介面說明：

開發平台 — Matlab

執行 — ① forward kinamic : stanford_forward.m

② inverse kinamic : standford_inverse.m

程式架構．數學運算說明：

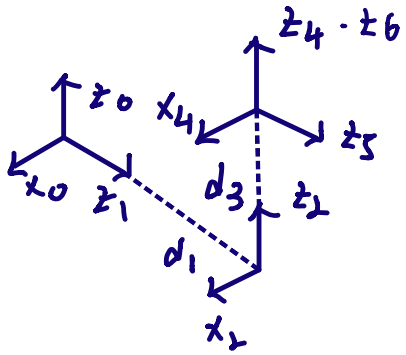
① stanford_forward.m

1° 輸入符合各軸工作範圍之參數

↳ function : InputParameter( )

2° 計算 forward kinamic

$$A_n = Rot(z,\theta_n)\cdot Trans(0,0,d_n)\cdot Trans(a_n,0,0)\cdot Rot(x,\alpha_n)$$



$$= \begin{pmatrix} c\theta_n & -s\theta_n\cdot c\alpha_n & s\theta_n\cdot s\alpha_n & a_n\cdot c\theta_n \\ s\theta_n & c\theta_n\cdot c\alpha_n & -c\theta_n\cdot s\alpha_n & a_n\cdot s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} c1 & 0 & -s1 & 0 \\ s1 & 0 & c1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad A_4 = \begin{pmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad A_5 = \begin{pmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad A_6 = \begin{pmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$T = A_1 A_2 A_3 A_4 A_5 A_6$

↳ function : TransMatrics( )

3° 從 T 得 $(x, y, z, \phi, \theta, \psi)$

↳ function : Euler_To_cartesian( )

② standford_inverse.m

1° 輸入 T

↳ Trans = input('')

2° 計算 inverse kinetic

$$A_1 A_2 = \begin{pmatrix} c_1 c_2 & -s_1 & c_1 s_2 & -s_1 d_2 \\ s_1 c_2 & c_1 & s_1 s_2 & c_1 d_2 \\ -s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & C_1S_2d_3 - S_1d_2 \\ S_1C_2 & C_1 & S_1S_2 & S_1S_2d_3 + C_1d_2 \\ -S_2 & 0 & C_2 & C_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 A_5 = \begin{bmatrix} C_4C_5 & -S_4 & C_4S_5 & 0 \\ S_4C_5 & C_4 & S_4S_5 & 0 \\ -S_5 & 0 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 A_5 A_6 = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & 0 \\ S_4C_5C_6 + C_4C_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1 A_2 A_3 A_4 A_5 A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_1^{-1} T = {}^1T_6$$

$$\begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \sim & \sim & \sim & S_2d_3 \\ \sim & \sim & \sim & -C_2d_3 \\ \sim & \sim & \sim & d_2 \\ \sim & \sim & \sim & 1 \end{bmatrix}$$

$$-S_1 p_x + C_1 p_y = d_2$$

let $\phi = \tan^{-1}\left(\frac{p_y}{p_x}\right)$, $\rho = \sqrt{p_x^2 + p_y^2}$

$$\longrightarrow \quad -S_1 \cdot P\cos\phi + C_1 \cdot P\sin\phi = d_2$$

$$\longrightarrow \quad P \cdot \sin(\phi - \theta_1) = d_2$$

$$\longrightarrow \quad \theta_1 = \phi - \sin^{-1}\left(\frac{d_2}{P}\right) \quad \cdots \text{ 2 solution}$$

$$= \tan^{-1}\left(\frac{P_y}{P_x}\right) - \tan^{-1}\frac{d_2}{\pm\sqrt{P^2 - d_2^2}}$$

$$C_1 P_x + S_1 P_y = S_2 d_3$$

$$-P_z = -C_2 d_3 \quad \text{or} \quad P_z = C_2 d_3 \quad \cdots \text{ 2 solution}$$

$$\longrightarrow \quad \frac{1}{\pm 1}\tan(\theta_2) = \frac{C_1 P_x + C_2 P_y}{-P_z} \quad \text{or} \quad \tan(\theta_2) = \frac{C_1 P_x + C_2 P_y}{P_z}$$

$$\longrightarrow \quad \theta_2 = \tan^{-1}\left(\frac{-(C_1 P_x + C_2 P_y)}{-P_z}\right) \text{ or } \tan^{-1}\left(\frac{(C_1 P_x + C_2 P_y)}{+P_z}\right)$$

$$-P_z = -C_2 d_3$$

$$\longrightarrow \quad d_3 = \frac{P_z}{C_2}$$

$$T_3^{-1} \cdot T = A_4 A_5 A_6$$

$$\begin{bmatrix} C_1C_2 & S_1C_2 & -S_2 & 0 \\ -S_1 & C_1 & 0 & 0 \\ C_1S_2 & S_1S_2 & C_2 & 0 \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & 0 \\ S_4C_5C_6 + C_4C_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_1c_2a_x + s_1c_2a_y - s_2a_z = c_4s_5$$

$$-s_1a_x + c_1a_y = s_4s_5$$ By T(1,4) and T(2,4)

if $s_5 > 0$

$$\longrightarrow \theta_4 = \tan^{-1}\left(\frac{-s_1a_x + c_1a_y}{c_1c_2a_x + s_1c_2a_y - s_2a_z}\right)$$

if $s_5 < 0$

$$\longrightarrow \theta_4 = \tan^{-1}\left(\frac{-(-s_1a_x + c_1a_y)}{-(c_1c_2a_x + s_1c_2a_y - s_2a_z)}\right)$$

$$-s_1a_x + c_1a_y = s_4s_5$$

$$c_1s_2a_x + s_1s_2a_y + c_2a_z = c_5$$ By T(2,4) and T(3,4)

$$s_4 \cdot \tan(\theta_5) = \frac{-s_1a_x + c_1a_y}{c_1c_2a_x + s_1s_2a_y + c_2a_z}$$

$$\longrightarrow \theta_5 = \tan^{-1}\left(\frac{-s_1a_x + c_1a_y}{(c_1s_2a_x + s_1s_2a_y + c_2a_z)\cdot s_4}\right)$$

$$-(c_1s_2n_x + s_1s_2n_y + c_2n_z) = s_5c_6$$

$$c_1s_2o_x + s_1s_2o_y + c_2o_z = s_5s_6$$ By T(3,1) and T(3,2)

$$\longrightarrow \frac{s_5}{s_5}\tan\theta_6 = \frac{c_1s_2o_x + s_1s_2o_y + c_2o_z}{-c_1s_2n_x - s_1s_2n_y - c_2n_z}$$

if $s_5 > 0$

$$\longrightarrow \theta_6 = \tan^{-1}\left(\frac{c_1s_2o_x + s_1s_2o_y + c_2o_z}{-c_1s_2n_x - s_1s_2n_y - c_2n_z}\right)$$

if $S_5 < 0$

$$\longrightarrow \theta_6 = \tan^{-1}\left(\frac{-(C_1 S_2 O_x + S_1 S_2 O_y + C_2 O_z)}{-(-C_1 S_2 n_x - S_1 S_2 n_y - C_2 n_z)} \right)$$

3. 輸出

(function : Inverse_output( )