Quantitative Method (I)

Department of Economics

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Fall 2017

Homework 1

(Due in Class on Oct. 11)

Consider a multiple linear regression model ($y = X\beta + u$) under the classical assumptions and answer the following questions. The notations here are defined as the same as the lecture notes. If you are not clear about the questions, please email me.

1. Use a unit vector $\underline{i}$ to show $\overline{y} = \frac{1}{n}\sum_{i=1}^{n}\widehat{y}_i$ in a model with an intercept.

2. Use $X = [\underline{i}, \underline{x}_2]$ to arrange the normal equation ($X'X\widehat{\beta} = X'y$) as two equations for a bivariate model with an intercept.

3. In a model with an intercept, $R^2 = \frac{\widehat{y}'m_0\widehat{y}}{y'm_0y}$ (centered R^2), where $m_0\widehat{y} = m_0X\widehat{\beta}$. Briefly explain why the first column is a null vector in $m_0X = [\underline{0}, \underline{x}_2 - \underline{\overline{x}}_2, \ldots, \underline{x}_k - \underline{\overline{x}}_k]$ and show that the R-squared can be rearranged as the sample correlation coefficient $r = \frac{\widehat{y}'m_0y}{\sqrt{y'm_0y}\sqrt{\widehat{y}'m_0\widehat{y}}}$. Hint: try to first show $\widehat{y}'m_0\widehat{y} = \widehat{y}'m_0y$.

4. $R^2 = \frac{\widehat{y}'\widehat{y}}{y'y}$ (uncentered R^2) is used in a model without an intercept, in which the original definition of the R-squared becomes problematic ($\frac{ESS}{TSS} \neq 1 - \frac{RSS}{TSS}$). More details can be found in showing $\sum_{i=1}^{n}(y_i - \overline{y})^2 \neq \sum_{i=1}^{n}(y_i - \widehat{y}_i)^2 + \sum_{i=1}^{n}(\widehat{y}_i - \overline{y})^2$.

5. The $100(1-\alpha)$ percent confidence interval for $E(y_f)$ is $\widehat{y}_f \pm s\sqrt{\underline{x}_f'(X'X)^{-1}\underline{x}_f}\; t_{\frac{\alpha}{2},(n-k)}$. Show that if $X = [\underline{i}, \underline{x}_2]$ and $\underline{x}_f' = [1, x_{f2}]$, then $\underline{x}_f'(X'X)^{-1}\underline{x}_f = \frac{1}{n} + \frac{(x_{f2} - \overline{x}_2)^2}{\sum_{i=1}^{n}(x_{i2} - \overline{x}_2)^2}$. Note: y_f, $\widehat{y}_f$, x_{f2}, x_{i2} and $\overline{x}_2$ are scalars.