### Topic

Prediction of the burial status of transmembrane residues of helical membrane proteins (with support vectors)

by Thorsten Will

### Overview

#### Introduction

- helical membrane proteins
- needed definitions: burial status ,(r)SASA
- a two step architecture: the TMX method

#### Statistical methods used

- prediction:
  - basic principles of a SVM
  - Support Vector Regression in detail
- assessment:
  - measuring regression performance
  - cross-validation

#### The practical part

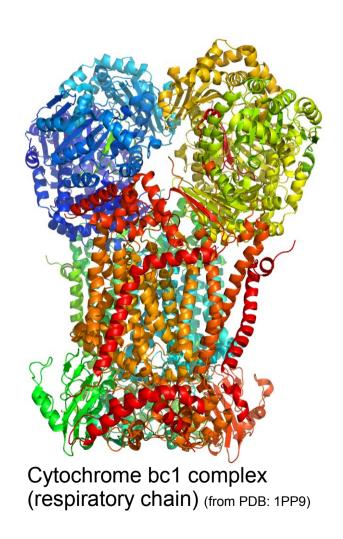
- the dataset and the problem
- workflow
- results and discussion

### Introduction Why helical membrane proteins are important

#### Facts:

 crucial role in fundamental cellular processes

- account for 20-30% of the ORFs of sequenced genomes



### Introduction Why helical membrane proteins are interesting

#### Facts:

 crucial role in fundamental cellular processes structure determination desirable

so far very difficult with current experimental techniques

- account for 20-30% of the ORFs of sequenced genomes

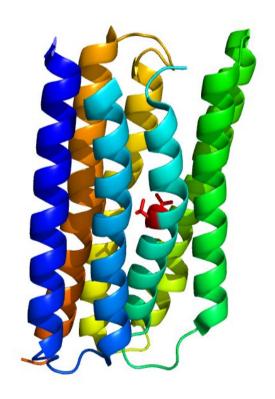
Less than 1% of the proteins with known structure are HMPs!

sequence-based predictors invaluable!

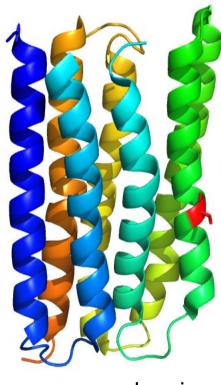
### Introduction What is the burial status and why is it useful

In the case of membrane proteins:

Buried in the protein core vs exposed to the membrane



two burried threonine



an exposed serine

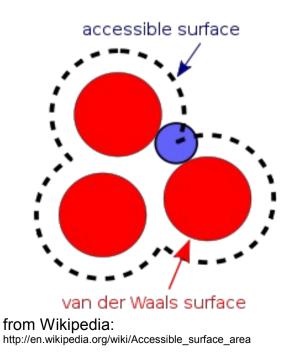
Example:

a bacterial rhodopsin from the dataset (from PDB: 1xio)

### Introduction Definition of the rSASA

#### (S)ASA: (Solvent) Accessible Surface Area

- the accessible area of the surface for a solvent of a specific size



rSASA: relative SASA

- normalized measurement for the SASA:
  - → dividation by reference values
- rSASA = 0.00 → residue defined as burried

### Introduction The TMX method

few words about...

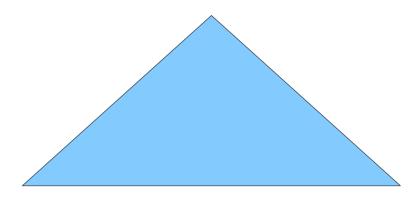
### **TMX**: TransMembrane eXposure

A two step approach

- 1. positional score
  - 2. classification

# Introduction SVM in general

**SVM**: Support Vector Machines



**SVC**: Support Vector Classifier

**SVR**: Support Vector Regression

### Statistical methods used SVM basics: the formal problem simplified

**SVC:** classification

 $\{(x_1, y_1), \dots, (x_n, y_n)\} \subset \mathcal{X} \times \{-1, +1\}$ 

Input / training data

SVR: regression

$$\{(x_1, y_1), \dots, (x_n, y_n)\}\subset X\times\mathbb{R}$$

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Search for a function  $f(x_i) \approx y_i$  for "many" i.  $\rightarrow$  construct specific **hyperplane H** 

H does best possible class separation

$$H:\langle w, x \rangle + b$$
  
 $w \in X, b \in \mathbb{R}$ 

H never deviates larger than  $\varepsilon$ , so called  $\varepsilon$ -regression

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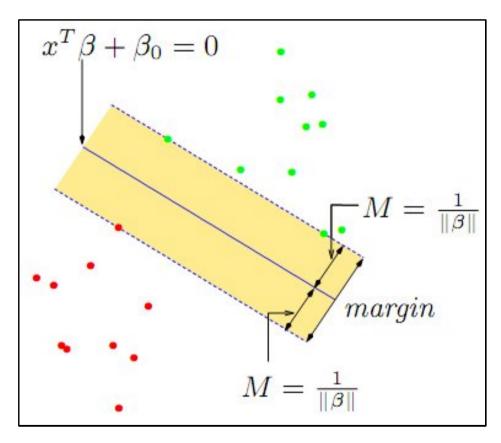
$$f(x) = sgn(\langle w, x \rangle + b)$$

Yielding predictions for unknown  $x \in X$ 

$$f(x) = \langle w, x \rangle + b$$

### Statistical methods used SVM basics: optimal hyperplanes

How to choose the **best** hyperplane



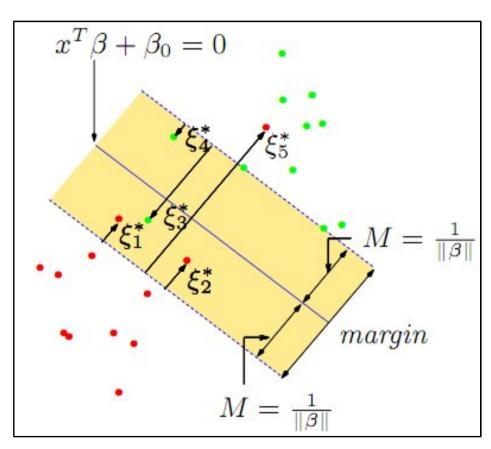
from "The elements of statistical learning"

Choose the hyperplane that maximizes the margin

 $\rightarrow$  minimize  $||\beta||$ 

### Statistical methods used SVM basics: introducing slack variables

#### Getting around with **non-feasible** problems



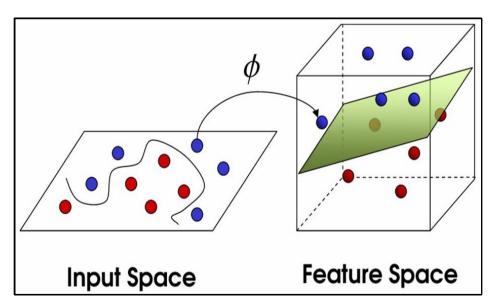
from "The elements of statistical learning"

**Allow violations** of the margin constraint but minimize the extent of the violations

### Statistical methods used SVM basics: the kernel-trick

#### Getting non-linear

Every dot product is replaced by a **non-linear kernel function**→ input space transported into a high-dimensional feature space

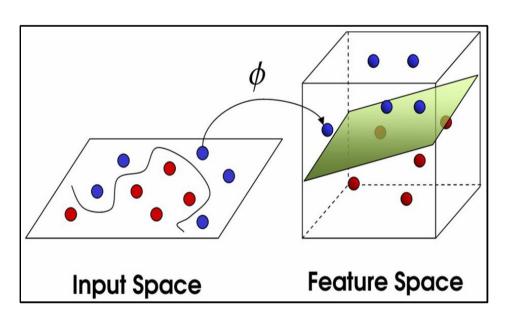


from www.imtech.res.in: rice blast prediction http://www.imtech.res.in/raghava/rbpred/svm.jpg

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from www.imtech.res.in: rice blast prediction http://www.imtech.res.in/raghava/rbpred/svm.jpg

#### Some kernel functions:

polynomial (homogenous):

$$k(x, x') = \langle x, x' \rangle^d$$

polynomial (inhomogeneous):

$$k(x, x') = (\langle x, x' \rangle + 1)^d$$

radial basis function:

$$k(x, x') = e^{-\gamma * \langle x - x', x - x' \rangle}$$
 for  $\gamma > 0$ 

sigmoid:

$$k(x, x') = \tanh(\kappa \langle x, x' \rangle + c)$$
for some  $\kappa > 0 \land c < 0$ 

### Statistical methods used SVR in particular: from abstract to application

#### simplified:

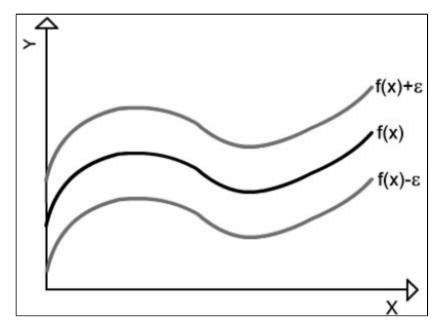
$$\{(x_1, y_1), \dots, (x_n, y_n)\} \subset X \times \mathbb{R}$$



#### Seeking for best w,b:

minimize 
$$\frac{1}{2}||w||^2$$

subject to 
$$\begin{cases} y_i - \langle w, x_i \rangle - b \leq \epsilon \\ \langle w, x_i \rangle + b - y_i \leq \epsilon \end{cases}$$



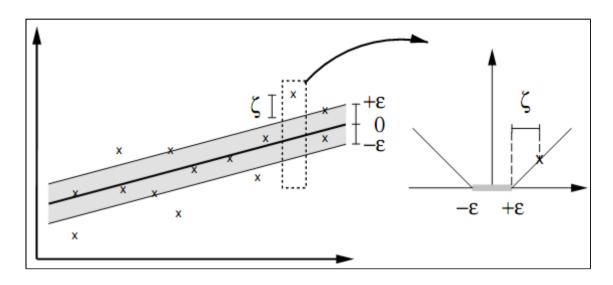
from "Online SVR", Parrella http://onlinesvr.altervista.org/Theory/Images/03-17.png

### Statistical methods used SVR in particular: from abstract to application

Adding the "soft-margin" loss function to obtain the ε-regression:

minimize 
$$\frac{1}{2} ||w||^2 + C \sum_{i=1}^{n} (\xi_i + \xi_i^*)$$

subject to 
$$\begin{cases} y_i - \langle w, x_i \rangle - b \leq \epsilon + \xi_i \\ \langle w, x_i \rangle + b - y_i \leq \epsilon + \xi_i^* \\ \xi_i, \xi_i^* \geq 0 \end{cases}$$



from "A Tutorial on SVR", Smola / Schölkopf

Quadratic Programming Problem

Behaviour: ε-insensitive loss-function:

$$|\xi|_{\epsilon} := \begin{cases} 0 & \text{for } |\xi| \le \epsilon \\ |\xi| - \epsilon & \text{otherwise} \end{cases}$$

### Statistical methods used SVR in particular: building the Lagrangian

$$L := \frac{1}{2} ||w||^2 + C \sum_{i=1}^{n} (\xi_i + \xi_i^*)$$
$$- \sum_{i=1}^{n} (\eta_i \xi_i + \eta_i^* \xi_i^*)$$
$$\sum_{i=1}^{n} \alpha_i (\zeta_i + \xi_i - y_i + w_i^* + y_i^*) + \frac{1}{2} \alpha_i^* (\zeta_i + \xi_i - y_i + w_i^* + y_i^*)$$

objective / primal function

$$-\sum_{i=1}^{n} \alpha_{i}(\epsilon + \xi_{i} - y_{i} + \langle w, x_{i} \rangle + b)$$

$$-\sum_{i=1}^{n}\alpha_{i}^{*}(\epsilon+\xi_{i}^{*}+y_{i}-\langle w,x_{i}\rangle-b)$$

 $\alpha_i, \alpha_i^*, \eta_i, \eta_i^* \ge 0$  are Langrangian multipliers  $w, b, \xi_i, \xi_i^*$  are the primal variables

$$\partial_b L = \sum_{i=1}^n (\alpha_i^* - \alpha_i) = 0$$

$$\partial_w L = w - \sum_{i=1}^n (\alpha_i - \alpha_i^*) x_i = 0$$

$$\partial_{\varepsilon_{i}^{(*)}} L = C - \alpha_{i}^{(*)} - \eta_{i}^{(*)} = 0$$

### Statistical methods used SVR in particular: gaining insight

$$\partial_w L = w - \sum_{i=1}^n (\alpha_i - \alpha_i^*) x_i = 0$$

can be rewritten as  $w = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) x_i$ 



$$f(x) = \sum_{i=1}^{n} (\alpha_{i} - \alpha_{i}^{*}) \langle x_{i}, x \rangle + b$$

w can be **completely** described by a linear combination of the training patterns!

### Statistical methods used SVR in particular: optimization problem

Substitution yields the dual optimization problem:

maximize 
$$\begin{cases} -\frac{1}{2} \sum_{i,j=1}^{n} (\alpha_{i} - \alpha_{i}^{*})(\alpha_{j} - \alpha_{j}^{*}) \langle x_{i}, x_{j} \rangle \\ -\epsilon \sum_{i=1}^{n} (\alpha_{i} + \alpha_{i}^{*}) + \sum_{i=1}^{n} y_{i}(\alpha_{i} - \alpha_{i}^{*}) \end{cases}$$
subject to 
$$\sum_{i=1}^{n} (\alpha_{i} - \alpha_{i}^{*}) = 0 \land \alpha_{i}, \alpha_{i}^{*} \in [0, C]$$

easier Quadratic Programming Problem (solvable by several optimization algorithms)

### Statistical methods used Model assessment

A performance measure for regression:

#### **Pearson's Correlation Coefficient**

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}} = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} \in [-1,1]$$

### Statistical methods used Model assessment

A performance measure for regression:

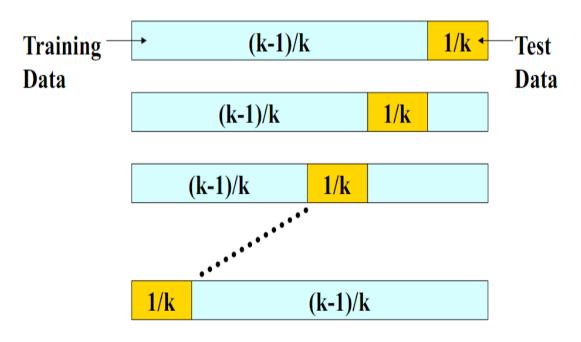
#### **Mean Squared Error**

$$MSE(f(X)) = E[(X - f(X))^{2}] = Var(f(X)) + Bias^{2}(f(X), X)$$
$$f(X): predictor for X$$

### Statistical methods used Model assessment

Estimating the prediction quality with limited data:

#### k-fold cross-validation



### The practical part The dataset and the problem

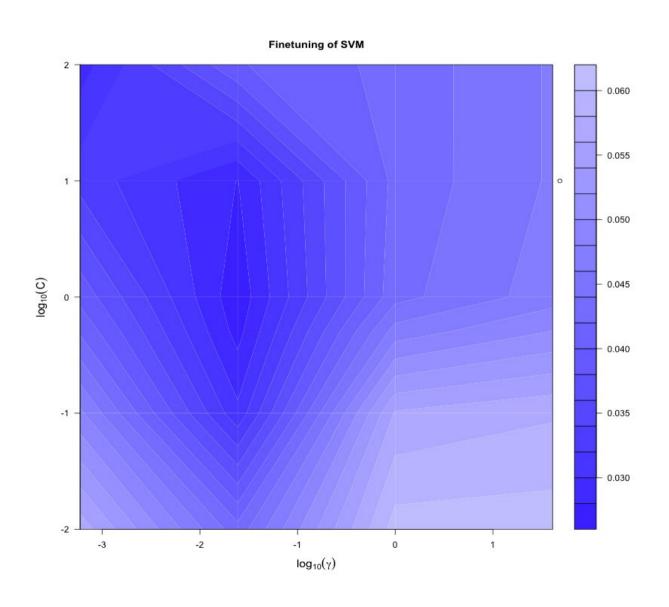
pdbid chain number type rsasa freq1 freq2 freq3 freq4 freq5 freq6 freq7 freq8 freq9 F 0.642 0.0 0.234043 0.0 0.0 0.531915 0.085106 0.148936 0.0

- 2595 residues with computed rSASA
  - of the transmembrane regions of 28 different proteins
- 41 features for each residue
  - frequencies per aa
  - PSIBLAST-score per aa
  - conservation-score

### The practical part Workflow

```
> summary(model)
Call:
svm.default(x = trainset[6:ncol(dataset)], y = trainset[5])
Parameters:
   SVM-Type: eps-regression
 SVM-Kernel: radial
      cost: 1
    gamma: 0.02439024
    epsilon: 0.1
Number of Support Vectors: 1914
```

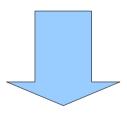
# The practical part SVR Setup



### The practical part Results

after 2595-fold cross-validation on training data:

Total Mean Squared Error: 0.02627267 Squared Correlation Coefficient: 0.4464628



Correlation of 0.668

# The practical part Accuracy per amino acid

Amino acid	Mean squared error [10 <sup>-2</sup> ]
R	0.055
Н	0.058
Q	0.100
D	0.243
N	0.530
Е	0.585
K	0.879
S	1.035
Т	1.307
G	1.789



Y	1.902
M	2.160
Р	2.571
W	2.603
С	2.872
Α	3.003
V	3.306
- 1	3.327
L	3.328
F	3.704



# The practical part Accuracy per amino acid

Amino acid	Mean squared error [10 <sup>-2</sup> ]	Burried [%]
R	0.055	100.0
Н	0.058	90.9
Q	0.100	100.0
V	3.306	39.9
- I	3.327	30.0
L	3.328	27.7
F	3.704	28.4

hydrophilic

hydrophobic

# The practical part Accuracy per amino acid

Amino acid	Mean squared error [10 <sup>-2</sup> ]	Burried [%]	Mean conservation score
R	0.055	100.0	1.673
Н	0.058	90.9	1.413
Q	0.100	100.0	1.040
V	3.306	39.9	-0.230
- 1	3.327	30.0	-0.255
L	3.328	27.7	-0.204
F	3.704	28.4	0.008

hydrophilic

hydrophobic

### The practical part Contribution of individual features

feature(s)	Correlation with rSASA
aa-freq.	0.0806
PSSM-score	0.1481
conservation	0.4393

Other probably interesting features:

hydrophobicity	0.2862
vdW - volumes	0.2044

# The practical part Adding new features to the model

Features used for prediction	Correlation of prediction and rSASA
standard	0.668
- conservation	0.644
+ hydrophobicity	0.671
+ hydrophobicity + VdW - volumes	0.669

### Summary

Membrane proteins are interesting for bioinformatics

Support Vector Machines are useful tools in machine learning

Features that are **most likely useful** in sequence based methods:

- → neighborhood (windowing)
- → profiles / PSSM
- → conservation

But, like always:

it is **not** about the more the better!

### Used literature

Hastie, Tibshirani, Friedman:

The Elements of Statistical Learning (sec. edition, 2008)

Park, Hayat, Helms:

Prediction of the burial status of transmembrane residues of helical membrane proteins (paper, BMC Bioinformatics, 2007)

Smola, Schölkopf:

A Tutorial on Support Vector Regression (paper, 2003)

# Thank you for your attention!

