# Multidimensional Modelling of Cross-Beam Energy Transfer for Direct-Drive Inertial Confinement Fusion

(Hopefully soon to be Dr.) Philip W. X. Moloney

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Department of Physics
Imperial College London
Prince Consort Road
London SW7 2AZ

# Contents

# List of Acronyms

**Rad-MHD** Radiative-Magnetohydrodynamics

**MHD** Magnetohydrodynamics

**HEDP** High Energy Density Physics

**LPIs** Laser-Plasma Instabilities

**CBET** Cross-Beam Energy Transfer

**ICF** Inertial Confinement Fusion

**PiC** Particle in Cell

**EPW** Electron Plasma Wave

**IAW** Ion Acoustic Wave

**Inv-Brem** Inverse-Bremsstrahlung

**MCF** Magnetic Confinement Fusion

**VFP** Vlasov-Fokker-Planck

**EoS** Equation of State

**UV** Ultraviolet

**EMW** Electro-Magnetic Wave

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# 1 The Interaction of Light with Plasma

This chapter shall introduce theoretical background relevant to the work conducted in this thesis. The main focus of the work is centred on improving the laser modelling in the CHIMERA Radiative-Magnetohydrodynamics (Rad-MHD) code. Therefore, the theoretical framework for modelling both the plasma and the interaction of light with it is introduced.

Initially, the plasma state is defined and important length- and time-scales are provided in Sec. 1.1. The kinetic and fluid descriptions of plasma are introduced in Sec. 1.2 and the validity domain of each framework is discussed, with particular reference to typical conditions for laser-plasma interactions. Additional physics packages of the fluid code, CHIMERA, which are utilised in later chapters, are introduced. A model for kinetic heatflow is desirable in fluid codes which model laser-plasma interactions, therefore although CHIMERA does not have this capability, some basic theory of kinetic heatflow is highlighted.

An additional aim of the work was to include a Cross-Beam Energy Transfer (CBET) model into the new CHIMERA laser package. Laser-Plasma Instabilities (LPIs) such as CBET are multi-wave coupling phenomena, and therefore a basic description of waves in plasma is provided in Sec. 1.3. The dispersion relation of the light waves in plasma is also derived. Beginning with the full wave equation and then introducing successive assumptions which are broadly satisfied in typical laser-produced Inertial Confinement Fusion (ICF) plasmas, Sec. 1.4 derives the equations of ray-tracing. In Sec. 1.5 important absorption processes are then outlined, particularly Inverse-Bremsstrahlung (Inv-Brem), which is the dominant mechanism on the largest ICF facilities in the world today. Finally, the basic theory of LPIs is provided in Sec. 1.6, particularly CBET and its relevance for direct-drive ICF.

## 1.1 Basic Plasma Physics

As stated in Chap. **??**, thermonuclear fusion requires the fuel to exist at significant temperatures, which are well above ionisation energies. Therefore, the fuel configuration in these fusion experiments is a plasma. Formally, a plasma is defined as a quasi-neutral, ionised gas which exhibits collective behaviour [1]. The charged particles within a plasma interact via the long-range Coulomb force, and thus undergo many simultaneous interactions with the other particles. This leads to a variety of collective phenomena such as the plasma-waves described in Sec. 1.3. Quasineutrality of the plasma means that, when observed at a length-scale $L$, the plasma has no net charge,

$$\Sigma_\alpha q_\alpha N_\alpha = 0, \tag{1.1}$$

where $N_\alpha$ is the number of particles of species $\alpha$, with charge $q_\alpha$, in the cube with volume $V = L^3$. For a 'single-species' plasma[1] with average ionisation state $Z$, this implies,

$$n_e = Z n_i, \tag{1.2}$$

where $n_e$ and $n_i$ are the number densities of electrons and ions respectively.

Quasineutrality arises because the particles in the plasma are free to move due to forces they experience. Thus, if a local charge imbalance occurs, the electrons, which respond faster than the ion population due to their lower mass, move to rebalance this field and restore quasineutrality. This electron relocation to eliminate local electric fields is known as Debye-screening. The length scale below the electron population cannot effectively screen the charges sets the length scale of quasineutrality, and is known as the Debye-length,

$$\lambda_D^2 = \frac{\epsilon_0 k_B T_e}{n_e e^2}, \tag{1.3}$$

where $T_e$ is the electron temperature, $\epsilon_0$ is the permittivity of free space, $k_B$ is the Boltzmann constant and $e$ is the electron charge. This screening can only occur if there is a large number of particles in the Debye sphere, $n_e L^3 \gg 1$.

The timescale of the charge relocation is of particular importance to the interaction of light with plasmas. Light waves are an oscillating electric field, which the charged particles in the plasma can respond to. If the particles are able to respond quickly enough, they can therefore influence the propagation of the light and ultimately force the plasma can become opaque to the propagating light wave. The oscillation timescale can be derived by considering a uniform assembly of quasineutral plasma and then displacing the electron population from the ions by a small distance $\delta x$ along the $x$-axis. The electric field which develops within the plasma is thus,

$$E_x = \frac{n_e e \delta x}{\epsilon_0}, \tag{1.4}$$

leading to a restoring force $F_x = -eE_x$ on each electron. Solving Newton's second Law demon-

[1] Single-species here means that there is only a single type of ion.

strates that, when thermal motion of the electron population is ignored[2], oscillations of the electrons occur at the 'plasma frequency',

$$\omega_p^2 = \frac{n_e e^2}{m_e \epsilon_0}. \tag{1.5}$$

If forcing oscillations occur at a frequency which is lower than $\omega_p$, then the electrons move rapidly enough to nullify the field. Consider light with frequency $\omega$, which is incident normal to a plasma density gradient Because $\omega_p \propto n_e$, the light is able to propagate until it reaches the density where $\omega = \omega_p$, which is known as the critical density,

$$n_{\mathrm{cr}} = \frac{m_e \epsilon_0 \omega^2}{e^2}. \tag{1.6}$$

After reaching the critical density, the field of the light decays exponentially as an evanescent wave, but cannot propagate.

## 1.2 The Kinetic and Fluid Formulations

Idealised computational modelling of a plasma state would solve the long-range electromagnetic interaction between every pair of particles at all times. However, this very rapidly becomes intractable due to the large number $N$ of particles and the $\mathcal{O}(N^2)$ scaling of interactions to solve. Reduced frameworks must thus be devised with which to analyse and predict the behaviour of the plasma state. Plasmas are divided into two broad classifications. When in local thermal equilibrium, the plasma is often described as 'thermal' and the fluid formulation is an adequate description. There are numerous situations when this is not true however, for instance if a particular subset of particles is heated at a rate, which is much faster than thermalising collision timescales. In this case, the subset of the system is described as 'non-thermal' or 'kinetic'. The fluid formulation becomes an inadequate description and higher fidelity, kinetic tools must be used to describe the evolution of the system.

When a kinetic description of a plasma is required, the distribution function, $f_\alpha(\mathbf{x}, \mathbf{v}, t)$ is used to describe the state of particle species $\alpha$. It provides a statistical description of the number density of particles, which inhabit a phase-space, $\mathbf{x}$-$\mathbf{v}$, at time $t$. If the system evolves on a timescale much lower than the collision time and plasma length scales are lower than the collisional mean free-path, then these collisions between particles act to locally relax the distribution function toward a Maxwellian,

$$f_{\alpha,\mathrm{Max.}}(v) = n_\alpha \left( \frac{1}{2\pi v_{\mathrm{th}}^2} \right)^{1.5} e^{-\left[v/(\sqrt{2} v_{\mathrm{th}})\right]^2}, \tag{1.7}$$

where $v_{\mathrm{th}} = \sqrt{k_B T_\alpha / m_\alpha}$ is the thermal speed of the species with temperature and mass $T_\alpha$ and $m_\alpha$, respectively. The fraction outside the exponential is set such the integral over velocity space yields the number density of the species. Typically, for bulk of the target config-

[2] Inclusion of thermal motion leads to pressure, which acts as a restoring force, and yields the dispersion relation for an Electron Plasma Wave (EPW).

uration throughout a laser-produced ICF implosion, the assumption of a Maxwellian distribution is close to accurate, although there are several notable exceptions. For instance, DT fusion products have energies much higher than thermal energies and are also monoenergetic. LPIs also generate energetic electron populations which are able to range through the implosion due to their low collisionality [2]. Additionally, laser-heated plasmas typically exhibit steep density and temperature gradients near the ablation surface, such that collisions, and therefore collision processes such as transport of thermal energy, do not act locally [3]. When performing fluid simulations of ICF experiments, accurate modelling of these phenomena requires specific non-local modelling techniques.

### 1.2.1 The Vlasov Equation

The evolution of the distribution function for each species individually, is described by the Vlasov equation,

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}.\nabla_x f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v}\times\mathbf{B}).\nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{collisions}} + \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{external}}, \tag{1.8}$$

where $q_\alpha$ is the charge of the species, $\mathbf{E}$ and $\mathbf{B}$ are the macroscopic electric and magnetic fields[3], respectively, and $\nabla_x$ and $\nabla_v$ are gradients with respect to position and velocity coordinates respectively [4]. The equation describes the conservation of phase-space particle density. The collision operator on the right-hand side of Eq. 1.8 describes the action of microscopic fields, which arise due to the random motion of the charged plasma particles. External forces, for example gravity, are also accounted for in the second right-hand side term. Evolution of the macroscopic fields is governed by Maxwell's equations,

$$\nabla.\mathbf{E} = \frac{\rho_{\text{charge}}}{\epsilon_0}, \tag{1.9}$$

$$\nabla.\mathbf{B} = 0, \tag{1.10}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \tag{1.11}$$

$$\nabla\times\mathbf{B} = \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} + \mu_0\mathbf{J}, \tag{1.12}$$

where $\mu_0$ is the permeability of free space, $\rho_{\text{charge}}$ is the charge density, $\mathbf{J}$ is the current density and $\nabla \equiv \nabla_x$. In the general case with three spatial dimensions, Eq. 1.8 is seven dimensional and evolves on small spatial and temporal scales. Therefore, it is typically too expensive to solve in its entirety for the relatively long times and large spatial scales of an entire ICF implosion. Solution methods to the Vlasov equation include Vlasov-Fokker-Planck (VFP) codes, which discretise the 6-D phase space of Eq. 1.8 and evolve the state forward in time [5]. The velocity space can be either directly discretised [6], or expanded in spherical harmonics [7]. Alternatively, the distribution function can be approximated by Monte Carlo techniques, which is done in Particle in Cell (PiC) codes [8]

[3]Macroscopic here means on a scale larger than the Debye length.

### 1.2.2 The Fluid Equations

Due to the expense of the Vlasov equation, the assumption of local thermodynamic equilibrium can be used to derive the fluid equations. By inserting Eq. 1.7 into Eq. 1.8, multiplying each side of the equation by functions of $\mathbf{v}$ and integrating over velocity space, the fluid equations can be derived. This process is known as taking moments of the Vlasov equation. A moment of the Vlasov equation at order $n$ of $\mathbf{v}$ yields an equation which depends upon the $n+1$ moment. A solvable system of equations thus requires an external closure. The inviscid hydrodynamic equations[4] are obtained from $n = [0, 1, 2]$ moments,

$$\left[\frac{\partial}{\partial t} + \mathbf{u}_\alpha.\nabla\right]\rho_\alpha + \rho_\alpha \nabla.\mathbf{u}_\alpha = 0, \tag{1.13}$$

$$\rho_\alpha \left[\frac{\partial}{\partial t} + \mathbf{u}_\alpha.\nabla\right]\mathbf{u}_\alpha = -\nabla P_\alpha + \mathbf{F}_{\alpha,L} + \mathbf{F}_{\alpha,\text{ext.}} + \mathbf{F}_{\alpha,\text{col.}}, \tag{1.14}$$

$$\left[\frac{\partial}{\partial t} + \mathbf{u}_\alpha.\nabla\right]U_\alpha + (U_\alpha + P_\alpha)\nabla.\mathbf{u}_\alpha = -\nabla.\mathbf{q}_\alpha + Q_{\alpha,\text{ext.}} + Q_{\alpha,\text{col.}}, \tag{1.15}$$

where $\rho_\alpha$ is the mass density of species $\alpha$, $\mathbf{u}_\alpha$ is the fluid velocity, $U_\alpha$ is the internal energy, $P_\alpha$ is the isotropic pressure, $\mathbf{F}_{\alpha,L} = n_\alpha q_\alpha(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B})$ is the Lorentz, $\mathbf{F}_{\alpha,\text{ext.}}$ is external forcing, $\mathbf{F}_{\alpha,\text{col.}}$ is collisional forcing, $\mathbf{q}_\alpha$ is the heat flux, $Q_{\alpha,\text{ext.}}$ is external heating or cooling and $Q_{\alpha,\text{col.}}$ is collisional heating or cooling [9]. A closure relation for the plasma pressure can be obtained from an Equation of State (EoS), such as the ideal gas law, which relates pressure to density, temperature and energy density. The heat flux can be obtained from Fourier's law,

$$\mathbf{q}_\alpha = -\kappa \nabla T_\alpha, \tag{1.16}$$

where the thermal conductivity, $\kappa$, is obtained from local transport theory [10, 11].

The single-fluid, two temperature hydrodynamic equations can be obtained by assuming quasineutrality ($Zn_i = n_e = n$), the electron mass is negligible to the ion mass ($m_i \gg m_e$) and modelling only a single ion-species[5]. The populations will therefore be co-moving, and it follows that,

$$\rho \equiv m_i n_i + m_e n_e \approx m_i n, \tag{1.17}$$

$$\mathbf{u} \equiv \frac{(\rho_i \mathbf{u}_i + \rho_e \mathbf{u}_e)}{\rho} \approx \mathbf{u}_i, \tag{1.18}$$

where the fluid density, $\rho$, and velocity, $\mathbf{u}$, have been defined. Noting also that collisions within a fluid will not affect their net momentum and that collisional forces between the electron and ion will cancel, the fluid equations for electrons and ions can be added to obtain

[4] Note that since the velocity dependence has been integrated out, all variables are now only functions of $\mathbf{x}$.
[5] When multiple ion-species compose the plasma, *e.g.* for CH ablator or DT fuel, then the average ion mass (by number density) must be used and frictional forces/ temperature separation between ion species ignored.

the single-fluid, two temperature equations,

$$\left[\frac{\partial}{\partial t}+\mathbf{u}.\nabla\right]\rho+\rho\nabla.\mathbf{u}=0, \tag{1.19}$$

$$\rho\left[\frac{\partial}{\partial t}+\mathbf{u}.\nabla\right]\mathbf{u}=-\nabla P+\mathbf{F}_L, \tag{1.20}$$

$$\left[\frac{\partial}{\partial t}+\mathbf{u}.\nabla\right]U_e+(U_e+P_e)\nabla.\mathbf{u}=-\nabla.\mathbf{q_e}+Q_{e,\text{ext.}}+Q_{e,\text{col.}}, \tag{1.21}$$

$$\left[\frac{\partial}{\partial t}+\mathbf{u}.\nabla\right]U_i+(U_i+P_i)\nabla.\mathbf{u}=-\nabla.\mathbf{q_i}+Q_{i,\text{ext.}}+Q_{i,\text{col.}}, \tag{1.22}$$

where the total pressure is defined, $P = P_e + P_i$, the external forcing has been dropped, and equilibration between the electron and ion temperatures occurs via the collisional heating terms $Q_{\alpha,\text{col.}}$. By defining the current from electron motion in the frame of the plasma, $\mathbf{J} = -n_e e(\mathbf{u}_e - \mathbf{u})$ and utilising quasineutrality, the Lorentz force term in the single fluid approximation,

$$\mathbf{F}_L = \mathbf{J}\times\mathbf{B}. \tag{1.23}$$

The CHIMERA code solves the two-temperature, single fluid equations. In the absence of magnetic fields, the Lorentz force term goes to zero. Macroscopic magnetic fields can be included under the extended-Magnetohydrodynamics (MHD) framework, and their evolution and effect on plasma transport and dynamics is discussed in Sec. 1.2.4. Further details of the numerical solution to the fluid equations can be found in Refs. [12, 13, 14, 15, 16, 17]. Additional physics packages are included by calculating contributions to the external forces and heating. For example, the non-thermal fusion products are modelled by a Monte Carlo treatment, which collide with the bulk plasma and thus deposit energy which contributes to the electron and ion $Q_{\alpha,\text{ext.}}$ [18]. The additional physics packages which are utilised in the subsequent work presented in this manuscript are discussed briefly below.

### 1.2.3 Radiation Transport

For this section, a distinction between coherent, approximately visible or Ultraviolet (UV) wavelength laser radiation, and higher frequency ($\omega \gg \omega_p$) x-ray radiation is drawn. The former is the main focus of Chap. **??**. It refracts significantly in coronal plasma density gradients and radiation at this wavelength is not significantly re-emitted by the thermal plasma. In contrast, the latter does not significantly refract and is re-emitted by in ICF plasma conditions. The latter is accounted for numerically in CHIMERA by a radiation transport algorithm, which is cursively described here.

In addition to driving the implosion of indirect-drive ICF experiments, x-ray radiation is also significant in a wide array of laser-driven High Energy Density Physics (HEDP) physics experiments. For example, the hot coronal plasma in direct-drive radiates a significant amount of energy as thermal Bremsstrahlung emission. This both lowers the coronal temperatures, reducing the thermal conduction drive efficiency and also preheats the fuel, making it harder to compress. Radiation acts both as a source and sink of energy, as it is radiated and emitted by the material, depending on the plasma conditions and the atomic properties. Thermal emission is wavelength dependent and atomic transitions create sharp resonances of emissivity and opacity in wavelength space. Therefore, the radiation-transport algorithm must be discretised in wavelength, depending on the properties of the material and the plasma conditions.

The radiative transfer equation describes the propagation, absorption, emission and scattering of photons with matter,

$$\frac{\partial I_\nu}{\partial t} + c\hat{\Omega}.\nabla i_\nu = \left(\frac{\partial I_\nu}{\partial t}\right)_{\text{collisions}} + \left(\frac{\partial I_\nu}{\partial t}\right)_{\text{source}}, \tag{1.24}$$

where $c$ is the speed of light in vacuum, $I_\nu$ is the spectrally resolved radiation intensity, $\hat{\Omega}$ is the photon direction of travel [9]. The left-hand side describes the advection of radiation at speed $c$ and the first and second terms on the right-hand side describe collisions between matter and photons, and emission or absorption of photons by the matter respectively. Eq. 1.24 is seven dimensional, and therefore is typically highly expensive to solve, so approximations are often employed to make solutions more tractable. In analogy to derivation of the fluid equations from the Vlasov equation, angular moments of Eq. 1.24 can be taken to reduce the dimensionality of the problem. This also leads to a requirement for a closure relation, and the approach taken in CHIMERA is to use the $P_{1/3}$ closure, which works well for highly isotropic radiation fields [19]. More detail of the CHIMERA implementation is provided in Refs. [20, 21].

### 1.2.4 Magnetohydrodynamics

The method used to evolve macroscopic magnetic fields and their impact upon the plasma dynamics and transport is discussed in this section. Magnetic fields are able to alter the evolution of the plasma state via the Lorentz force in Eq. 1.20 and by altering the conductivities in Eq. 1.16. The relative importance of magnetisation can typically be broadly assessed by calculating dimensionless numbers. For instance, the plasma $\beta$ describes the ratio of thermal pressure to the magnetic pressure[6],

$$\beta = \frac{2\mu_0 P}{|\mathbf{B}|^2}. \tag{1.25}$$

This gives an order of magnitude estimate for when it is necessary to include the Lorentz force in the momentum equation, *i.e.* for high beta plasmas, thermal pressure dominates over magnetic pressure. Similarly, the Hall parameter, $\omega_\alpha \tau_\alpha$, describes the ratio of the gyrofrequency to the collision frequency for a species, $\alpha$. For example, the electron Hall parameter,

$$\omega_e \tau_e = \frac{e|\mathbf{B}|}{m_e} \frac{3\sqrt{m_e}(k_B T_e)^{1.5}}{4\sqrt{2\pi} e^4 Z^2 n_i \ln\Lambda}, \tag{1.26}$$

where $\ln\Lambda$ is the Coulomb logarithm, which is an important parameter for collisional phenomena that is related to the impact parameter of collisions [22, 23, 24]. When $\omega_\alpha \tau_\alpha \sim 1$,

[6] Using Ampère's law with $(\partial \mathbf{E}/\partial t) \ll c^2$, the Lorentz force in Eq, 1.23 can be recast as the sum of magnetic pressure and tension [16].

then collisional phenomena, such as thermal conduction, are significantly affected by the magnetic field.

The evolution of the magnetic field is governed by Faraday's law (Eq. 1.11) and the electric field by Ampère's law (Eq. 1.12). This is not a closed system of equations due to the appearance of **J** in Ampère's law however, and therefore an additional equation is required to evolve the fields. This can be derived from the momentum equations for ions and electrons, Eq. 1.14 [15]. In the limit of no electron inertia, the left-hand side of Eq. 1.14 is zero, leading to the 'generalised Ohm's law',

$$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \frac{\mathbf{J}\times\mathbf{B}}{n_e e} - \frac{\nabla P_e}{n_e e} + \frac{1}{n_e e}\left(\frac{\overline{\alpha}.\mathbf{J}}{n_e e} - n_e\overline{\beta}.\nabla T_e\right), \tag{1.27}$$

where $\overline{\alpha}$ and $\overline{\beta}$ are transport coefficient tensors, the precise form of which is described in, for example, Ref. [16]. Taking the curl of Eq. 1.27 and combining with Faraday's law yields the magnetic induction equation,

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{u}\times\mathbf{B} - \frac{\mathbf{J}\times\mathbf{B}}{n_e e} + \frac{\nabla P_e}{n_e e} - \frac{\overline{\alpha}.\mathbf{J}}{n_e^2 e^2} + \frac{\overline{\beta}.\nabla T_e}{e}\right). \tag{1.28}$$

Each term on the right-hand side of the Eq. 1.28 is ascribed a physical affect. The first term describes advection of the field with the movement of the plasma. When this term dominates, the 'ideal MHD' framework is obtained, in which the field is said to be frozen-in to the flow. Ideal MHD can often be applied in fields such as space physics [25, 26] and Magnetic Confinement Fusion (MCF) [27], where many phenomena featuring highly conductive plasmas are well described by this limit. In order from the second term on the right-hand side, the subsequent terms are related to, the Hall effect (collisionless advection of field with current), the Biermann effect (generation of field), resistive phenomena and thermoelectric phenomena. The non-ideal terms of relevance to the work conducted in this thesis are discussed below.

#### 1.2.4.1 Resistive MHD

When only the first term and the resistive term from Eq. 1.28 are significant (and using a simple form for the transport coefficient, $\overline{\alpha}$), then the induction equation can be written,

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) - \nabla\times\left(\frac{\eta}{\mu_0}\nabla\times\mathbf{B}\right), \tag{1.29}$$

where $\eta$ is the resistivity of the plasma [15]. As already discussed, the first term describes advection of the plasma with the fluid, whereas the second term describes resistive diffusion of the plasma. This effect moves **B** from regions of high- to low-field, via the damping of currents due to collisions between electrons and ions. Resistive diffusion can lead to phenomena such as magnetic reconnection [28, 29]. The relative importance of these two terms is described by the magnetic Reynolds number, which is the ratio field advection to diffusion,

$$R_m = \frac{\mu_0 |\mathbf{u}| L}{\eta}, \tag{1.30}$$

where $L$ is the characteristic length scale of the system.

#### 1.2.4.2 The Nernst Effect

The Nernst effect arises from the thermoelectric term in Eq. 1.28, and makes a contribution to the field induction,

$$\left(\frac{\partial \mathbf{B}}{\partial t}\right)_{\text{Nernst}} = \nabla \times \left(-\frac{\beta_\wedge}{e|\mathbf{B}|} \nabla T_e \times \mathbf{B}\right), \tag{1.31}$$

where $\beta_\wedge$ is a transport coefficient. This contribution looks like an advection of the field in the direction of $-\nabla T_e$ with a velocity,

$$\mathbf{v}_N = -\frac{\beta_\wedge}{e|\mathbf{B}|} \nabla T_e. \tag{1.32}$$

Nernst-advection of magnetic field is often of particular importance to magnetised HEDP experiments, acting to move field down temperature gradients, even when the plasma is highly conductive [30, 13]. As this is a collisional phenomenon, it is most important in regions where the electron Hall parameter is low, $\omega_e \tau_e \ll 1$.

#### 1.2.4.3 Magnetised Thermal Conduction

Magnetic fields can affect collisional transport anisotropically by forcing particles to gyrate around field lines, which changes their collisional behaviour anisotropically. Because the Lorentz force does not affect particle motion in the direction parallel to field lines, transport along the direction, $\hat{\mathbf{b}} = \mathbf{B}/|\mathbf{B}|$, is unaffected by magnetisation. However, the gyration around field lines restricts transport in the perpendicular direction, compared to the parallel direction. Local transport analysis of the heat flux demonstrates that the electron thermal conduction heatflow can be expanded,

$$\mathbf{q}_e = -\kappa_\parallel \hat{\mathbf{b}} \left(\hat{\mathbf{b}}.\nabla T_e\right) - \kappa_\perp \hat{\mathbf{b}} \times \left(\nabla T_e \times \hat{\mathbf{b}}\right) - \kappa_\wedge \hat{\mathbf{b}} \times \nabla T_e, \tag{1.33}$$

where the $\kappa_\parallel$ term describes thermal conduction parallel to the field, $\kappa_\perp$ perpendicular to the field and $\kappa_\wedge$ perpendicular to the field and the temperature gradient [11]. The wedge term, $\kappa_\wedge$, describes 'Righi-Leduc' heatflow, which occurs along isotherms. Considering the first two terms, when unmagnetised, $\kappa_\parallel = \kappa_\perp$ and conduction is isotropic. As the magnetisation increases, however, $\kappa_\perp$ decreases with respect to $\kappa_\parallel$, for example, $\kappa_\perp/\kappa_\parallel \sim 1$ at $\omega_e \tau_e = 1$, which anisotropises the thermal conduction.

### 1.2.5 Kinetic Heatflow

While the CHIMERA implementation of thermal transport is simply to use the local limit described in Eq. 1.16, this is not always a valid approximation in laser produced plasmas. When laser-heating is applied to a plasma, laser energy is transferred to the electron population mostly by Inv-Brem, as is described in Sec. 1.5, heating the electron fluid to significant temperatures. For laser-solid interactions, this often results in sharp temperature and density gradients in the conduction zone, where the thermal energy of the hot corona is transported via thermal conduction to an ablation surface. To assess whether local transport theory is valid, the Knudsen number is a convenient parameter which compares the mean free path of electrons, $\lambda_{\mathrm{mfp}}$ to the length scale of the gradient, $L$. Specifically, the Knudsen number is defined,

$$\mathrm{Kn} = \frac{\lambda_{\mathrm{mfp}}}{L} = \frac{3(k_B^2 T_e^2)}{4\sqrt{2\pi} e^4 Z^2 n_i \ln \Lambda L}, \tag{1.34}$$

and transport effects, become significantly non-local when Kn ~ 0.1, which is often observed for high-power laser solid conduction zones [31]. Physically, more energetic particles within a plasma are less collisional and also play a more significant role in heat flux because it is obtained a higher order moment quantity than density or fluid velocity. Therefore, the fast heat carrying particles in the tail of the distribution function are able to range through longer length scales and preheat the fuel more than a local treatment would predict. Although not included within CHIMERA, models for non-local transport exist, which can be included in fluid models, such as SNB [32, 33, 34, 35], Fast-VFP [36] and RKM [37]. They obtain an improved heat flux estimate, **q**, which accounts for non-local conduction effects such including pre-heat, ad can be included in the fluid framework. Implementation of one of these models into CHIMERA could significantly improve the modelling capability for simulations involving laser-solid interactions, including direct-drive calculations.

## 1.3 Waves in Plasma

LPIs are a class of multi-wave coupling phenomena that occur when a light wave excites additional plasma-waves. Therefore, understanding LPIs requires some background theoretical knowledge of the relevant waves. In the absence of a macroscopic magnetic field, three classes of wave can propagate in a plasma: the Ion Acoustic Wave (IAW), the EPW and the Electro-Magnetic Wave (EMW), *i.e.* a light wave. This section shall describe the basic theory of these waves and provide dispersion relations for their propagation.

### 1.3.1 Plasmas as a Dielectric Medium

Talk about how plasmas are dielectric, therefore described by susceptibilties. Can get dispersion relations by susceptibilities and outline process. Multi-species effects.

### 1.3.2 Plasma Waves

Get disp rels of IAW and EPW and say what they both are physically.

### 1.3.3 Light Waves

These are transverse waves and therefore don't create space-charge separation, unlike longitudinal waves. Get disp rel and show wpe comes out. Give physical interpretation.

## 1.4 Propagation of Light in a Plasma

Want to described light propagating through plasma in limit of typical ICF configurations. Weakly focussing, moderate intensities etc.

### 1.4.1 Paraxial Approximation

Weakly focussing limit. Give equations, interpretations and validity.

### 1.4.2 WKB Approximation

Uniform medium limit. Give equations, interpretations and validity. Give Airy example - ie not valid nearby turning point.

### 1.4.3 Ray Tracing

Give equations, interpretations and validity. State can be used for any kind of wave where valid. Say how and why used for direct drive and typical frozen plasma assumption.

Talking about validity region, give extra bits can solve like ray amplitude.

## 1.5 Absorption of Light in a Plasma

ICF we want to give laser energy to plasma to drive implosion, therefore need to talk about absorption.

### 1.5.1 Inv Brem

Introduce all bits to get NRL formaularly equation. Talk about Langdon as well.

### 1.5.2 Resonance Absorption

Introduce

### 1.5.3 ICF Relevant Absorption comparison

Say inv brem goes up relatively at larger scales and shorter wavelengths. Prefered to resonance absorption because bulk population gets energy. Therefore use frequency tripled light.

## 1.6 Laser Plasma instabilities

### 1.6.1 Ponderamotive Force

Introduce and say why it happens roughly.

### 1.6.2 Three-Wave Coupling

Give the general picture, i.e. ponderamotive, perturbation, driven plasma wave. Give momentum and energy conservation. List all types seeded by an EMW.

### 1.6.3 Cross-Beam Energy Transfer

Derive something to an appropriate level of detail. Talk about how it is in frame of plasma, flow velocities change this, mach 1 surface etc. Give general picture, i.e. sidescatter and backscatter and what it does in ICF.

#### 1.6.3.1 Linear Gain Theory

Say that we use this for raytracing. Assume uniform plasma and can solve plasma response either by linearising fluid or kinetic equations.

#### 1.6.3.2 Effect of Polarisation

Say that LPIs are affected by polarisation via ponderatmotive beat. Only parallel polarisations interact. Important on OMEGA due to polarisation smoothing, leads to mode-1.

#### 1.6.3.3 Langdon Effect on CBET

Say that Langdon affects cbet. Reduced model to alter linear gain. Could potentially explain why indirect ICF models require a clamp.

### 1.6.4 Mitigation of Laser Plasma Instabilities

Say its coherence spatially, temporally and spectrally, so break this to stop LPIs. Mention stud pulses and say zooming for CBET.

Mainstream approach is bandwidth. Talk about studies showing that bandwidth should mitigate CBET. Talk about experimental progress eg FLUX laser at LLE.

## 1.7 Summary

Summarise that introduced descriptions of plasmas, particularly fluid framework solved by CHIMERA. Talked about waves in plasmas and their physical interpareatations. Talked about how light propagates, assumptions etc, used for raytracing in next section. LPIs, particularly CBET, modelling it is focus of next chapter.

# Appendices

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