000
0.0
0.2
0.4
0.6
0.8
1.0
ydum

Exercise 2 OLS

- Correlation between Y and X1

Correlation between Y and X1 is approximately 0.2018198 which is way less than the actual value of 1.2. This is because correlation reflects only how much Y and X1 move in accordance with each other, not the marginal effect of X1. From exercise 1, we know that $Y = 1.2X1 - 0.9X2 + 0.1X3 + eps)$, X1 explains only some portion of Y, so the correlation of 0.2018 is appropriate. In addition, the value of correlation is between -1 to 1.

- OLS Coefficients

Using formula $\hat{\beta} = (X'X)^{-1}X'Y$, estimation result is shown in column 1 of Figure 1. The second column of the figure is the result obtained from R built-in package. Both of the estimates for dependent variables, X1, X2 and X3, are very similar, and they are also close to the actual value. Notice that the coefficient for the constant term is close to 2.5 which is 0.5, the actual constant, plus 2, the mean of eps we used to construct Y.

Figure 1: OLS Coefficient

	Standard formular	R OLS package
c	2.4866583	2.48667726
X1	1.2181458	1.21809470
X2	-0.8984368	-0.89860058
X3	0.0810096	0.08064399

- OLS Standard error

Standard error from each method is reported in Figure 2. Overall, all methods provide similar results. R built-in package gives the closest SE to the Standard formula $(SE(\hat{\beta}) = \sigma^2(X'X)^{-1})$. For bootstrap, the larger the number of replications is the closer the value of SE to the one from Standard formula. This is expected as the standard formula is derived from asymptotic distribution assumption which is comparable to Bootstrap with replication $\rightarrow \infty$.

Figure 2: OLS Standard error

	Standard formular	Bootstrap 49	Bootstrap 499	R OLS package
c	0.040600003	0.039812673	0.03919052	0.040606137
X1	0.017439030	0.016871321	0.01750368	0.017441654
X2	0.002894556	0.003140135	0.00302345	0.002895197
X3	0.021873127	0.019786333	0.02143240	0.021876482

Exercise 3 Numerical Optimization

The function that return probit's likelihood and implementation of steepest ascent algorithm is written in the associated R script file. The result of the optimization is reported in Figure 3. The optimization result in a coefficients that have correct sign. However, the relative magnitudes differ from the actual parameter, which is probably caused by my algorithm having large tolerance level, to limit run time, so the steepest ascent stop searching before the true maximum is found.

Figure 3: Probit Coefficient from steepest ascent optimization

	Steepest ascent	Expected Coefficient
intercept	1.0182453	2.5
X1	1.3810255	1.2
X2	-0.6464752	-0.9
X3	0.1988064	0.1

Exercise 4 Discrete Choice

After I wrote functions for calculating probit likelihood, logit likelihood and least square loss, I optimize the models by nlm package. Then I compare the coefficients and SE from optimization with those obtained from from R built-in packages, glm and lm. Coefficients and SE from these 2 processes are extremely close as reported in Figure 4.

In terms of sign of coefficients, coefficient for each variable have same sign in all 3 models. X1 and X2 have positive effect on probability of ydum = 1; if X1 or X2 increase, ceteris paribus, ydum will have **more** chance of being 1. On the contrary, X3 has negative coefficient; if X3 increases, ydum will have **less** chance of being 1.

For significance, all of the coefficients in all 3 models are more than 1.96 times their respective SE. Hence, they are all significant at 95% confidence level.

Magnitude wise, without marginal effect calculation, we can only interpret the magnitude of linear probability model because its coefficient reflects marginal effect while probit and logit coefficient reflect the effect on latent variable. From linear probability model, an increase of 1 unit of X1 **increases** the chance that ydum will be 1 by 0.142%, and an increase in X3 by 1 unit **increases** the chance that ydum will be 1 by 0.02%. In contrast, a unit increase in X2 is associated with the **reduction** in the chance ydum will be 1 by 0.104%.

Figure 4: Comparing optimized model's coefficient and R built-in packages

	(Intercept)	X1	X2	X3
Probit from optimization:Coefficient	2.98529288	1.12754028	-0.88193206	0.09332700
Probit from optimization:SE	0.09886172	0.04248176	0.01768615	0.04669540
Probit from glm package	2.98527659	1.12753085	-0.88192555	0.09332476
Probit from glm SE	0.09883073	0.04248464	0.01773748	0.04670450

	(Intercept)	X1	X2	X3
Logit from optimization:Coefficient	5.3809672	2.03684861	-1.59082523	0.16723885
Logit from optimization:SE	0.1864089	0.07905283	0.03567548	0.08438981
Logit from glm package	5.3809347	2.03683039	-1.59081262	0.16723335
Logit from glm SE	0.1863976	0.07905583	0.03568045	0.08438846

	(Intercept)	X1	X2	X3
LP from optimization:Coefficient	0.89460771	0.142395476	-0.1040294247	0.019792208
LP from optimization:SE	0.01315352	0.005649866	0.0009377730	0.007086417
LP from glm package	0.89459087	0.142398938	-0.1040276778	0.019792000
LP from glm SE	0.01348599	0.005792672	0.0009615445	0.007265554

Exercise 5 Marginal Effects

Compute marginal effect

Using average marginal effects (ME) in sample method, ME can be compute by this process.

- Compute $y\hat{dum}_i = X_i\beta$ for each observatons.
- Compute pdf, $F'(X_i\beta)$, which is the first derivative of $F(X\beta)$ for each value of $X_i\beta$. Probit = normal distribution pdf, and Logit = logistic distribution pdf.
- Compute average value of all pdf $\frac{1}{n}\sum(F'(X_i\beta))$
- Compute marginal effect by $ME = \frac{1}{n}\sum(F'(X_i\beta))\beta$

The result is demonstrated in Figure 5. As expected, marginal effect of both probit and logit model are close to each other.

Figure 5: Marginal Effect of Probit and Logit model

	Probit ME	Logit ME
Intercept	0.36780907	0.36639762
X1	0.13892089	0.13869189
X2	-0.10866023	-0.10832153
X3	0.01149854	0.01138753

Compute standard deviation

The SD for marginal effect calculated by both delta method and bootstrap is reported in Figure 6. Overall, both practices give similar result. However, delta method give smaller SD, which is expected because delta method calculates asymptotic SD which assume n approaches infinity. The steps taken to obtain result from delta method is outlined below.

- Delta method
  1. Compute $y\hat{dum}_i = X_i\beta$ for each observatons.
  2. Compute pdf, $F'(X_i\beta)$, which is the first derivative of $F(X\beta)$ for each value of $X_i\beta$. Probit = normal distribution pdf, and Logit = logistic distribution pdf.
  3. Compute Jacobian matrix, $J = \frac{1}{n}F'(X\beta)X$
  4. Compute coefficient var-cov matrix from Probit and Logit model, V
  5. Compute Var(ϵ), $\sigma^2 = JVJ^t$
  6. Compute SD of each coefficient β_i ME by $SD(\beta_i) = \sqrt{\frac{\sigma^2}{Var(x_i)}}$

Figure 6: SD of Marginal Effect

	Probit SD Delta	Probit SD Bootstrap	Logit SD Delta	Logit SD Bootstrap
X1	0.0045610996	0.004319791	0.0025214989	0.004375671
X2	0.0007570338	0.001133105	0.0004185087	0.001121910
X3	0.0057204524	0.005831670	0.0031624204	0.005794622