Dynamics of Neural Systems

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Exercise Sheet 3 Due Nov 20^{th} 2014

Please send the plots and code (latest) on the due date to the email address given above and bring your plots to the exercises. If possible, bring your laptop to show the solutions via beamer.

1 Exercise 1 Branching. Credits: 4

1.1

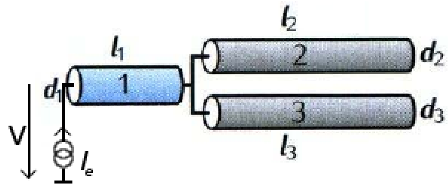

Figure 1: Multi-compartment model with two-way branch and input current.

Given a multi-compartment model with two-way branching as in Fig. 1, assume cylindrical compartments with identical electrical properties: specific membrane resistance $\tilde{r}_m = 1\Omega\text{m}^2$, specific membrane capacitance $\tilde{c}_m = 10^{-2}F/\text{m}^2$, specific axial resistance $\tilde{r}_a = 1\Omega\text{m}$. (Lecture naming convention (and Dayan textbook). Don't confuse with r_m and r_a or R_m and R_a from lecture or books.)

The compartments' geometrical dimensions and terminals are given in the table below. Assume that a constant current $I_0 = 1nA$ is injected and that the system has relaxed to a stationary solution. Assume $E_m = 0V$.

compartment Nr.	length l	diameter d	terminating segment
1	180 μm	12 μm	leaky[1]
2	100 μm	5 μm	killed
3	100 μm	4 μm	sealed

[1]) A compartment that branches into other compartments is classified as having a *leaky* terminating segment. The leak resistance R_L is computed by calculating the total resistance of the parallel input resistances of all branches.

Compute input resistance $R_{\text{in},j}$ for each compartment j. *(Hint: start at the terminals. Leak resistance R_L of parent compartment 1 is the inverse of the sum of the inverses of the input resistances R_{in} of the child branches.)* Use the formulas for the length constant λ and the input resistance for a semi-infinite cable R_∞:

$$\lambda = \sqrt{\frac{\tilde{r}_m d}{4\tilde{r}_a}} = \sqrt{\frac{r_m}{r_a}} \tag{1}$$

$$R_\infty = \frac{\tilde{r}_m}{\pi d\lambda} = \sqrt{\frac{4\tilde{r}_m\tilde{r}_a}{\pi^2 d^3}} = \sqrt{r_m r_a} \tag{2}$$

Do not confuse the length l with the electrotonic length $L = \frac{l}{\lambda}$. *(Hint: See also slides 11 and 12 of Lecture 2.)*

1.2

Compute the potential V as specified in Fig. 1.

1.3

Compute the currents I_2, I_3 flowing through each child-compartment. *(Hint: Remember voltage divider or current divider formula.)*

1.4

Assuming compartment 2 is terminated by a sealed end, compute the membrane potential V.

2 Exercise 2 Equivalent Cylinder. Credits: 4

2.1

Given the model in Fig. 1, assume the electrical properties as above and the geometric dimensions given in the table below:

compartment Nr.	length l	diameter d	terminating segment
1	200 μm	16 μm	leaky/branch
2	100 μm	4 μm	sealed
3	100 μm	2 μm	sealed

Can this branching model of a neuron be simplified by an equivalent cylinder model (without branches)? If yes, calculate the geometric properties of the equivalent cylinder. If not, what are the rules that are violated?

2.2

Given the model in Fig. 1, assume the electrical properties as above and the geometric dimensions given in the table below:

compartment Nr.	length l	diameter d	terminating segment
1	260 μm	? μm	leaky/branch
2	140 μm	4 μm	sealed
3	100 μm	? μm	sealed

What diameters should compartments 1 and 3 have, in order to allow the simplification of the two-way branch model in Fig. 1 to an equivalent cylinder? What is the length l_e of the equivalent cylinder?

2.3

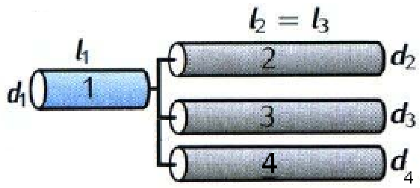

Figure 2: Multi-compartment model with three-way branch.

compartment Nr.	length l	diameter d	terminating segment
1	210 μm	16 μm	leaky/branch
2	150 μm	? μm	sealed
3	100 μm	? μm	sealed
4	120 μm	? μm	sealed

Assume the given geometrical dimensions in this table and the electrical properties as above. Analogous to the previous question, calculate the diameters the child compartments 2-4 must have, in order to allow the simplification of the three-way branch model in Fig. 2 to a single equivalent cylinder? What is the length l_e of the equivalent cylinder?

3 Exercise 3 Simulation of Two-compartment model of passive membrane. Credits: 4

For the implementation, choose your preferred programming language (e.g. Python, Matlab, ...). Implement all parameters to ease later value changes.

Send your code (and plots) on the due date to the email address given above, and bring the plots to the exercises (digital or printed).

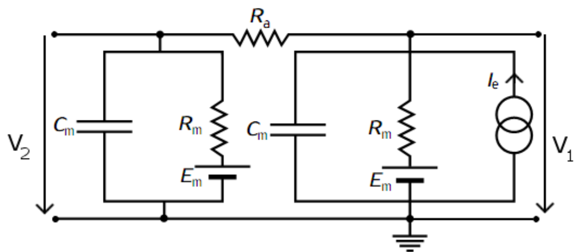

Figure 3: Two compartment model with input current.

Assume $E_m = 0V$, membrane resistance $R_m = 265M\Omega$, axial resistance $R_a = 7M\Omega$ and membrane capacitance $C_m = 75pF$.

3.1

Given the two-compartment model of a neuron in Fig. 3, derive the DEQ system for V_1 and V_2. *(Hint: Use Kirchoff's current law.)*

3.2

Approximate $\dot{V}_1$ and $\dot{V}_2$ with $\dot{V} = \frac{\mathrm{d}V}{\mathrm{d}t} \approx \frac{V(t+\Delta t)-V(t)}{\Delta t}$ and solve for $V_1(t+\Delta t)$ and $V_2(t+\Delta t)$. Implement this in your preferred programming language and simulate with $\Delta t = 0.1ms$ (or smaller), $R_a = 7M\Omega$, $t_e = 0.4$s, $t_s = 0.44$s and input current $I_e(t)$:

$$I_e(t) = \begin{cases} 0 & t < t_e \\ -100pA & t_e \leq t < t_s \\ 0 & t_s \leq t \end{cases} \tag{3}$$

Simulate with $R_{\mathrm{a},1} = 7M\Omega$, $R_{\mathrm{a},2} = 265M\Omega$ and $R_{\mathrm{a},3} = 30G\Omega$. Plot $V_1(t)$ and $V_2(t)$ against t. What happens depending on R_a? What does this mean for axons or dendrites?

3.3

Set the axial resistance to $R_a = 300M\Omega$. Simulate with input current $I_e(t) = 100pA\sin(2\pi f t)$. Plot amplitudes of $\log V_1(t)$ and $\log V_2(t)$ after relaxing to sinusoidal output voltage, e.g. $t > 1s$, against $\log f$ for the following frequencies (Bode Diagram):

- $f_1 = 1$Hz, $f_2 = 2$Hz, $f_3 = 5$Hz
- $f_4 = 10$Hz, $f_5 = 20$Hz, $f_6 = 50$Hz
- $f_7 = 100$Hz, $f_8 = 200$Hz, $f_9 = 500$Hz
- $f_{10} = 1$kHz, $f_{11} = 2$kHz, $f_{12} = 5$kHz