Homework 8 Dynamics of Neural Systems

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Task 1

1.3

At temporal steady state, the spatial Fourier transform of the neural field output is the following:

$$\tilde{u}(k) = \frac{e^{-d^2k^2}}{1 - a\left[e^{-b^2(k-k_0)^2} + e^{-b^2(k+k_0)^2}\right]} \tag{0.1}$$

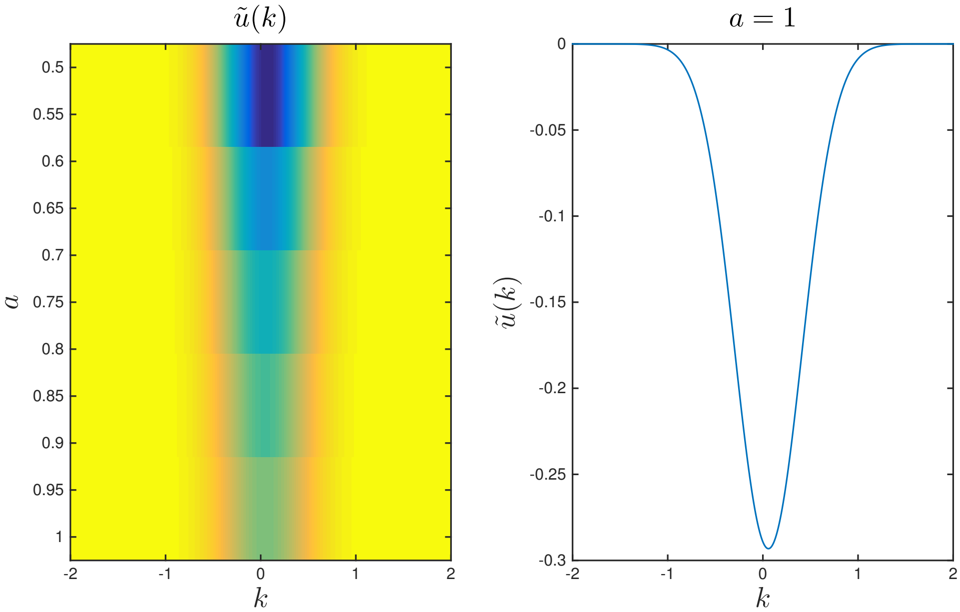

Figure 0.1: Effect of parameter a in the spacial Fourier transformation of u at temporal steady state. Left: the magnitude of low frequency components increases as $a \to 1$ (less attenuation of low frequencies). Right: Fourier transformation of u when $a = 1$

1.4

When simulating the neural field as it is shown in the following figures, the steady state is reached after 50 time units and it does not change for long time intervals. The spacial stable state is a Gaussian with a peak at zero. In the Gaussian noise case, the solution of the equation takes a similar form as in the previous so one could fairly say that the system is robust to noise in the input. This can be explained by the fact that the filtering of the output signal with $w(x)$ attenuates high frequency changes in $u(x, t)$ and therefore it cleans the signal avoiding the amplification of noise. The noise observed in the output is then a consequence of the noise added in the input at each time step.

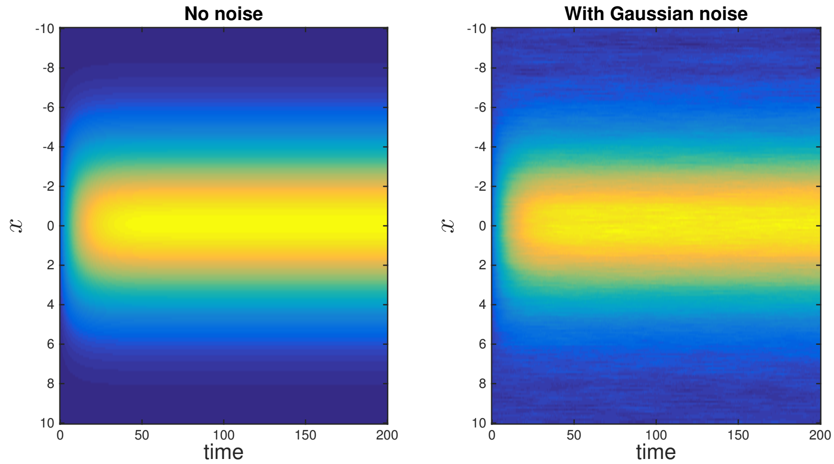

Figure 0.2: Simulation of the output of the neural field $u(x,t)$. Left: without noise in the input $s(x)$. Right: Gaussian noise with 0 mean and $\sigma^2 = 0.01^2$

Now, if one changes $a = 0.7$ the result does not differ much with $a = 1$ since the solution is stable but reaches another value. However when $a = 1.5$, the solution becomes unstable. For large t the system oscillates with a non-bounded growing amplitude given by the frequency k_0. The neural field becomes then oscillatory in space generating active regions where the highest amplitude is reached around 0.

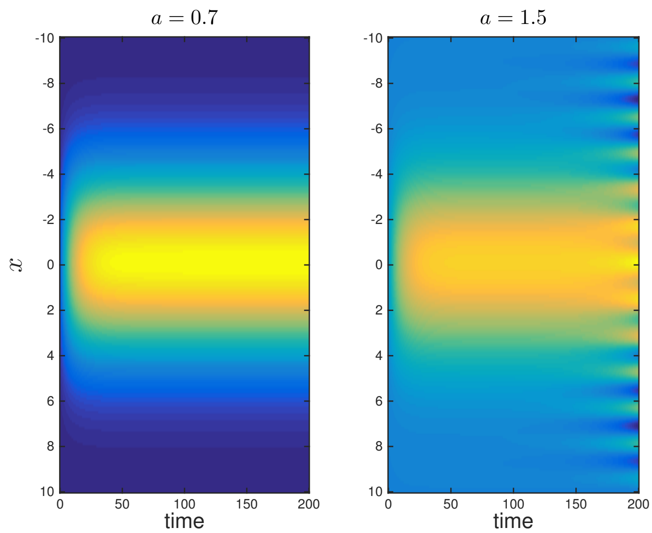

Figure 0.3: Simulation of the output of the neural field $u(x,t)$ with parameter $a = 0.7$ (Left) and $a = 1.5$ (Right)

If now one changes the oscillation frequency of $w(x)$ cosine component k_0, for $a = 1.5$, the spatial frequency of the response for large t increases to k_0 accordingly as shown in the following:

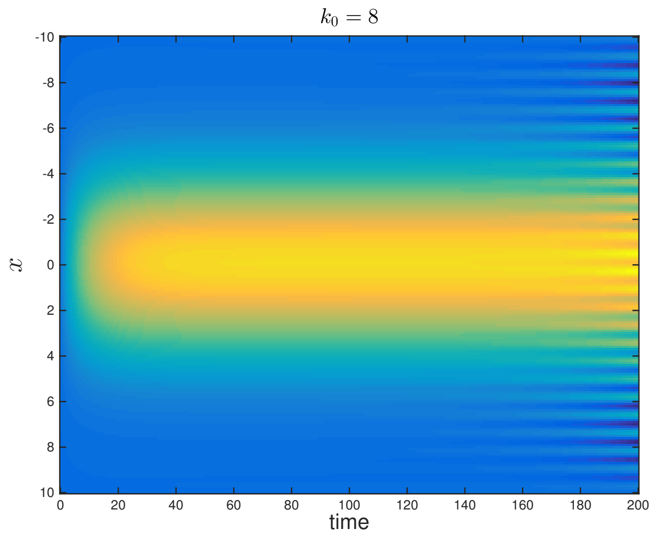

Figure 0.4: Simulation of the output of the neural field $u(x,t)$ with parameter $a = 1.5$ and $k_0 = 8$

1.6

The new interaction kernel given by $w(x) = sign(x)e^{-c|x|}$ and a Gaussian input where the mean depends on time with the established parameters produce the following figures for different values of v (the travelling speed of stimulus peak). The field looks like a spacial edge detector in a one dimensional spacial stimulus from low to high changes in stimulus. For positive v the stimulus will move the preferred spacial selective low and high peaks in the positive direction. On the contrary, por negative v the higher responses will be shifted in the negative spatial direction.

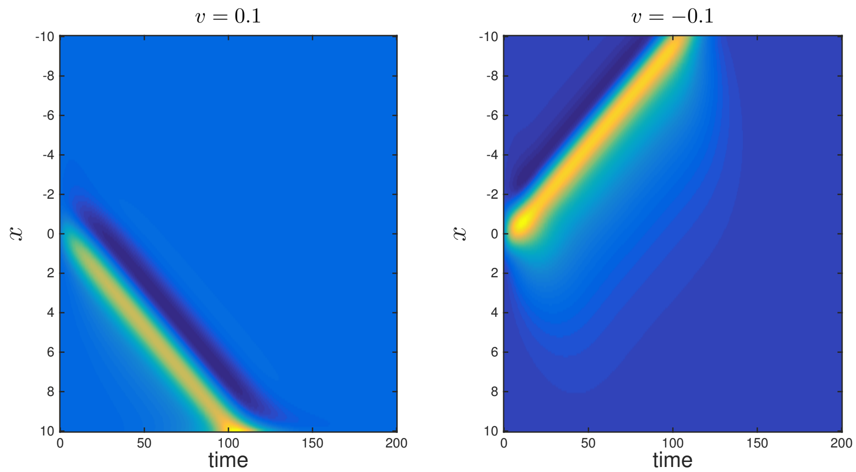

Figure 0.5: Simulation of the output of the neural field $u(x,t)$ with travelling speeds $v = 0.1$ (left) and $v = -0.1$ (Right)

Task 2

2.1

When doing the simulation of the new Amari neural field, the solution for different initial conditions is the following:

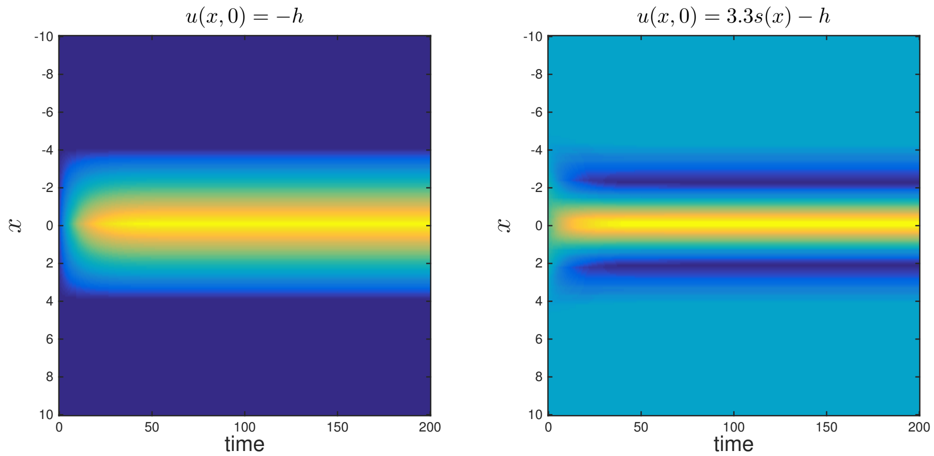

Figure 0.6: Neural Amari field for different initial conditions. $u(x,0) = -h$ (left) and $u(x,0) = 3.3s(x) - h$ (right)

The stable state for both cases is shown in the following figure. In the case of $u(x,0) = -h$, the field does not reach active regions ($u(x) > 0$) because the stimulus is not strong enough to go from that low initial condition to another stable fixed point. On the contrary, for $u(x,0) = 3.3s(x) - h$, there are active regions around $-1 < x < 1$ where $u(x) > 0$ since the initial condition is located in a region that converged to stable fixed points where active regions are plausible.

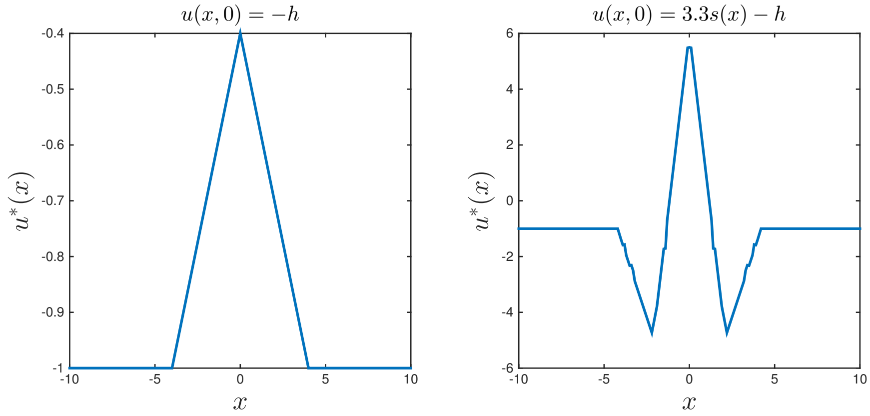

Figure 0.7: Stable state for Neural Amari field for different initial conditions. $u(x,0) = -h$ (left) and $u(x,0) = 3.3s(x) - h$ (right)

Now, if one changes $w(x) \leftarrow 0.1w(x)$ the resulting field changes and it is shown in the following figures. It is evident now, that the kernel function is not strong enough now to elicit activated regions in the steady state for both initial conditions that are considered. In the transient state there are positive values for the field in the case of $u(x,0) = 3.3s(x) - h$, however it reaches the same fixed points reached by the case when the initial condition is $u(x,0) = -h$.

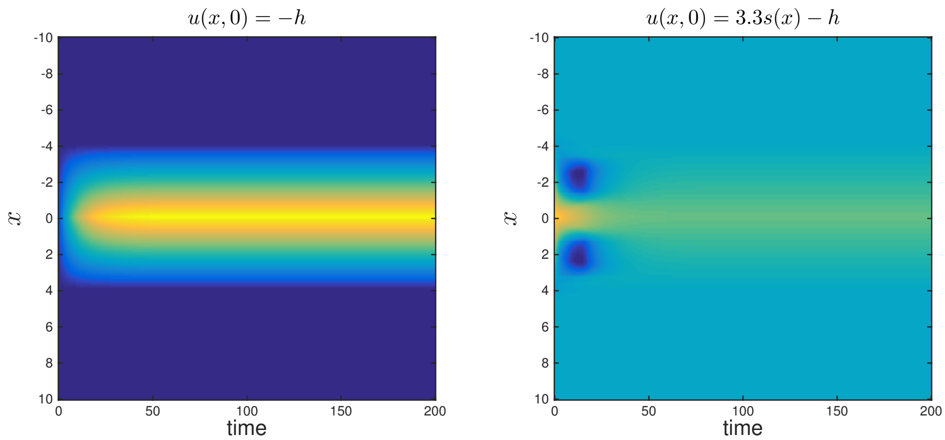

Figure 0.8: Neural Amari field for different initial conditions when $w(x) \leftarrow 0.1w(x)$. $u(x,0) = -h$ (left) and $u(x,0) = 3.3s(x) - h$ (right)

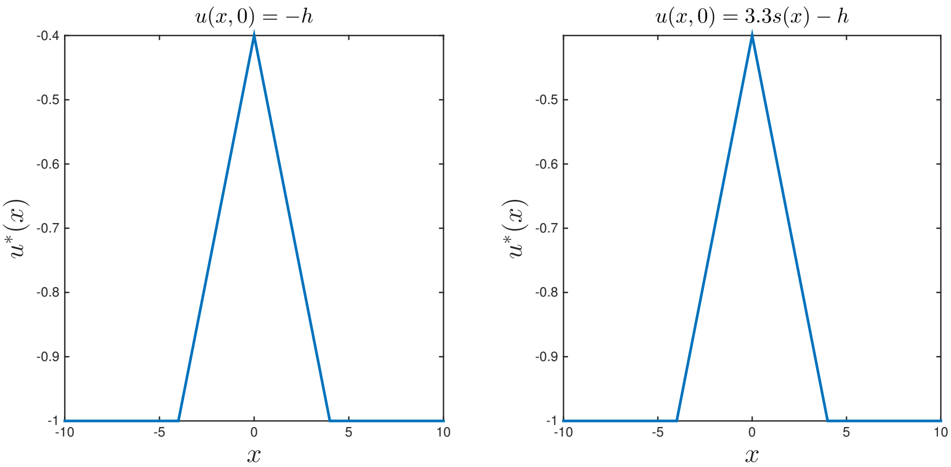

Figure 0.9: Stable state for Neural Amari field for different initial conditions when $w(x) \leftarrow 0.1w(x)$. $u(x,0) = -h$ (left) and $u(x,0) = 3.3s(x) - h$ (right)

2.4

When the initial condition is set to $u(x,0) = 3.3s(x) - h$ but during time $s(x) = 0$, the solution is the following:

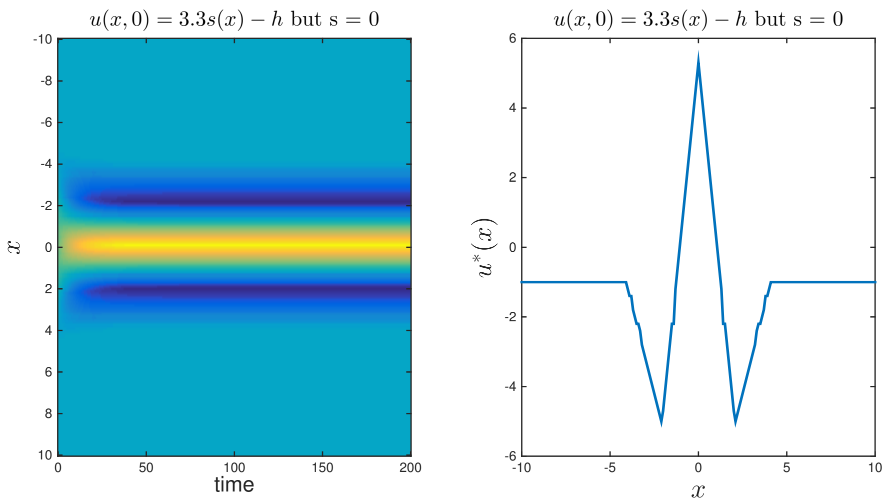

Figure 0.10: Neural Amari field for $u(x,0) = 3.3s(x) - h$ but no input ($s = 0$) Right: Spatio-temporal solution of the field. Left: Steady state solution

This stationary solution emerges because the initial condition is enough to set the field for convergence to the already known fixed point.