

V_g is a vector in global and V_l is a vector in local

M is a rotation matrix in create from euler of local in global

$$\vec{v}_{global} = M_{l|g} \cdot \vec{v}_{local}$$

V_0 is a vector in Zero coordinate system and v_1 is vector we have.

M_{10} is a rotation matrix of vector

$$\vec{v}_0 = M_{rotation} \cdot \vec{v}_1$$

We have:

$$\vec{v}_0 = M_{global|virtual} \cdot \vec{v}_{global_0} = M_{global|virtual} \cdot M_{euler_0} \cdot \vec{v}_{local_0}$$

$$\vec{v}_1 = M_{global|virtual} \cdot \vec{v}_{global_1} = M_{global|virtual} \cdot M_{euler_1} \cdot \vec{v}_{local_1}$$

Thence inferred

\vec{v}_T is a vector in local coordinate at T pose with euler in IMU is x_T, y_T, z_T

\vec{v}_1 is a vector in local coordinate at 1 pose with euler in IMU is x_1, y_1, z_1

When we roll IMU from T pose to 1 pose we have a rotation matrix D_{T-1}

And we have $\vec{v}_T = D_{T-1} \cdot \vec{v}_1$ (1)

In global (earth coordinate system) \vec{v}_T, \vec{v}_1 is \vec{v}_g

$$\begin{aligned} \vec{v}_g &= D_T \vec{v}_T = D_1 \vec{v}_1 \\ \vec{v}_T &= D_T^{-1} \cdot D_T \cdot \vec{v}_T = D_T^{-1} D_1 \vec{v}_1 \end{aligned} \quad (2)$$

From (1) and (2) we have:

$$D_{T-1} = D_T^{-1} D_1$$

But D_T, D_1, D_{T-1} is rotation matrices in global coordinate.

We need rotation matrix in virtual coordinate (Ox: left, Oy: up, Oz: forward)

Multi by rotation matrix from global to virtual ([90, 0, 90])

$$D_{g-l} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Therefor: $D = D_{g-l} \cdot D_{T-1} = D_{g-l} \cdot D_T^{-1} D_1$

Set $D_0 = D_{g-l} \cdot D_T^{-1}$ we have: $D = D_0 \cdot D_1$