Programming for Economic Modeling: Signaling

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Spence's education game

- Spence (1973)
  - ✓ here, MWG §13.C
- Key ingredients: senders (job applicants) *vs.* receivers (employers)
  - ✓ employers care about the "quality," or productivity of job applicants
  - ✓ before job application, applicants choose years/level of education
  - ✓ education does *not* change productivity of an applicant
  - ✓ employers only observe years/level of education, not productivity of an applicant
  - ✓ costs of education also depend on the quality, and so productivity of an applicant
  - $\Rightarrow$ different types of applicant may choose different levels of educations
  - ✓ higher quality/productivity applicants $\Rightarrow$ lower costs $\Rightarrow$ more years of education
  - $\Rightarrow$ employers may want to infer productivity from years of education
- What do you think will happen?

Can employers correctly infer productivity from years of education?

- If applicants do not behave strategically
  - ✓ by Bayesian updating, more years of education indicate higher productivity
  - $\Rightarrow$ (by model setting) employers offer higher wage
- Strategic play
  - ✓ more education $\Rightarrow$ higher wage
  - ✓ applicants with lower quality/productivity may distort schooling decision
  - $\Rightarrow$ more education may also come from less-capable applicants
  - ✓ what constitutes an equilibrium?
- A perfect Bayesian equilibrium consists of strategy and *belief*
  - ✓ strategy: no unilateral deviation, given others' strategy and *belief*
  - ✓ belief: consistent with strategy, i.e., using Bayesian updating when applicable
  - ✓ here, applicant's years of education maximizes his/her payoffs
  - ✓ employer's wage offer justified by his/her belief
  - ✓ belief derived from applicant's equilibrium strategy: $\Pr(\text{type}|12 \text{ year}) =$

$$\text{if} \begin{cases} \text{only type}-\alpha \text{ would choose 12 years of education in equilibrium} \\ \text{two types } (\alpha \text{ and } \beta) \text{ would choose 12 years} \\ \text{no type would choose 12 years} \end{cases}$$

Two (major) types of equilibrium

- In equilibrium, what employers could learn from education
- Separating equilibrium
  - ✓ different types/qualities/productivities of applicants choose different years of education
  - $\Rightarrow$ employers learn perfectly true type/productivity of an applicant $\Rightarrow$ no lyin *in equilibrium*
  - ✓ incentive-compatibility condition: to prevent lying/deviation
- Pooling equilibrium
  - ✓ applicants of different types choose the same years of education
  - ✓ no learning at all, employers maintain *ex ante* belief
  - ✓ no deviation: what would happen if choosing other years of education? $\Rightarrow$ off-path event
- Semi-pooling or semi-separating equilibrium
  - ✓ partial revelation

$$\text{e.g.,} \quad \begin{cases} \text{type-}\alpha \text{ chooses 12 years in equilibrium} \\ \text{type-}\beta \text{ choose 12 years and 16 years with equal probability} \end{cases}$$

$$\Rightarrow \quad \begin{cases} \Pr(\text{type}|12 \text{ years}) = \\ \Pr(\text{type}|16 \text{ years}) = \end{cases}$$

  - ✓ type-β plays mixed strategy $\Rightarrow$ indifference between mixing strategies

Model illustration

- Applicants: *ex ante* identical with mass one, or only one applicant
  - ✓ two types: productivity $\theta_H > \theta_L > 0$, and *ex ante* belief $Pr(\theta = \theta_H) = \lambda \in (0,1)$
  - ✓ high- *vs.* low-type
  - ✓ payoff u = wage (w) − cost of education (c)
  - ✓ cost of education $c(e,\theta)$, with $c(0,\cdot) = 0$ and

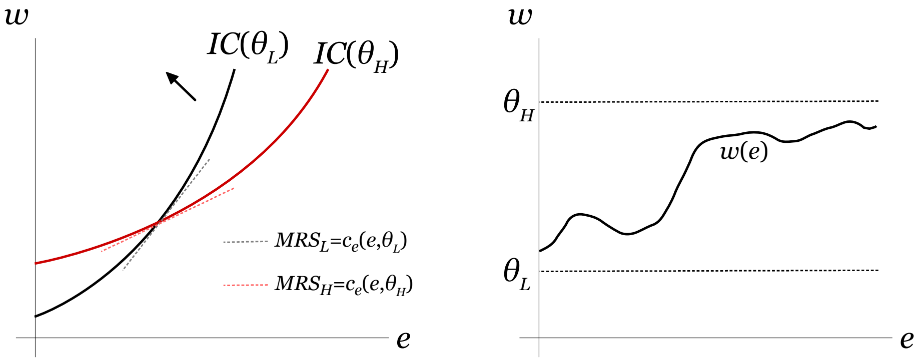
$$\begin{cases} c_e \text{ and } c_{ee} > 0 & \Rightarrow & \text{interpretation?} \\ c_\theta \text{ and } c_{e\theta} < 0 & \Rightarrow & \text{interpretation?} \end{cases}$$

  - ✓ single-crossing property: check marginal rate of substitution and indifference curves

$$MRS = \left|\frac{\partial u/\partial e}{\partial u/\partial w}\right| = c_e(e,\theta),\ \uparrow \text{in } e \text{ and} \downarrow \text{in } \theta$$

- Employers
  - ✓ productivity θ generates revenue $\theta \Rightarrow$ maximal wage offer $\mathbb{E}(\theta|\cdot)$
  - ✓ no bargaining power: Bertrand competition in labor market or applicants propose wage
  - $\Rightarrow$ $w = \mathbb{E}(\theta|\text{observation})$
  - ✓ the same (updated) beliefs $\mu(\cdot) \equiv \Pr(\theta_H|\cdot) \in [0,1]$ both on and off equilibrium path
  - $\Rightarrow$ $w = \mu(\cdot)\theta_H + [1 - \mu(\cdot)]\theta_L \in [\theta_L, \theta_H]$
- (weakly) Perfect Bayesian equilibrium
  - ✓ e_H^*, e_L^*, μ^*, and $w^* = \mu^*\theta_H + (1-\mu^*)\theta_L$

Illustration of model structure



First best

- Employers directly observe θ
  - ✓ $\mu^*(\theta_H) = 1$ and $\mu^*(\theta_L) = 0$
  - $\Rightarrow$ $w^*(\theta_H) = \theta_H$ and $w^*(\theta_L) = \theta_L$
- Equilibrium education level
  - ✓ $e_H^* = e_L^* = 0$

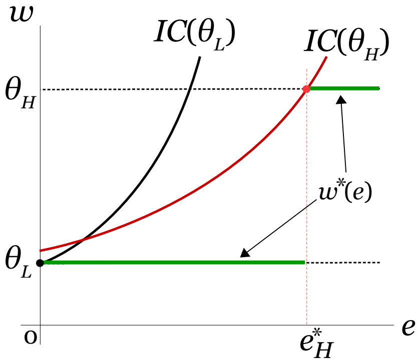
Second best: Separating equilibrium

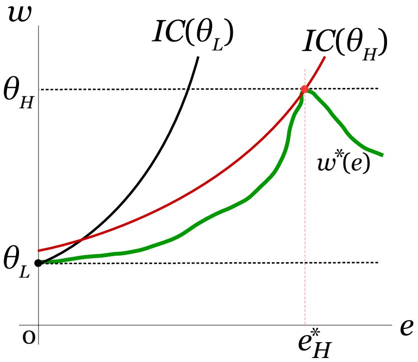
- $e_H^* \neq e_L^*$
  - ✓ $\mu^*(e_H^*) = 1$ and $\mu^*(e_L^*) = 0 \Rightarrow w^*(e_H^*) = \theta_H$ and $w^*(e_L^*) = \theta_L$
  - ✓ e_H^* and $e_L^* = ?$
- LEMMA: IN ANY SEPARATING EQUILIBRIUM, $e_L^* = 0$
  - ✓ in any separating equilibrium, $w^*(\theta_L) = \theta_L$, the lowest possible wage
  - ✓ suppose that $e_L^* > 0$, any incentives to deviate?
  - ✓ if the low-type deviates to a smaller $e \Rightarrow$ cost of education $\downarrow$, and wage cannot $\downarrow$
- Determination of e_H^*: no unilateral deviation
  - ✓ incentive compatibility for high-type: $\theta_H - c(e_H^*, \theta_H) \geqslant \theta_L - c(e_L^*, \theta_H) = \theta_L - 0$
  - ✓ incentive compatibility for low-type: $\theta_L - 0 \geqslant \theta_H - c(e_H^*, \theta_L)$
  - $\Rightarrow$ $c(e_H^*, \theta_L) \geqslant \theta_H - \theta_L \geqslant c(e_H^*, \theta_H)$, possible? why?
  - ✓ other deviations: for $e \neq \{e_H^*, 0\}$,

$$\begin{cases} e > e_H^* & \Rightarrow \quad \mu^* = 1 \text{ and so } w^*(e) = \theta_H \\ e \in (0, e_H^*) & \Rightarrow \quad \mu^* = 0 \qquad\qquad w^*(e) = \theta_L \end{cases} \Rightarrow \text{ verify}$$

  - ✓ unique off-path belief? unique e_H^*?

Illustration of separating equilibrium





Second best: pooling equilibrium

注意，separated, pooling 都是 second-bast

- $e_H^* = e_L^* = e^*$
  - ✓ $\mu^*(e^*) = \lambda \Rightarrow w^*(e^*) = \lambda\theta_H + (1-\lambda)\theta_L \equiv \mathbb{E}(\theta)$, *ex ante* average
  - ✓ no deviation: for all $e \neq e^*$ and $\mu^*(e)$,

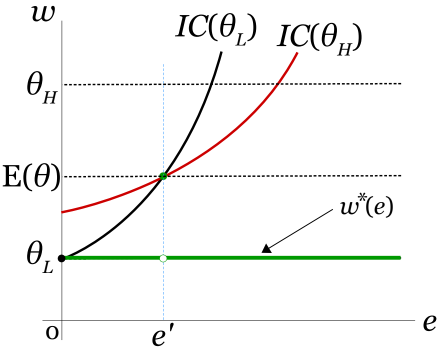
$$\begin{cases} \text{high-type:} & \mathbb{E}(\theta) - c(e^*, \theta_H) \geqslant w^*(e) - c(e, \theta_H) \\ \text{low-type:} & \mathbb{E}(\theta) - c(e^*, \theta_L) \geqslant w^*(e) - c(e, \theta_L) \end{cases}$$

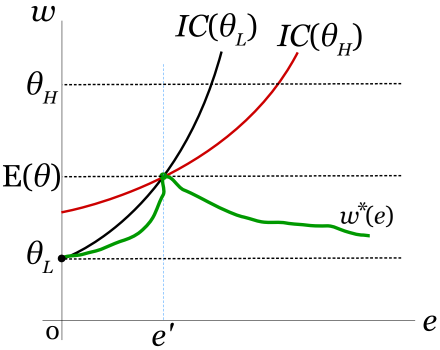
- Off-path belief: for all $e \neq e^*$, $\mu^*(e) = 0 \Rightarrow w^*(e) = \theta_L$
  - ✓ for $e > e^*$, wage ↓ but cost ↑; for $e < e^*$, wage ↓ and cost ↓
  - ✓ only need to consider deviation with least cost: $e = 0$, with $c(0, \theta_H) = c(0, \theta_L) = 0$

$$\begin{cases} \text{high-type:} & \mathbb{E}(\theta) - c(e^*, \theta_H) \geqslant \theta_L - 0 \\ \text{low-type:} & \mathbb{E}(\theta) - c(e^*, \theta_L) \geqslant \theta_L - 0 \qquad * \text{ (relevant condition)} \end{cases}$$

  - ✓ maximal possible equilibrium education e': $\mathbb{E}(\theta) - c(e', \theta_L) \equiv \theta_L - 0$
  - ✓ e' is robust to off-path beliefs (why?)
  - ✓ reasonable off-path belief? other off-path belief supporting the same equilibrium e^*?
  - ✓ multiple equilibria and Pareto ranking

Illustration of pooling equilibrium





Equilibrium refinement

- Often, multiple equilibria caused by freedom to choose off-path belief
  - ✓ e.g., in pooling equilibrium, reasonable to think that only low-type would deviate?
- Refinement
  - ✓ exclude "unreasonable" off-path beliefs $\Rightarrow$ delete corresponding equilibrium
  - ✓ domination-based refinement: a sender should not deviate to a strictly dominated strategy
  - ✓ intuitive criterion: should not deviate to a strategy dominated by *equilibrium* strategy
  - ✓ other refinements
  - ✓ not always working...