Homework 3

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$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_{i2} + \hat{\beta}_3 X_{i3} + \hat{\beta}_4 X_{i4} + \hat{u}_i \quad \sim ①$$

$$\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 \bar{X}_2 + \hat{\beta}_3 \bar{X}_3 + \hat{\beta}_4 \bar{X}_4 \quad \sim ②$$

$$① - ② \Rightarrow \quad Y_i - \bar{Y} = \hat{\beta}_1 - \hat{\beta}_1 + \hat{\beta}_2 (X_{i2} - \bar{X}_2) + \beta_3 (X_{i3} - \bar{X}_3) + \beta_4 (X_{i4} - \bar{X}_4) + \hat{u}_i$$

$$y_i = \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i3} + \hat{\beta}_4 x_{i4} + \hat{u}_i$$

$$\hat{\beta} = (x'x)^{-1} x'y = \begin{bmatrix} \sum x_{i2}^2 & \sum x_{i2}x_{i3} & \sum x_{i2}x_{i4} \\ \sum x_{i2}x_{i3} & \sum x_{i3}^2 & \sum x_{i3}x_{i4} \\ \sum x_{i2}x_{i4} & \sum x_{i3}x_{i4} & \sum x_{i4}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum x_{i2} y_i \\ \sum x_{i3} y_i \\ \sum x_{i4} y_i \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 10 & 5 \\ 10 & 30 & 15 \\ 5 & 15 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ -7 \\ -26 \end{bmatrix} = \begin{bmatrix} 0.15 & -0.05 & 0 \\ -0.05 & 0.07 & -0.04 \\ 0 & -0.04 & 0.08 \end{bmatrix} \begin{bmatrix} 7 \\ -7 \\ -26 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 0.2 \\ -1.8 \end{bmatrix}$$

$$S^2 = \frac{\hat{u}'\hat{u}}{n-k} = \frac{y'y - \hat{y}'\hat{y}}{n-k} = \frac{y'y - \hat{\beta}'x'(y-\hat{u})}{n-k} = \frac{y'y - \hat{\beta}'x'y}{n-k}$$

$$= \frac{60 - 55.2}{24 - 4} = 0.24$$

$$\widehat{Var}(\hat{\beta}) = S^2 (x'x)^{-1} = \begin{bmatrix} 0.036 & -0.012 & 0 \\ -0.012 & 0.0168 & -0.0096 \\ 0 & -0.0096 & 0.0196 \end{bmatrix}$$

$$H_0: \beta_2 = 1, \quad t = \frac{\hat{\beta}_2 - 1}{\sqrt{\widehat{Var}(\hat{\beta}_2)}} = \frac{1.4 - 1}{\sqrt{0.036}} = 2.108 > 2.086 = t_{\frac{\alpha}{2}=0.025,\, df=20}$$

$$H_0: \beta_3 = 1, \quad t = \frac{\hat{\beta}_3 - 1}{\sqrt{\widehat{Var}(\hat{\beta}_3)}} = \frac{0.2 - 1}{\sqrt{0.0168}} = -6.172 < -2.086$$

$$H_0: \beta_4 = -2, \quad t = \frac{\hat{\beta}_4 + 2}{\sqrt{\widehat{Var}(\hat{\beta}_4)}} = \frac{-1.8 + 2}{\sqrt{0.0196}} = 1.4434$$

We reject $H_0: \beta_2 = 1$ and $H_0: \beta_3 = 1$ but fail to reject $H_0: \beta_4 = -2$.

$H_0: \beta_2 = 1, \ \beta_3 = 1, \ \beta_4 = -2$

$$F = \frac{(R\hat{\beta} - r)'[R(X'X)^{-1}R']^{-1}(R\hat{\beta} - r)/4}{s^2}$$

$$R\hat{\beta} - r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.4 \\ 0.2 \\ -1.8 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 0.2 \\ -1.8 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ -0.8 \\ 0.2 \end{bmatrix}$$

$$[R(X'X)^{-1}R']^{-1} = [I_3(X'X)^{-1}I_3']^{-1} = [(X'X)^{-1}]^{-1} = X'X$$

$$= \frac{1}{0.24 \times 3} [0.4 \ \ -0.8 \ \ 0.2] \begin{bmatrix} 10 & 10 & 5 \\ 10 & 30 & 15 \\ 5 & 15 & 20 \end{bmatrix} \begin{bmatrix} 0.4 \\ -0.8 \\ 0.2 \end{bmatrix}$$

$$= \frac{11.2}{0.72}$$

$$= 15.556 > 3.10 = F_{1-\alpha = 0.95} \ (df1 = 3, \ df2 = 20)$$

We reject $H_0: \beta_2 = 1, \ \beta_3 = 1, \ \beta_4 = -2$.