Dynamics of Neural Systems

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Exercise Sheet 1 due Nov 13^{th} 2014

Please send plots and code on the due date to the email address given above and bring your plots to the exercises. If possible, bring your laptop to show the solutions on a projector.

1 Exercise 1 Green's function. Credits: 4

1.1

Assume a linear point neuron model. In order to study its properties in an electrophysiological experiment, a current $I_e(t)$ is injected via an intracellular electrode (current clamp experiment). The neuron can be described by the circuit in Fig. 1.

Derive the differential equation (DEQ) for the membrane potential for an input current $I_e(t)$. Assume a δ-pulse input of the form in Eq. (1). Use the substitution $V' = V - E_M$ and derive the DEQ for V'.

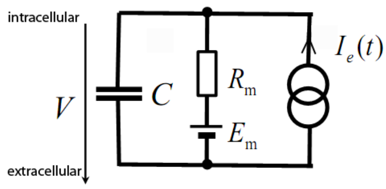

Figure 1: Equivalent circuit with electrode current I_e.

$$I_e(t) = Q_0 \delta(t) \tag{1}$$

Variables:

- Q_0 = The charge injected.

1.2

Apply the Fourier Transform to the last DEQ and solve for $\tilde{V}(\omega) = F[V'(t)](\omega)$. Remember Eqs. (2)-(4).

$$F[f](\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \,\mathrm{d}t = \tilde{f}(\omega) \tag{2}$$

$$F\left[\frac{\mathrm{d}f(t)}{\mathrm{d}t}\right](\omega) = i\omega F[f(t)](\omega) \tag{3}$$

$$F[\delta(t)](\omega) = 1 \tag{4}$$

Variables:

- i = The imaginary unit, i.e. $i^2 = -1$.
- ω = The angular frequency.

1.3

Using the last result, compute the backtransform of $\tilde{V}(\omega)$ from the frequency to the time domain. Use the following back-transforms, Eq. (5): *(Hint: When you input a δ peak into a linear time-invariant system, you* *receive the impulse response $h(t)$.)*

$$F^{-1}\left[\frac{1}{c+i\omega}\right](t) = e^{-ct}\mathbf{1}(t) \tag{5}$$

$$V'(t) = F^{-1}\left[\tilde{V}(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{V}(\omega)e^{i\omega t}\,\mathrm{d}\omega \tag{6}$$

1.4

What does $V'(t)$ look like for time-shifted input, i.e. current injected at time $t = t_0$ instead of $t = 0$?

1.5

Compute membrane voltage response $V'(t)$ for an arbitrary input current $I_e(t)$. *(Remember the superposition property of linear systems: The input can be described as a superposition of weighted δ-peaks, Eq.* (7)*, and the result is itself a superposition of impulse responses $h(t)$.)*

$$I_e(t) = \int_{-\infty}^{\infty}\delta(t-t')I_e(t')\,\mathrm{d}t' \tag{7}$$

1.6

Compute the membrane voltage response $V'(t)$ for a sinusoidal input current, Eq. (8).*(Hint:Sine and cosine Fourier transforms given in Eq.* (9) *and* (10)*.)*

$$I_e(t) = I_0\sin(\omega_0 t) \tag{8}$$

$$F[\sin(\omega_0 t)](\omega) = \frac{\pi}{i}(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)) \tag{9}$$

$$F[\cos(\omega_0 t)](\omega) = \pi(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)) \tag{10}$$

2 Exercise 2 Passive Axon properties. Credits: 2

The passive properties of axons depend on their geometry. Assume a cylindrical axon of diameter d and length L. Assume specific intracellular resistivity $\tilde{r}_a = 1\,\Omega\,\mathrm{m}$, specific membrane resistance $\tilde{r}_m = 1\,\Omega\,\mathrm{m}^2$ and specific membrane capacitance $\tilde{c}_m = 10^{-2}F\mathrm{m}^{-2}$. (See units Lecture 2 slide 19.)

	length L	**diameter d**
Axon 1	10mm	2μm
Axon 2	20mm	2μm
Axon 3	10mm	4μm

For axons with the parameters given in the table above, compute the following parameters:

2.1

the axial resistance R_a.

2.2

the membrane capacitance C_m.

2.3

the time constant $\tau = c_m r_m$.

2.4

How do the different parameters change with L and d. What could this imply for the signal propagation along the axon?

3 Exercise 3 Simulation of single compartment model. Credits: 4

For the implementation, choose your preferred programming language (e.g. Python, Matlab, ...).

Since we have calculated the response of a point neuron with a passive membrane we can implement this model in order to investigate its properties numerically. In later exercises we will extend this neuron model, embedding nonlinear elements.

For the exercises it will also be helpful to bring relevant plots of the results to our class.

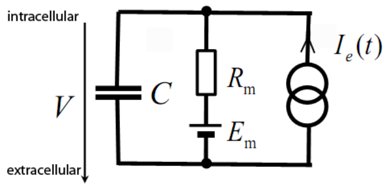

Figure 2: Equivalent circuit with electrode current.

Fig. 2 shows the familiar circuit discussed above.

Hint: Approximate $V(t)$ by sampling with timesteps $\Delta t = 0.1ms$. Approximate $\frac{dV}{dt}\big|_t$ by e.g. the backward Euler method:

$$\frac{V(t) - V(t - \Delta t)}{\Delta t}$$

The backward Euler method and better approximations can be found in Appendix B.1 of Sterrat 2011. Keep Δt as a parameter in your program that can be varied later on.

The cylindrical compartment is specified by:

- membrane equilibrium potential: $E_m = 0V$.
- length: $L = 100\mu$m.
- diameter: $d = 2\mu$m.
- specific membrane resistance: $\tilde{r}_m = 1\Omega\text{m}^2$.
- specific axial resistivity: $\tilde{r}_a = 1\Omega$m.
- specific membrane capacitance: $\tilde{c}_m = 10^{-2} F\,\text{m}^{-2}$

(Implement these parameters in a way that eases later value changes.)

3.1

Simulate the circuit in Fig. 2 and determine the time course for $V(t)$ for a step input $I_e(t)$, Eq. (11)

$$I_e(t) = \begin{cases} 0 & t < 0 \\ -50pA & 0 \leq t \end{cases} \tag{11}$$

3.2

Set Δt to the values 1ms, 10ms. What changes in the simulation?

3.3

Set Δt back to 0.1ms. Change the specific membrane capacitance to $\tilde{c}_m = 10^{-1} F\,\text{m}^{-2}$ and examine the changes in the simulation. Now reset $\tilde{c}_m$ to its original value and set the specific membrane resistance to $\tilde{r}_m = 10\Omega\text{m}^2$.

What happens in the simulation and why? What does this mean from a neuron's viewpoint?

3.4

Reset $\tilde{r}_m$ to its original value and implement the following sinusoidal input signal, Eq. (12).

$$I_e(t) = 100pA \sin(2\pi f t) \tag{12}$$

and test the model for the frequencies:

- $f_1 = 0.5$Hz

- $f_2 = 1\text{Hz}$
- $f_3 = 2\text{Hz}$
- $f_4 = 8\text{Hz}$
- $f_5 = 100\text{Hz}$
- $f_6 = 1000\text{Hz}$

For the converged state, plot the log of the amplitude of the voltage $V(t)$ against the log of the frequency (Bode diagram). Explain the result.