Linear Algebra Homework 1

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Solve using Gauss-Jordan elimination

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -3 & -2 & -5 \\ 3 & 1 & 4 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -3 & -2 & -5 \\ 0 & -1 & -1 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} -3 & 0 & -3 \\ 0 & -1 & -1 \end{bmatrix}, \begin{bmatrix} R1 - 2\,R2 \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & -4 & 0 \\ 6 & -8 & -4 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 2 & -4 & 0 \\ 0 & 4 & -4 \end{bmatrix}, \begin{bmatrix} \\ R2 - 3\,R1 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} 2 & 0 & -4 \\ 0 & 4 & -4 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -1 & 4 & 7 \\ 1 & -5 & -9 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -1 & 4 & 7 \\ 0 & -1 & -2 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -2 \end{bmatrix}, \begin{bmatrix} R1 + 4\,R2 \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -2 & 4 & -8 \\ 2 & 0 & -4 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -2 & 4 & -8 \\ 0 & 4 & -12 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} -2 & 0 & 4 \\ 0 & 4 & -12 \end{bmatrix}, \begin{bmatrix} R1 - R2 \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -1 & 4 & -4 & -4 \\ -1 & 0 & -2 & 0 \\ -3 & 0 & -4 & -4 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -1 & 4 & -4 & -4 \\ 0 & -4 & 2 & 4 \\ 0 & -12 & 8 & 8 \end{bmatrix}, \begin{bmatrix} \\ R2 - R1 \\ R3 - 3\,R1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & -4 & -4 \\ 0 & -4 & 2 & 4 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 - 3\,R2 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} -1 & 4 & 0 & -12 \\ 0 & -4 & 0 & 8 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R1 + 2\,R3 \\ R2 - R3 \\ \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & -4 \\ 0 & -4 & 0 & 8 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 0 \\ 6 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 3 & -6 & 6 & -6 \\ -3 & 9 & -8 & 0 \\ 3 & -9 & 6 & 6 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 3 & -6 & 6 & -6 \\ 0 & 3 & -2 & -6 \\ 0 & -3 & 0 & 12 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R3 - R1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -6 & 6 & -6 \\ 0 & 3 & -2 & -6 \\ 0 & 0 & -2 & 6 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + R2 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} 3 & -6 & 0 & 12 \\ 0 & 3 & 0 & -12 \\ 0 & 0 & -2 & 6 \end{bmatrix}, \begin{bmatrix} R1 + 3\,R3 \\ R2 - R3 \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & -12 \\ 0 & 3 & 0 & -12 \\ 0 & 0 & -2 & 6 \end{bmatrix}, \begin{bmatrix} R1 + 2\,R2 \\ \\ \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} -4 \\ -4 \\ -3 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}, b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 4 & -6 & 8 & -2 \\ -4 & 3 & -2 & -1 \\ 4 & 0 & -2 & 0 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 4 & -6 & 8 & -2 \\ 0 & -3 & 6 & -3 \\ 0 & 6 & -10 & 2 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R3 - R1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -6 & 8 & -2 \\ 0 & -3 & 6 & -3 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + 2\,R2 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} 4 & -6 & 0 & 14 \\ 0 & -3 & 0 & 9 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R1 - 4\,R3 \\ R2 - 3\,R3 \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 0 & -4 \\ 0 & -3 & 0 & 9 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R1 - 2\,R2 \\ \\ \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}, b = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -3 & 1 & 2 & -3 \\ 9 & -4 & -8 & 3 \\ -9 & 5 & 8 & 1 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -3 & 1 & 2 & -3 \\ 0 & -1 & -2 & -6 \\ 0 & 2 & 2 & 10 \end{bmatrix}, \begin{bmatrix} \\ R2 + 3\,R1 \\ R3 - 3\,R1 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 & 2 & -3 \\ 0 & -1 & -2 & -6 \\ 0 & 0 & -2 & -2 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + 2\,R2 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} -3 & 1 & 0 & -5 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & -2 & -2 \end{bmatrix}, \begin{bmatrix} R1 + R3 \\ R2 - R3 \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} -3 & 0 & 0 & -9 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & -2 & -2 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ \\ \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

Solve using Gauss-Jordan eliminiation (None/Many Solutions)

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 6 & -4 \\ 2 & -4 & 2 \\ 2 & -4 & 2 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -6 \\ -2 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -2 & 6 & -4 & -8 \\ 2 & -4 & 2 & -6 \\ 2 & -4 & 2 & -2 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -2 & 6 & -4 & -8 \\ 0 & 2 & -2 & -14 \\ 0 & 2 & -2 & -10 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R1 + R3 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 6 & -4 & -8 \\ 0 & 2 & -2 & -14 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 - R2 \end{bmatrix}$$

3. Solution: None!

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -3 & -9 & 12 \\ 3 & 9 & -12 \\ 3 & 9 & -12 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 15 \\ 9 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -3 & -9 & 12 & -6 \\ 3 & 9 & -12 & 15 \\ 3 & 9 & -12 & 9 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -3 & -9 & 12 & -6 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R1 + R3 \end{bmatrix}$$

3. Solution: None!

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 4 & -2 & 2 \\ 4 & 0 & 4 \\ 16 & -16 & 0 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 4 & -2 & 2 & 4 \\ 4 & 0 & 4 & 8 \\ 16 & -16 & 0 & 0 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 4 & -2 & 2 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & -8 & -8 & -16 \end{bmatrix}, \begin{bmatrix} \\ R2 - R1 \\ R3 - 4\,R1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 2 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + 4\,R2 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} 4 & 0 & 4 & 8 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_3 \cdot \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -2 & -6 & -4 \\ 6 & 18 & 12 \\ 2 & 6 & 4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 12 \\ 4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -2 & -6 & -4 & -4 \\ 6 & 18 & 12 & 12 \\ 2 & 6 & 4 & 4 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -2 & -6 & -4 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \\ R2 + 3\,R1 \\ R1 + R3 \end{bmatrix}$$

3. Diagonalize
4. Normalize

$$\begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_2 \cdot \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} + t_3 \cdot \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 3 & 2 & -2 & 8 \\ 12 & 7 & -12 & 19 \\ -12 & -6 & 18 & 0 \\ -9 & -4 & 18 & 14 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 13 \\ -6 \\ 2 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 3 & 2 & -2 & 8 & 3 \\ 12 & 7 & -12 & 19 & 13 \\ -12 & -6 & 18 & 0 & -6 \\ -9 & -4 & 18 & 14 & 2 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 3 & 2 & -2 & 8 & 3 \\ 0 & -1 & -4 & -13 & 1 \\ 0 & 2 & 10 & 32 & 6 \\ 0 & 2 & 12 & 38 & 11 \end{bmatrix}, \begin{bmatrix} \\ R2 - 4\,R1 \\ R3 + 4\,R1 \\ R4 + 3\,R1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & -2 & 8 & 3 \\ 0 & -1 & -4 & -13 & 1 \\ 0 & 0 & 2 & 6 & 8 \\ 0 & 0 & 4 & 12 & 13 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + 2\,R2 \\ R4 + 2\,R2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & -2 & 8 & 3 \\ 0 & -1 & -4 & -13 & 1 \\ 0 & 0 & 2 & 6 & 8 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix}, \begin{bmatrix} \\ \\ \\ R4 - 2\,R3 \end{bmatrix}$$

3. Solution: None!

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 2 & 2 & 4 & 2 \\ -4 & -5 & -9 & -6 \\ 8 & 10 & 18 & 12 \end{bmatrix}, b = \begin{bmatrix} -4 \\ -4 \\ 6 \\ -14 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & 3 & 5 & 4 & -4 \\ 2 & 2 & 4 & 2 & -4 \\ -4 & -5 & -9 & -6 & 6 \\ 8 & 10 & 18 & 12 & -14 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 2 & 3 & 5 & 4 & -4 \\ 0 & -1 & -1 & -2 & 0 \\ 0 & 1 & 1 & 2 & -2 \\ 0 & -2 & -2 & -4 & 2 \end{bmatrix}, \begin{bmatrix} \\ R2 - R1 \\ R3 + 2\,R1 \\ R4 - 4\,R1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 5 & 4 & -4 \\ 0 & -1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + R2 \\ R4 - 2\,R2 \end{bmatrix}$$

3. Solution: None!

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 2 & 2 & -4 \\ 2 & -2 & -2 & 4 \\ 8 & -8 & -8 & 16 \\ -4 & 4 & 4 & -8 \end{bmatrix}, b = \begin{bmatrix} 6 \\ -10 \\ -8 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -2 & 2 & 2 & -4 & 6 \\ 2 & -2 & -2 & 4 & -10 \\ 8 & -8 & -8 & 16 & -8 \\ -4 & 4 & 4 & -8 & -4 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} -2 & 2 & 2 & -4 & 6 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & -16 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R3 + 4\,R1 \\ R4 - 2\,R1 \end{bmatrix}$$

3. Solution: None!

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ -6 & -7 & -4 & -1 \\ -2 & -6 & -6 & 8 \\ -2 & -3 & -1 & 0 \end{bmatrix}, b = \begin{bmatrix} 4 \\ -11 \\ 4 \\ -7 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 4 \\ -6 & -7 & -4 & -1 & -11 \\ -2 & -6 & -6 & 8 & 4 \\ -2 & -3 & -1 & 0 & -7 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 4 \\ 0 & -1 & -1 & 2 & 1 \\ 0 & -4 & -5 & 9 & 8 \\ 0 & -1 & 0 & 1 & -3 \end{bmatrix}, \begin{bmatrix} \\ R2 + 3\,R1 \\ R1 + R3 \\ R4 + R1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 4 \\ 0 & -1 & -1 & 2 & 1 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 1 & -1 & -4 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 - 4\,R2 \\ R4 - R2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 4 \\ 0 & -1 & -1 & 2 & 1 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \\ \\ \\ R4 + R3 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} 2 & 2 & 0 & 2 & 8 \\ 0 & -1 & 0 & 1 & -3 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R1 + R3 \\ R2 - R3 \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 4 & 2 \\ 0 & -1 & 0 & 1 & -3 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R1 + 2\,R2 \\ \\ \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_4 \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ -4 \\ 0 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 1 & 2 & 10 \\ -2 & -2 & 2 & -12 \\ -2 & -3 & 6 & -14 \\ 4 & 0 & 12 & 16 \end{bmatrix}, b = \begin{bmatrix} -7 \\ 10 \\ 13 \\ -8 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & 1 & 2 & 10 & -7 \\ -2 & -2 & 2 & -12 & 10 \\ -2 & -3 & 6 & -14 & 13 \\ 4 & 0 & 12 & 16 & -8 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 2 & 1 & 2 & 10 & -7 \\ 0 & -1 & 4 & -2 & 3 \\ 0 & -2 & 8 & -4 & 6 \\ 0 & -2 & 8 & -4 & 6 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R1 + R3 \\ R4 - 2\,R1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 2 & 10 & -7 \\ 0 & -1 & 4 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 - 2\,R2 \\ R4 - 2\,R2 \end{bmatrix}$$

3. Diagonalize

$$\begin{bmatrix} 2 & 0 & 6 & 8 & -4 \\ 0 & -1 & 4 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ \\ \\ \\ \end{bmatrix}$$

4. Normalize

$$\begin{bmatrix} 1 & 0 & 3 & 4 & -2 \\ 0 & 1 & -4 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_3 \cdot \begin{bmatrix} -3 \\ 4 \\ 1 \\ 0 \end{bmatrix} + t_4 \cdot \begin{bmatrix} -4 \\ -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \\ 0 \\ 0 \end{bmatrix}$$

Solve A x = b using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 6 & 2 & 2 \\ 4 & 12 & 4 & 4 \\ -6 & -18 & -6 & -6 \\ 2 & 6 & 2 & 2 \end{bmatrix}, b = \begin{bmatrix} -4 \\ -8 \\ 12 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & 6 & 2 & 2 & -4 \\ 4 & 12 & 4 & 4 & -8 \\ -6 & -18 & -6 & -6 & 12 \\ 2 & 6 & 2 & 2 & -4 \end{bmatrix}$$

2. Triangularize

$$\begin{bmatrix} 2 & 6 & 2 & 2 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \\ R2 - 2\,R1 \\ R3 + 3\,R1 \\ R4 - R1 \end{bmatrix}$$

3. Diagonalize
4. Normalize

$$\begin{bmatrix} 1 & 3 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_2 \cdot \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t_3 \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t_4 \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Overconstrained system: Best approximate solution

Find the best approximate solution of A x = b

$$A = \begin{bmatrix} 4 & -3 \\ -1 & 1 \\ -4 & -3 \end{bmatrix}, b = \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

1. Formula for the best approximate solution

$$x = \left(A^t A\right)^{-1} A^t b$$

2. Best approximate solution

$$x = \begin{bmatrix} -0.535 \\ -0.660 \end{bmatrix}$$

Find the best approximate solution of A x = b

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 3 \\ 1 & -1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -4 \\ -3 \end{bmatrix}$$

1. Formula for the best approximate solution

$$x = \left(A^t A\right)^{-1} A^t b$$

2. Best approximate solution

$$x = \begin{bmatrix} -2.443 \\ 1.800 \end{bmatrix}$$

Find the best approximate solution of A x = b

$$A = \begin{bmatrix} -2 & -3 \\ -1 & -4 \\ -4 & -1 \\ -4 & -1 \end{bmatrix}, b = \begin{bmatrix} -2 \\ -2 \\ 2 \\ 4 \end{bmatrix}$$

1. Formula for the best approximate solution

$$x = \left(A^t A\right)^{-1} A^t b$$

2. Best approximate solution

$$x = \begin{bmatrix} -0.933 \\ 0.919 \end{bmatrix}$$

Find the best approximate solution of A x = b

$$A = \begin{bmatrix} -3 & 4 \\ 1 & 4 \\ 3 & -1 \\ -2 & -2 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 4 \\ -3 \\ 2 \end{bmatrix}$$

1. Formula for the best approximate solution

$$x = \left(A^t A\right)^{-1} A^t b$$

2. Best approximate solution

$$x = \begin{bmatrix} -0.388 \\ 0.440 \end{bmatrix}$$

Find the best approximate solution of A x = b

$$A = \begin{bmatrix} -3 & 4 & 3 \\ -3 & -2 & -4 \\ 4 & 2 & -1 \\ 3 & 1 & -3 \end{bmatrix}, b = \begin{bmatrix} 2 \\ -3 \\ 1 \\ 1 \end{bmatrix}$$

1. Formula for the best approximate solution

$$x = \left(A^t A\right)^{-1} A^t b$$

2. Best approximate solution

$$x = \begin{bmatrix} 0.239 \\ 0.473 \\ 0.266 \end{bmatrix}$$

Find the best approximate solution of A x = b

$$A = \begin{bmatrix} -3 & -1 & 2 \\ -1 & 2 & -2 \\ 4 & -4 & 1 \\ -1 & -3 & -3 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 4 \\ 4 \\ -4 \end{bmatrix}$$

1. Formula for the best approximate solution

$$x = \left(A^t A\right)^{-1} A^t b$$

2. Best approximate solution

$$x = \begin{bmatrix} 1.428 \\ 0.831 \\ -0.192 \end{bmatrix}$$

Find the best approximate solution of A x = b

$$A = \begin{bmatrix} 3 & -3 & 2 \\ 4 & -1 & -1 \\ -2 & -1 & 2 \\ 4 & -1 & 3 \\ 2 & -1 & -2 \end{bmatrix}, b = \begin{bmatrix} -3 \\ -2 \\ -1 \\ 4 \\ 3 \end{bmatrix}$$

1. Formula for the best approximate solution

$$x = \left(A^t A\right)^{-1} A^t b$$

2. Best approximate solution

$$x = \begin{bmatrix} 0.560 \\ 1.318 \\ 0.327 \end{bmatrix}$$

Find the best approximate solution of A x = b

$$A = \begin{bmatrix} 3 & -1 & -1 \\ -3 & 2 & -3 \\ 2 & -4 & -2 \\ -4 & -3 & -1 \\ 3 & 1 & -1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \\ 4 \end{bmatrix}$$

1. Formula for the best approximate solution

$$x = \left(A^t A\right)^{-1} A^t b$$

2. Best approximate solution

$$x = \begin{bmatrix} 0.307 \\ 0.232 \\ -1.318 \end{bmatrix}$$

Underconstrained system: Smallest solution

Find the smallest solution of A x = b

$$A = \begin{bmatrix} 4 & -3 \end{bmatrix}, b = \begin{bmatrix} -1 \end{bmatrix}$$

1. Formula for the smallest solution

$$x = A^t \left(A A^t\right)^{-1} b$$

2. Smallest solution

$$x = \begin{bmatrix} -0.160 \\ 0.120 \end{bmatrix}$$

Find the smallest solution of A x = b

$$A = \begin{bmatrix} 4 & -3 & -1 \\ 1 & -4 & -3 \end{bmatrix}, b = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

1. Formula for the smallest solution

$$x = A^t \left(A\,A^t\right)^{-1} b$$

2. Smallest solution

$$x = \begin{bmatrix} 0.206 \\ 0.454 \\ 0.463 \end{bmatrix}$$

Find the smallest solution of A x = b

$$A = \begin{bmatrix} 4 & -3 & -1 & 1 \\ -4 & -3 & -1 & -3 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

1. Formula for the smallest solution

$$x = A^t \left(A A^t\right)^{-1} b$$

2. Smallest solution

$$x = \begin{bmatrix} 0.315 \\ -0.861 \\ -0.287 \\ -0.130 \end{bmatrix}$$

Find the smallest solution of A x = b

$$A = \begin{bmatrix} 4 & -3 & -1 & 1 \\ -4 & -3 & -1 & -3 \\ 4 & 2 & 3 & 3 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

1. Formula for the smallest solution

$$x = A^t \left(A\, A^t\right)^{-1} b$$

2. Smallest solution

$$x = \begin{bmatrix} 0.279 \\ -0.555 \\ -0.277 \\ -0.058 \end{bmatrix}$$

Determinant

Compute the determinant of A using the definition

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$$

0. Prepare

$$det(a_{1,1}\, e_1 + a_{2,1}\, e_2, a_{1,2}\, e_1 + a_{2,2}\, e_2)$$

1. Apply the multilinearity

Sum of

$+\ a_{1,1}\, a_{1,2}\, det(e_1, e_1)$

$+\ a_{1,1}\, a_{2,2}\, det(e_1, e_2)$

$+\ a_{2,1}\, a_{1,2}\, det(e_2, e_1)$

$+\ a_{2,1}\, a_{2,2}\, det(e_2, e_2)$

2. Apply the antisymmetry

Sum of

$+\ a_{1,1}\, a_{2,2}\, det(e_1, e_2)$

$+\ a_{2,1}\, a_{1,2}\, det(e_2, e_1)$

3. Apply the antisymmetry again

Sum of

$+\ a_{1,1}\, a_{2,2}\, det(e_1, e_2)$

-- $a_{2,1}\, a_{1,2}\, det(e_1, e_2)$

4. Apply the normality

Sum of

$+\ a_{1,1}\, a_{2,2}$

-- $a_{2,1}\, a_{1,2}$

Compute the determinant of A using the definition

$$A = \begin{bmatrix} -1 & -8 \\ 5 & -5 \end{bmatrix}$$

0. Prepare

$$det\left(-e_1 + 5\,e_2, -8\,e_1 - 5\,e_2\right)$$

1. Apply the multilinearity

Sum of

$8\,det\left(e_1, e_1\right)$

$5\,det\left(e_1, e_2\right)$

$-40\,det\left(e_2, e_1\right)$

$-25\,det\left(e_2, e_2\right)$

2. Apply the antisymmetry

Sum of

$5\,det\left(e_1, e_2\right)$

$-40\,det\left(e_2, e_1\right)$

3. Apply the antisymmetry again

Sum of

$5\,det\left(e_1, e_2\right)$

$40\,det\left(e_1, e_2\right)$

4. Apply the normality

Sum of

5

40

Compute the determinant of A using the definition

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

0. Prepare

$$det(a_{1,1} e_1 + a_{2,1} e_2 + a_{3,1} e_3, a_{1,2} e_1 + a_{2,2} e_2 + a_{3,2} e_3, a_{1,3} e_1 + a_{2,3} e_2 + a_{3,3} e_3)$$

1. Apply the multilinearity

Sum of

$$\begin{aligned}
&+ a_{1,1} a_{1,2} a_{1,3} \, det(e_1, e_1, e_1) \\
&+ a_{1,1} a_{1,2} a_{2,3} \, det(e_1, e_1, e_2) \\
&+ a_{1,1} a_{1,2} a_{3,3} \, det(e_1, e_1, e_3) \\
&+ a_{1,1} a_{2,2} a_{1,3} \, det(e_1, e_2, e_1) \\
&+ a_{1,1} a_{2,2} a_{2,3} \, det(e_1, e_2, e_2) \\
&+ a_{1,1} a_{2,2} a_{3,3} \, det(e_1, e_2, e_3) \\
&+ a_{1,1} a_{3,2} a_{1,3} \, det(e_1, e_3, e_1) \\
&+ a_{1,1} a_{3,2} a_{2,3} \, det(e_1, e_3, e_2) \\
&+ a_{1,1} a_{3,2} a_{3,3} \, det(e_1, e_3, e_3) \\
&+ a_{2,1} a_{1,2} a_{1,3} \, det(e_2, e_1, e_1) \\
&+ a_{2,1} a_{1,2} a_{2,3} \, det(e_2, e_1, e_2) \\
&+ a_{2,1} a_{1,2} a_{3,3} \, det(e_2, e_1, e_3) \\
&+ a_{2,1} a_{2,2} a_{1,3} \, det(e_2, e_2, e_1) \\
&+ a_{2,1} a_{2,2} a_{2,3} \, det(e_2, e_2, e_2) \\
&+ a_{2,1} a_{2,2} a_{3,3} \, det(e_2, e_2, e_3) \\
&+ a_{2,1} a_{3,2} a_{1,3} \, det(e_2, e_3, e_1) \\
&+ a_{2,1} a_{3,2} a_{2,3} \, det(e_2, e_3, e_2) \\
&+ a_{2,1} a_{3,2} a_{3,3} \, det(e_2, e_3, e_3) \\
&+ a_{3,1} a_{1,2} a_{1,3} \, det(e_3, e_1, e_1) \\
&+ a_{3,1} a_{1,2} a_{2,3} \, det(e_3, e_1, e_2) \\
&+ a_{3,1} a_{1,2} a_{3,3} \, det(e_3, e_1, e_3) \\
&+ a_{3,1} a_{2,2} a_{1,3} \, det(e_3, e_2, e_1) \\
&+ a_{3,1} a_{2,2} a_{2,3} \, det(e_3, e_2, e_2) \\
&+ a_{3,1} a_{2,2} a_{3,3} \, det(e_3, e_2, e_3) \\
&+ a_{3,1} a_{3,2} a_{1,3} \, det(e_3, e_3, e_1) \\
&+ a_{3,1} a_{3,2} a_{2,3} \, det(e_3, e_3, e_2)
\end{aligned}$$

$+ \ a_{3,1} a_{3,2} a_{3,3} \, det(e_3, e_3, e_3)$

2. Apply the antisymmetry

Sum of

$+ \ a_{1,1} a_{2,2} a_{3,3} \, det(e_1, e_2, e_3)$
$+ \ a_{1,1} a_{3,2} a_{2,3} \, det(e_1, e_3, e_2)$
$+ \ a_{2,1} a_{1,2} a_{3,3} \, det(e_2, e_1, e_3)$
$+ \ a_{2,1} a_{3,2} a_{1,3} \, det(e_2, e_3, e_1)$
$+ \ a_{3,1} a_{1,2} a_{2,3} \, det(e_3, e_1, e_2)$
$+ \ a_{3,1} a_{2,2} a_{1,3} \, det(e_3, e_2, e_1)$

3. Apply the antisymmetry again

Sum of

$+ \ a_{1,1} a_{2,2} a_{3,3} \, det(e_1, e_2, e_3)$
-- $a_{1,1} a_{3,2} a_{2,3} \, det(e_1, e_2, e_3)$
-- $a_{2,1} a_{1,2} a_{3,3} \, det(e_1, e_2, e_3)$
$+ \ a_{2,1} a_{3,2} a_{1,3} \, det(e_1, e_2, e_3)$
$+ \ a_{3,1} a_{1,2} a_{2,3} \, det(e_1, e_2, e_3)$
-- $a_{3,1} a_{2,2} a_{1,3} \, det(e_1, e_2, e_3)$

4. Apply the normality

Sum of

$+ \ a_{1,1} a_{2,2} a_{3,3}$
-- $a_{1,1} a_{3,2} a_{2,3}$
-- $a_{2,1} a_{1,2} a_{3,3}$
$+ \ a_{2,1} a_{3,2} a_{1,3}$
$+ \ a_{3,1} a_{1,2} a_{2,3}$
-- $a_{3,1} a_{2,2} a_{1,3}$

Compute the determinant of A using the definition

$$A = \begin{bmatrix} 5 & -6 & 3 \\ 7 & 5 & 8 \\ -6 & -8 & -2 \end{bmatrix}$$

0. Prepare

$$det\left(5\,e_1 + 7\,e_2 - 6\,e_3, -6\,e_1 + 5\,e_2 - 8\,e_3, 3\,e_1 + 8\,e_2 - 2\,e_3\right)$$

1. Apply the multilinearity

Sum of

$$-90\,det\left(e_1, e_1, e_1\right)$$
$$-240\,det\left(e_1, e_1, e_2\right)$$
$$60\,det\left(e_1, e_1, e_3\right)$$
$$75\,det\left(e_1, e_2, e_1\right)$$
$$200\,det\left(e_1, e_2, e_2\right)$$
$$-50\,det\left(e_1, e_2, e_3\right)$$
$$-120\,det\left(e_1, e_3, e_1\right)$$
$$-320\,det\left(e_1, e_3, e_2\right)$$
$$80\,det\left(e_1, e_3, e_3\right)$$
$$-126\,det\left(e_2, e_1, e_1\right)$$
$$-336\,det\left(e_2, e_1, e_2\right)$$
$$84\,det\left(e_2, e_1, e_3\right)$$
$$105\,det\left(e_2, e_2, e_1\right)$$
$$280\,det\left(e_2, e_2, e_2\right)$$
$$-70\,det\left(e_2, e_2, e_3\right)$$
$$-168\,det\left(e_2, e_3, e_1\right)$$
$$-448\,det\left(e_2, e_3, e_2\right)$$
$$112\,det\left(e_2, e_3, e_3\right)$$
$$108\,det\left(e_3, e_1, e_1\right)$$
$$288\,det\left(e_3, e_1, e_2\right)$$
$$-72\,det\left(e_3, e_1, e_3\right)$$
$$-90\,det\left(e_3, e_2, e_1\right)$$
$$-240\,det\left(e_3, e_2, e_2\right)$$
$$60\,det\left(e_3, e_2, e_3\right)$$
$$144\,det\left(e_3, e_3, e_1\right)$$
$$384\,det\left(e_3, e_3, e_2\right)$$
$$-96\,det\left(e_3, e_3, e_3\right)$$

2. Apply the antisymmetry

Sum of

$-50\ det(e_1, e_2, e_3)$

$-320\ det(e_1, e_3, e_2)$

$84\ det(e_2, e_1, e_3)$

$-168\ det(e_2, e_3, e_1)$

$288\ det(e_3, e_1, e_2)$

$-90\ det(e_3, e_2, e_1)$

3. Apply the antisymmetry again

Sum of

$-50\ det(e_1, e_2, e_3)$

$320\ det(e_1, e_2, e_3)$

$-84\ det(e_1, e_2, e_3)$

$-168\ det(e_1, e_2, e_3)$

$288\ det(e_1, e_2, e_3)$

$90\ det(e_1, e_2, e_3)$

4. Apply the normality

Sum of

-50

320

-84

-168

288

90

Solve using Cramer's Rule

Solve A x = b using Cramer's rule

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

1. Denominator

$$det\left(\begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}\right) = 3$$

2. Numerators

$$det\left(\begin{bmatrix} -5 & -2 \\ 4 & 1 \end{bmatrix}\right) = 3$$

$$det\left(\begin{bmatrix} -3 & -5 \\ 3 & 4 \end{bmatrix}\right) = 3$$

3. Solution

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Solve A x = b using Cramer's rule

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

1. Denominator

$$det\left(\begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}\right) = 8$$

2. Numerators

$$det\left(\begin{bmatrix} 0 & -4 \\ -4 & -8 \end{bmatrix}\right) = -16$$

$$det\left(\begin{bmatrix} 2 & 0 \\ 6 & -4 \end{bmatrix}\right) = -8$$

3. Solution

$$x = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

Solve A x = b using Cramer's rule

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$$

1. Denominator

$$det\left(\begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}\right) = 1$$

2. Numerators

$$det\left(\begin{bmatrix} 7 & 4 \\ -9 & -5 \end{bmatrix}\right) = 1$$

$$det\left(\begin{bmatrix} -1 & 7 \\ 1 & -9 \end{bmatrix}\right) = 2$$

3. Solution

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solve A x = b using Cramer's rule

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

1. Denominator

$$det\left(\begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}\right) = -8$$

2. Numerators

$$det\left(\begin{bmatrix} -8 & 4 \\ -4 & 0 \end{bmatrix}\right) = 16$$

$$det\left(\begin{bmatrix} -2 & -8 \\ 2 & -4 \end{bmatrix}\right) = 24$$

3. Solution

$$x = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

Solve A x = b using Cramer's rule

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix}$$

1. Denominator

$$det\left(\begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}\right) = 8$$

2. Numerators

$$det\left(\begin{bmatrix} -4 & 4 & -4 \\ 0 & 0 & -2 \\ -4 & 0 & -4 \end{bmatrix}\right) = 32$$

$$det\left(\begin{bmatrix} -1 & -4 & -4 \\ -1 & 0 & -2 \\ -3 & -4 & -4 \end{bmatrix}\right) = -16$$

$$det\left(\begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & 0 \\ -3 & 0 & -4 \end{bmatrix}\right) = -16$$

3. Solution

$$x = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}$$

Solve A x = b using Cramer's rule

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 0 \\ 6 \end{bmatrix}$$

1. Denominator

$$det\left(\begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}\right) = -18$$

2. Numerators

$$det\left(\begin{bmatrix} -6 & -6 & 6 \\ 0 & 9 & -8 \\ 6 & -9 & 6 \end{bmatrix}\right) = 72$$

$$det\left(\begin{bmatrix} 3 & -6 & 6 \\ -3 & 0 & -8 \\ 3 & 6 & 6 \end{bmatrix}\right) = 72$$

$$det\left(\begin{bmatrix} 3 & -6 & -6 \\ -3 & 9 & 0 \\ 3 & -9 & 6 \end{bmatrix}\right) = 54$$

3. Solution

$$x = \begin{bmatrix} -4 \\ -4 \\ -3 \end{bmatrix}$$

Solve A x = b using Cramer's rule

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}, b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

1. Denominator

$$det\left(\begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}\right) = -24$$

2. Numerators

$$det\left(\begin{bmatrix} -2 & -6 & 8 \\ -1 & 3 & -2 \\ 0 & 0 & -2 \end{bmatrix}\right) = 24$$

$$det\left(\begin{bmatrix} 4 & -2 & 8 \\ -4 & -1 & -2 \\ 4 & 0 & -2 \end{bmatrix}\right) = 72$$

$$det\left(\begin{bmatrix} 4 & -6 & -2 \\ -4 & 3 & -1 \\ 4 & 0 & 0 \end{bmatrix}\right) = 48$$

3. Solution

$$x = \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}$$

Solve A x = b using Cramer's rule

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}, b = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

1. Denominator

$$det\left(\begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}\right) = -6$$

2. Numerators

$$det\left(\begin{bmatrix} -3 & 1 & 2 \\ 3 & -4 & -8 \\ 1 & 5 & 8 \end{bmatrix}\right) = -18$$

$$det\left(\begin{bmatrix} -3 & -3 & 2 \\ 9 & 3 & -8 \\ -9 & 1 & 8 \end{bmatrix}\right) = -24$$

$$det\left(\begin{bmatrix} -3 & 1 & -3 \\ 9 & -4 & 3 \\ -9 & 5 & 1 \end{bmatrix}\right) = -6$$

3. Solution

$$x = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

Invert matrix using determinants

Invert A using determinant

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}$$

1. Determinant of A

$$det\left(\begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}\right) = 3$$

2. Adjugate matrix of A

$$\begin{bmatrix} det(\begin{bmatrix} 1 \end{bmatrix}) & -det(\begin{bmatrix} -2 \end{bmatrix}) \\ -det(\begin{bmatrix} 3 \end{bmatrix}) & det(\begin{bmatrix} -3 \end{bmatrix}) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -3 \end{bmatrix}$$

3. Divide

$$inverse(A) = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -1 & -1 \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}$$

1. Determinant of A

$$det\left(\begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}\right) = 8$$

2. Adjugate matrix of A

$$\begin{bmatrix} det\left(\begin{bmatrix} -8 \end{bmatrix}\right) & -det\left(\begin{bmatrix} -4 \end{bmatrix}\right) \\ -det\left(\begin{bmatrix} 6 \end{bmatrix}\right) & det\left(\begin{bmatrix} 2 \end{bmatrix}\right) \end{bmatrix} = \begin{bmatrix} -8 & 4 \\ -6 & 2 \end{bmatrix}$$

3. Divide

$$inverse(A) = \begin{bmatrix} -1 & \frac{1}{2} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}$$

1. Determinant of A

$$det\left(\begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}\right) = 1$$

2. Adjugate matrix of A

$$\begin{bmatrix} det([\ -5\ ]) & -det([\ 4\ ]) \\ -det([\ 1\ ]) & det([\ -1\ ]) \end{bmatrix} = \begin{bmatrix} -5 & -4 \\ -1 & -1 \end{bmatrix}$$

3. Divide

$$inverse(A) = \begin{bmatrix} -5 & -4 \\ -1 & -1 \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}$$

1. Determinant of A

$$det\left(\begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}\right) = -8$$

2. Adjugate matrix of A

$$\begin{bmatrix} det(\begin{bmatrix} 0 \end{bmatrix}) & -det(\begin{bmatrix} 4 \end{bmatrix}) \\ -det(\begin{bmatrix} 2 \end{bmatrix}) & det(\begin{bmatrix} -2 \end{bmatrix}) \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ -2 & -2 \end{bmatrix}$$

3. Divide

$$inverse(A) = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}$$

1. Determinant of A

$$det\left(\begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}\right) = 8$$

2. Adjugate matrix of A

$$\begin{bmatrix} det\left(\begin{bmatrix} 0 & -2 \\ 0 & -4 \end{bmatrix}\right) & -det\left(\begin{bmatrix} 4 & -4 \\ 0 & -4 \end{bmatrix}\right) & det\left(\begin{bmatrix} 4 & -4 \\ 0 & -2 \end{bmatrix}\right) \\ -det\left(\begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}\right) & det\left(\begin{bmatrix} -1 & -4 \\ -3 & -4 \end{bmatrix}\right) & -det\left(\begin{bmatrix} -1 & -4 \\ -1 & -2 \end{bmatrix}\right) \\ det\left(\begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}\right) & -det\left(\begin{bmatrix} -1 & 4 \\ -3 & 0 \end{bmatrix}\right) & det\left(\begin{bmatrix} -1 & 4 \\ -1 & 0 \end{bmatrix}\right) \end{bmatrix} = \begin{bmatrix} 0 & 16 & -8 \\ 2 & -8 & 2 \\ 0 & -12 & 4 \end{bmatrix}$$

3. Divide

$$inverse(A) = \begin{bmatrix} 0 & 2 & -1 \\ \frac{1}{4} & -1 & \frac{1}{4} \\ 0 & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}$$

1. Determinant of A

$$det\left(\begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}\right) = -18$$

2. Adjugate matrix of A

$$\begin{bmatrix} det\left(\begin{bmatrix} 9 & -8 \\ -9 & 6 \end{bmatrix}\right) & -det\left(\begin{bmatrix} -6 & 6 \\ -9 & 6 \end{bmatrix}\right) & det\left(\begin{bmatrix} -6 & 6 \\ 9 & -8 \end{bmatrix}\right) \\ -det\left(\begin{bmatrix} -3 & -8 \\ 3 & 6 \end{bmatrix}\right) & det\left(\begin{bmatrix} 3 & 6 \\ 3 & 6 \end{bmatrix}\right) & -det\left(\begin{bmatrix} 3 & 6 \\ -3 & -8 \end{bmatrix}\right) \\ det\left(\begin{bmatrix} -3 & 9 \\ 3 & -9 \end{bmatrix}\right) & -det\left(\begin{bmatrix} 3 & -6 \\ 3 & -9 \end{bmatrix}\right) & det\left(\begin{bmatrix} 3 & -6 \\ -3 & 9 \end{bmatrix}\right) \end{bmatrix} = \begin{bmatrix} -18 & -18 & -6 \\ -6 & 0 & 6 \\ 0 & 9 & 9 \end{bmatrix}$$

3. Divide

$$inverse(A) = \begin{bmatrix} 1 & 1 & \frac{1}{3} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}$$

1. Determinant of A

$$det\left(\begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}\right) = -24$$

2. Adjugate matrix of A

$$\begin{bmatrix} det\left(\begin{bmatrix} 3 & -2 \\ 0 & -2 \end{bmatrix}\right) & -det\left(\begin{bmatrix} -6 & 8 \\ 0 & -2 \end{bmatrix}\right) & det\left(\begin{bmatrix} -6 & 8 \\ 3 & -2 \end{bmatrix}\right) \\ -det\left(\begin{bmatrix} -4 & -2 \\ 4 & -2 \end{bmatrix}\right) & det\left(\begin{bmatrix} 4 & 8 \\ 4 & -2 \end{bmatrix}\right) & -det\left(\begin{bmatrix} 4 & 8 \\ -4 & -2 \end{bmatrix}\right) \\ det\left(\begin{bmatrix} -4 & 3 \\ 4 & 0 \end{bmatrix}\right) & -det\left(\begin{bmatrix} 4 & -6 \\ 4 & 0 \end{bmatrix}\right) & det\left(\begin{bmatrix} 4 & -6 \\ -4 & 3 \end{bmatrix}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -12 & -12 \\ -16 & -40 & -24 \\ -12 & -24 & -12 \end{bmatrix}$$

3. Divide

$$inverse(A) = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{5}{3} & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}$$

1. Determinant of A

$$det\left(\begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}\right) = -6$$

2. Adjugate matrix of A

$$\begin{bmatrix} det\left(\begin{bmatrix} -4 & -8 \\ 5 & 8 \end{bmatrix}\right) & -det\left(\begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix}\right) & det\left(\begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix}\right) \\ -det\left(\begin{bmatrix} 9 & -8 \\ -9 & 8 \end{bmatrix}\right) & det\left(\begin{bmatrix} -3 & 2 \\ -9 & 8 \end{bmatrix}\right) & -det\left(\begin{bmatrix} -3 & 2 \\ 9 & -8 \end{bmatrix}\right) \\ det\left(\begin{bmatrix} 9 & -4 \\ -9 & 5 \end{bmatrix}\right) & -det\left(\begin{bmatrix} -3 & 1 \\ -9 & 5 \end{bmatrix}\right) & det\left(\begin{bmatrix} -3 & 1 \\ 9 & -4 \end{bmatrix}\right) \end{bmatrix} = \begin{bmatrix} 8 & 2 & 0 \\ 0 & -6 & -6 \\ 9 & 6 & 3 \end{bmatrix}$$

3. Divide

$$inverse(A) = \begin{bmatrix} -\frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & 1 \\ -\frac{3}{2} & -1 & -\frac{1}{2} \end{bmatrix}$$

Volume of linear object using determinant (generalized Pythagorean Theorem)

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 25 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 25$$

4. Compute the square-root of D

$$5$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} a_1^2 + a_2^2 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = a_1^2 + a_2^2$$

4. Compute the square-root of D

$$\sqrt{a_1^2 + a_2^2}$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 21 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 21$$

4. Compute the square-root of D

$$\sqrt{21}$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} -5 \\ 5 \\ -4 \\ 5 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -5 \\ 5 \\ -4 \\ 5 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 91 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 91$$

4. Compute the square-root of D

$$\sqrt{91}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -1 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -2 & -4 \\ 3 & 1 \\ 2 & -1 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 17 & 9 \\ 9 & 18 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 225$$

4. Compute the square-root of D

$$15$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 5 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 5 & 2 \\ 5 & 2 \\ -2 & 2 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 54 & 16 \\ 16 & 12 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 392$$

4. Compute the square-root of D

$$14\sqrt{2}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 4 \\ -5 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ -4 \\ -1 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 4 & 4 \\ -5 & -4 \\ -3 & -4 \\ -2 & -1 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 54 & 50 \\ 50 & 49 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 146$$

4. Compute the square-root of D

$$\sqrt{146}$$

Find the 3-volume (volume) of the linear object defined by the vectors

$$\begin{bmatrix} -5 \\ -2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ -5 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ -1 \\ -3 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -5 & 5 & -5 \\ -2 & -5 & -2 \\ -1 & -3 & -1 \\ 5 & 5 & -3 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 55 & 13 & 15 \\ 13 & 84 & -27 \\ 15 & -27 & 39 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 104064$$

4. Compute the square-root of D

$$8\sqrt{1626}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 1 \\ -4 \\ 5 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \\ -2 \\ 5 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 1 & 3 \\ -4 & 3 \\ 5 & 3 \\ 2 & -2 \\ -3 & 5 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 55 & -13 \\ -13 & 56 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 2911$$

4. Compute the square-root of D

$$\sqrt{2911}$$

Find the 3-volume (volume) of the linear object defined by the vectors

$$\begin{bmatrix} -3 \\ -4 \\ 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ -1 \\ -5 \\ 1 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -3 & 3 & 4 \\ -4 & 1 & -4 \\ 3 & 2 & -1 \\ -4 & -3 & -5 \\ 5 & 3 & 1 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 75 & 20 & 26 \\ 20 & 32 & 24 \\ 26 & 24 & 59 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 78128$$

4. Compute the square-root of D

$$4\sqrt{4883}$$

Find the 4-volume of the linear object defined by the vectors

$$\begin{bmatrix} -2 \\ -1 \\ 5 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ -4 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 4 \\ 1 \\ -5 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -2 & 2 & -4 & 1 \\ -1 & -4 & -1 & -1 \\ 5 & 4 & -4 & 4 \\ -2 & 1 & -2 & 1 \\ -3 & 3 & 5 & -5 \end{bmatrix}$$

2. Compute B = A^t A

$$B = \begin{bmatrix} 43 & 9 & -22 & 32 \\ 9 & 46 & -7 & 8 \\ -22 & -7 & 62 & -46 \\ 32 & 8 & -46 & 44 \end{bmatrix}$$

3. Compute D = det(B)

$$\mathrm{D} = 277200$$

4. Compute the square-root of D

$$60\sqrt{77}$$