Dynamics of Neural Systems

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Exercise Sheet 5 due Dec 4^{th} 2014

Please send the plots and code on the due date to the email address given above and bring your plots to the exercises. If possible, bring your laptop to show the solutions via beamer.

1 Exercise 1 Simulation of multi compartment HH model. Credits: 4

The Hodgkin-Huxley (HH) model for an active neurite with input current $I_e(j,t)$ for a single cylindrical compartment j is given in Eqs. (2)-(12). *(The unit of V in the equations is [mV], Eq. [1] exists to clarify the unit in the exponent of the gating probabilities.)*

$$V' = \frac{V}{mV} \tag{1}$$

$$I_e(t) = C_m \frac{\mathrm{d}V_j'}{\mathrm{d}t} + g_L\left(V_j' - E_L\right) + g_{\mathrm{Na},j}\left(V_j' - E_{\mathrm{Na}}\right) + g_{\mathrm{K},j}\left(V_j' - E_{\mathrm{K}}\right) + g_{\mathrm{ax}}\left(V_j' - V_{j-1}'\right) + g_{\mathrm{ax}}\left(V_j' - V_{j+1}'\right) \tag{2}$$

$$g_{\mathrm{Na},j} = \bar{g}_{\mathrm{Na}} m_j^3 h_j, \quad g_{\mathrm{K},j} = \bar{g}_{\mathrm{K}} n_j^4 \tag{3}$$

Sodium channel:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(1-m) - \beta_m m \tag{4}$$

$$\alpha_m = \begin{cases} 0.1\frac{V'-25}{1-\exp\left(-\frac{V'-25}{10}\right)} & V \neq 25mV \\ 1 & V = 25mV \end{cases} \tag{5}$$

$$\beta_m = 4\exp\left(-\frac{V'}{18}\right) \tag{6}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(1-h) - \beta_h h \tag{7}$$

$$\alpha_h = 0.07\exp\left(-\frac{V'}{20}\right) \tag{8}$$

$$\beta_h = \frac{1}{1+\exp\left(-\frac{V'-30}{10}\right)} \tag{9}$$

Potassium channel:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(1-n) - \beta_n n \tag{10}$$

$$\alpha_n = \begin{cases} 0.01\frac{V'-10}{1-\exp\left(-\frac{V'-10}{10}\right)} & V \neq 10mV \\ 0.1 & V = 10mV \end{cases} \tag{11}$$

$$\beta_n = 0.125\exp\left(-\frac{V'}{80}\right) \tag{12}$$

The electrical and geometrical properties are:

- $C_m = 1\mu F$, membrane capacitance.
- $E_{\mathrm{Na}} = 115mV$, sodium equilibrium potential.
- $E_{\mathrm{K}} = -12mV$, potassium equilibrium potential.

- $E_\mathrm{L} = 10.6mV$, leak equilibrium potential.
- $V(0) = 0mV$, (starting and) membrane equilibrium potential.
- $\bar{g}_\mathrm{Na} = 120mS$, maximum conductance for sodium channel.
- $\bar{g}_\mathrm{K} = 36mS$, maximum conductance for potassium channel.
- $g_\mathrm{L} = 0.3mS$, leak conductance.
- $g_\mathrm{ax} = 0.5mS$, axial conductance.
- $N = 100$, number of compartments.

Assume the first compartment as a "sealed end" terminal and the last compartment as a "killed end" terminal. Remember the way to model these types of terminals from the last exercise sheet.

1.1

Given the Hodgkin-Huxley model in Eqs. (2)-(12), approximate all DEQs and derive $V(j,t)$ for an arbitrary compartment j.

1.2

$$I_e(j,t) = \begin{cases} 0 & (t < t_e) \vee (t_s \leq t) \vee (j \neq j_e) \\ I_0 & (t_e \leq t < t_s) \wedge (j = j_e) \end{cases} \tag{13}$$

Assume the rectangle impulse input given in Eq. (13) with $j_e = 14$, $t_e = 60ms$, $t_s = 260ms$ and different amplitudes $I_0 = 6\mu A$, $I_0 = 8\mu A$, $I_0 = 15\mu A$, $I_0 = 20\mu A$. Simulate and 3D-plot the resulting potential $V(j,t)$ (as a function of j and t).

2 OPTIONAL: Exercise 2 HH model extension: A-Type current, single compartment model. Credits: 3

Extending the Hodgkin Huxley model given on the last sheet in Ex. 2, we incorporate the A-type potassium conductance g_A with associated equilibrium potential E_A. This, as well as experimental differences (e.g. room temp.), have an effect on the parameters of the model:

$$V' = \frac{V}{mV} \tag{14}$$

$$I_e(t) = C_m \frac{\mathrm{d}V'}{\mathrm{d}t} + g_L(V' - E_L) + g_\mathrm{Na}(V' - E_\mathrm{Na}) + g_\mathrm{K}(V' - E_\mathrm{K}) + g_\mathrm{A}(V' - E_\mathrm{A}) \tag{15}$$

$$g_\mathrm{Na} = \bar{g}_\mathrm{Na} m^3 h, \quad g_\mathrm{K} = \bar{g}_\mathrm{K} n^4, \quad g_\mathrm{A} = \bar{g}_\mathrm{A} a^3 b \tag{16}$$

Sodium channel:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(1-m) - \beta_m m \tag{17}$$

$$\alpha_m = \begin{cases} 3.8 \frac{0.1(V'+34.7)}{1-\exp\left(-\frac{V'+34.7}{10}\right)} & V \neq -34.7mV \\ 3.8 & V = -34.7mV \end{cases} \tag{18}$$

$$\beta_m = 3.8 * 4 \exp\left(-\frac{V'+59.7}{18}\right) \tag{19}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(1-h) - \beta_h h \tag{20}$$

$$\alpha_h = 3.8 * 0.07 \exp\left(-\frac{V'+53}{20}\right) \tag{21}$$

$$\beta_h = 3.8 \frac{1}{1+\exp\left(-\frac{V'+23}{10}\right)} \tag{22}$$

Potassium channel:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(1-n) - \beta_n n \tag{23}$$

$$\alpha_n = \begin{cases} \frac{3.8}{2}\frac{0.01(V'+50.7)}{1-\exp\left(-\frac{V'+50.7}{10}\right)} & V \neq -50.7mV \\ \frac{3.8}{2} * 0.1 & V = -50.7mV \end{cases} \tag{24}$$

$$\beta_n = \frac{3.8}{2} 0.125 \exp\left(-\frac{V'+60.7}{80}\right) \tag{25}$$

A-type potassium channel:

$$a_\infty = \left(\frac{0.0761\exp\left(\frac{V'+99.22}{31.84}\right)}{1+\exp\left(\frac{V'+6.17}{28.93}\right)}\right)^{\frac{1}{3}} \tag{26}$$

$$\tau_a = 0.3632 + \frac{1.158}{1+\exp\left(\frac{V'+60.96}{20.12}\right)} \tag{27}$$

$$b_\infty = \frac{1}{\left(1+\exp\left(\frac{V'+58.3}{14.54}\right)\right)^4} \tag{28}$$

$$\tau_b = 1.24 + \frac{2.678}{1+\exp\left(\frac{V'-55}{16.027}\right)} \tag{29}$$

where:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{a_\infty - a}{\tau_a}$$

The electrical properties for this neurite are:

- $C_m = 1\mu F$, membrane capacitance.
- $E_{\mathrm{Na}} = 50mV$, sodium equilibrium potential.
- $E_{\mathrm{K}} = -77mV$, potassium equilibrium potential.
- $E_{\mathrm{A}} = -80mV$, A-type potassium equilibrium potential.
- $E_{\mathrm{L}} = -22mV$, leak equilibrium potential.
- $V(0) = -73mV$, (starting) membrane equilibrium potential.
- $\bar{g}_{\mathrm{Na}} = 120mS$, maximum conductance for sodium channel.
- $\bar{g}_{\mathrm{K}} = 20mS$, maximum conductance for potassium channel.
- $\bar{g}_{\mathrm{A}} = 47.7mS$, maximum conductance for A-type potassium channel.
- $g_{\mathrm{L}} = 0.3mS$, maximum leak conductance.

$$I_e(j,t) = \begin{cases} 0 & (t < t_e) \vee (t_s \leq t) \\ I_0 & (t_e \leq t < t_s)) \end{cases} \tag{30}$$

Assume the rectangle impulse input given in Eq. (30) with $t_e = 60ms$ and, e.g. $t_s = 460ms$. (Increase t_s if stationary state is not yet reached or clearly visible.)

2.1

Simulate with different input current amplitudes: from $I_0 = 5\mu A$ to $I_0 = 20\mu A$ in steps of e.g. $\Delta I = 0.5\mu A$. Plot the firing rate in the stationary state as function of the input current $I_e(t)$. *(The threshold for calculating the firing rate is now lower than for the resting potential at 0mV, use e.g. 20mV.)*