Multidimensional Modelling of Cross-Beam Energy Transfer for Direct-Drive Inertial Confinement Fusion

(Hopefully soon to be Dr.) Philip W. X. Moloney

IMPERIAL

June 2024

Submitted in partial fulfilment of the requirements for the degree of
Doctor of Philosophy of Imperial College London

Department of Physics
Imperial College London
Prince Consort Road
London SW7 2AZ

Contents

List of Acronyms

List of Tables

List of Figures

1 The Interaction of Light with Plasma

This chapter shall introduce theoretical background relevent to the work conducted in this thesis. The main focus of the work is centered on improving the laser modelling in the CHIMERA **Rad-MHD!** (**Rad-MHD!**) code. Therefore, the theoretical framework for modelling both the plasma and the interaction of light with it is introduced.

Initially, the plasma state is defined and important length- and time-scales are provided. The kinetic and fluid descriptions of plasma are introduced and the validity domain of each framework is discussed, with particular reference to typical conditions for laser-plasma interactions. Additional physics packages of the fluid code, CHIMERA, which are utilised in later chapters, are introduced. A model for kinetic heatflow is desirable in fluid codes which model laser-plasma interactions, therefore although CHIMERA does not have this capability, some basic theory of kinetic heatflow is highlighted.

An additional aim of the work was to include a **CBET!** (**CBET!**) model into the new CHIMERA laser package. **LPIs!** (**LPIs!**) such as **CBET!** are multi-wave coupling phenomena, and therefore a basic description of waves in plasma is provided. The dispersion relation of the light waves in plasma is also derived. Beginning with the full wave equation and then introducing successive assumptions which are broadly satiisfied in typical laser-produced **ICF!** (**ICF!**) plasmas, the equations of ray-tracing are derived. Important absorption processes are then outlined, particularly **Inv-Brem!** (**Inv-Brem!**), which is the dominant mechanism on the largest **ICF!** facilities in the world today. Finally, the basic theory of **LPIs!** is provided, particularly **CBET!** and its relevance for direct-drive **ICF!**.

1.1 Basic Plasma Physics

As stated in Chap. **??**, thermonuclear fusion requires the fuel to exist at significant temperatures, which are well above ionisation energies. Therefore, the fuel configuration in these fusion expeiments is a plasma. Formally, a plasma is defined as a quasi-neutral, ionised gas which exhibits collective behaviour. The charged particles within a plasma interact via the long-range Coulomb force, and thus undergo many simultaneuos interactions with the other particles. This leads to a variety of collective phenomena such as the plasma-waves described in Sec. **??**. Quasineutrailty of the plasma means that, when observed at a length-scale L, the plasma has no net charge,

$$\Sigma_\alpha q_\alpha N_\alpha = 0, \tag{1.1}$$

where N_α is the number of particles of species α, with charge q_α, in the cube with volume $V = L^3$. For a 'single-species' plasma[1] with average ionisation state Z, this implies,

$$n_e = Z n_i, \tag{1.2}$$

where n_e and n_i are the number densities of electrons and ions respectively.

Quasineutrailty because the particles in the plasma are free to move due to forces they experience. Thus if a local charge imbalance occurs, the electrons, which respond faster than the ion population due to their lower mass, move to rebalance this field and restore quasineutrailty. This electron relocation to eliminate local electric fields is known as Debye-screening. The length scale below the electron population cannot effectively screen the charges sets the length scale of quasineutrality and is known as the Debye-length,

$$\lambda_D^2 = \frac{\epsilon_0 k_B T_e}{n_e e^2}, \tag{1.3}$$

where T_e is the electron temperature, ϵ_0 is the permitivity of free space, k_B is the Boltzmann constant and e is the electron charge. This is valid if there is a large number of particles in the Debye sphere, $n_e L^3 \gg 1$. The timescale of the charge relocation is of particular importance to the interaction of light with plasmas. Light waves are an oscillating electric field, which charged particles in the plasma can respond to. If the particles are able to respond quickly enough, they can therefore influence the propagation of the light and ultimately the plasma can become opaque. The oscillation timescale can be derived by considering a uniform assembly of quasineutral plasma and then displacing the electron population from the ions by a small distance δx along the x-axis. The electric field which develops within the plasma is thus,

$$E_x = \frac{n_e e \delta x}{\epsilon_0}, \tag{1.4}$$

leading to a restoring force $F_x = -eE_x$ on each electron. Solving Newton's second Law demontsrates that, when thermal motion of the electron population is ignored[2], oscillations of

[1] Single-species here means that there is only a single type of ion.

[2] Inclusion of thermal motion leads to pressure, which acts as an restoring force, and yields the dispersion relation for an **EPW!** (**EPW!**).

the electrons occur at the 'plasma frequency',

$$\omega_p^2 = \frac{n_e e^2}{m_e \epsilon_0}. \tag{1.5}$$

If forcing oscillations occur at a frequency which is lower than ω_p, then the electrons move rapidly enough to nullify the field. Consider light with frequency ω, which is incident normal to a plasma density gradient Because $\omega_p \propto n_e$, the light is able to propagate until it reaches the density where $\omega = \omega_p$, which is known as the ciritcal density,

$$n_{\text{cr}} = \frac{m_e \epsilon_0 \omega^2}{e^2}. \tag{1.6}$$

After reaching the critical density, the field of the light decays exponentially as an evanescent wave, but cannot propagate.

1.2 Kinetic and Fluid Formulations of Plasmas

Idealised computational modelling of a plasma state would solve the long-range electromagnetic interaction between every pair of particles at all times. However, this very rapidly becomes intractable due to the large number N of particles and the $\mathcal{O}(N^2)$ scaling of interactions to solve. Reduced frameworks must thus be devised with which to analyse and predict the behaviour of the plasma state. Plasmas are divided into two broad classifications. When in local thermal equilibrium, the plasma is often described as 'thermal' and the fluid formulation is an adequate description. When this is not true however, for instance a particular subset of particles are heated at a rate, which is much faster than thermalising collision timescales, this subset of the system is described as 'non-thermal' ot kinetic. In this case the fluid formulation is an inadequate description and higher fidelity tools must be used to describe the evloution of the system.

When a kinetic description of a plasma is required, the distribution function, $f_\alpha(\mathbf{x}, \mathbf{v}, t)$ is used to describe the state of particle species α. It provides a statistical description of the number density of particles, which inhabit a phase-space, $\mathbf{x}$-$\mathbf{v}$, at time t. If the system evolves on a time-scale much lower than the collision time, then these collisions between particles act to relax the distribution function toward a Maxwellian,

$$f_{\alpha,\text{Max.}}(v) = n_\alpha \left(\frac{1}{2\pi v_{\text{th}}^2} \right)^{1.5} e^{-v^2/(2v_{\text{th}}^2)}, \tag{1.7}$$

where $v_{\text{th}} = \sqrt{k_B T_\alpha / m_\alpha}$ is the thermal speed of the species with temperature and mass T_α and m_α, respectively. The fraction outside the exponential is set such the integral over velocity space yields the number density of the species. Typically, for bulk of the target configuration throughout a laser-produced **ICF!** implosion, the assumption of a Maxwellian distribution is close to accurate although there are notable exceptions. For instance, DT fusion products have energies much higher than thermal energies and are also monoenergetic. **LPIs!**

also generate energetic electron populations which are able to range through the implosion due to their low collisionality. Laser-heated plasmas also typically exhibit steep density and temperature gradients near the ablation surface, such that collisions, and therefore collision processes such as transport of thermal energy, do not act locally.

1.2.1 The Vlasov Equation

The evolution of the distribution function for each species indiviudally, is described by the Vlasov equation,

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}.\nabla_x f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{collisions}}, \tag{1.8}$$

where q_α is the charge of the species, **E** and **B** are the macroscopic electric and magnetic fields[3], respectively, and ∇_x and ∇_v are gradients with respect to postiion and velocity coordinates respectively. The equation describes the conservation of phase-space particle density. The collision operator on the right hand side of Eq. **??** describes the action of microscopic fields, which arise due to the random motion of the charged plasma particles. Evolution of the macroscopic fields is governed by Maxwell's equations,

$$\nabla.\mathbf{E} = \rho_{\text{charge}}\epsilon_0, \tag{1.9}$$

$$\nabla.\mathbf{B} = 0, \tag{1.10}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1.11}$$

$$\nabla \times \mathbf{E} = \frac{1}{\mu_0\epsilon_0}\frac{\partial \mathbf{E}}{\partial t} + \mu_0\mathbf{j}, \tag{1.12}$$

$$\tag{1.13}$$

where μ_0 is the permeability of free space, ρ_{charge} is the charge density and **j** is the current density.

x and v of all particles described by f. bulk collision operator and lorentz force give vlasov equation. Solved by VFP.

1.2.2 The Fluid Equations

Take moments of vlasov, assuming equilibrium, to get fluid equations. Give assumptions. Describe closure problem. Say how solved in Chimera. Knudsen number describes how kinetic it is.

1.2.3 Radiation Transport

Additional mechanism which described emission and absorption of radiation by plasma. Important for high temperatures and densities of ICF. Say how solved in Chimera.

[3] Macroscopic here means on a scale larger than Debye shielding.

1.2.4 Magnetohydrodynamics

Say electric fields on large scales don't exist in plasma because they are dielectric. If magnetic field are important, then extend the fluid equations to include the effect of B on the above equations.

1.2.4.1 Ideal MHD

Get lorentz force on plasma. Solve for B using Maxwell. Solve for j using Ohms law. Magnetic tension and pressure. Give plasma beta.

1.2.4.2 Magnetised Heatflow

Give Hall parameters. Say they describe transport processes like heatflow.

1.2.4.3 Resistive MHD

Plasma often not perfectly conducting so B field moves without flow. Describe Rm. Say when important.

1.2.4.4 The Nernst Effect

Hmmm, describe and say when important.

1.2.4.5 The Biermann Battery Effect

Hmmm, describe and say when important.

1.2.5 Kinetic Heatflow

Say that in laser heated plasmas, often have Knudsen significant due to high temps and low densities. Therefore, accurate modes include kinetic model for heatflux. Either VFP or SNB etc, say roughly what they do.

1.3 Waves in Plasma

3 waves can exist in plasma without B. First look at plasma waves, i.e. not a light wave, both of which are longitudinal.

1.3.1 Plasmas as a Dielectric Medium

Talk about how plasmas are dielectric, therefore described by susceptibilties. Can get dispersion relations by susceptibilities and outline process. Multi-species effects.

1.3.2 Plasma Waves

Get disp rels of IAW and EPW and say what they both are physically.

1.3.3 Light Waves

These are transverse waves and therefore don't create space-charge separation, unlike longitudinal waves. Get disp rel and show wpe comes out. Give physical interpretation.

1.4 Propagation of Light in a Plasma

Want to described light propagating through plasma in limit of typical ICF configurations. Weakly focussing, moderate intensities etc.

1.4.1 Paraxial Approximation

Weakly focussing limit. Give equations, interpretations and validity.

1.4.2 WKB Approximation

Uniform medium limit. Give equations, interpretations and validity. Give Airy example - ie not valid nearby turning point.

1.4.3 Ray Tracing

Give equations, interpretations and validity. State can be used for any kind of wave where valid. Say how and why used for direct drive and typical frozen plasma assumption.

Talking about validity region, give extra bits can solve like ray amplitude.

1.5 Absorption of Light in a Plasma

ICF we want to give laser energy to plasma to drive implosion, therefore need to talk about absorption.

1.5.1 Inv Brem

Introduce all bits to get NRL formaularly equation. Talk about Langdon as well.

1.5.2 Resonance Absorption

Introduce

1.5.3 ICF Relevant Absorption comparison

Say inv brem goes up relatively at larger scales and shorter wavelengths. Prefered to resonance absorption because bulk population gets energy. Therefore use frequency tripled light.

1.6 Laser Plasma instabilities

1.6.1 Ponderamotive Force

Introduce and say why it happens roughly.

1.6.2 Three-Wave Coupling

Give the general picture, i.e. ponderamotive, perturbation, driven plasma wave. Give momentum and energy conservation. List all types seeded by an EMW.

1.6.3 Cross-Beam Energy Transfer

Derive something to an appropriate level of detail. Talk about how it is in frame of plasma, flow velocities change this, mach 1 surface etc. Give general picture, i.e. sidescatter and backscatter and what it does in ICF.

1.6.3.1 Linear Gain Theory

Say that we use this for raytracing. Assume uniform plasma and can solve plasma response either by linearising fluid or kinetic equations.

1.6.3.2 Effect of Polarisation

Say that LPIs are affected by polarisation via ponderatmotive beat. Only parallel polarisations interact. Important on OMEGA due to polarisation smoothing, leads to mode-1.

1.6.3.3 Langdon Effect on CBET

Say that Langdon affects cbet. Reduced model to alter linear gain. Could potentially explain why indirect ICF models require a clamp.

1.6.4 Mitigation of Laser Plasma Instabilities

Say its coherence spatially, temporally and spectrally, so break this to stop LPIs. Mention stud pulses and say zooming for CBET.

Mainstream approach is bandwidth. Talk about studies showing that bandwidth should mitigate CBET. Talk about experimental progress eg FLUX laser at LLE.

1.7 Summary

Summarise that introduced descriptions of plasmas, particularly fluid framework solved by CHIMERA. Talked about waves in plasmas and their physical interpareatations. Talked about how light propagates, assumptions etc, used for raytracing in next section. LPIs, particularly CBET, modelling it is focus of next chapter.

Appendices

Permissions