Quantitative Method (I)

Department of Economics

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Fall 2017

Homework 2

(Due in Class on Oct. 25)

1. Consider the binomial variable Y, which takes on the values zero or one according to the probability density function

$$f(Y) = \theta^Y (1-\theta)^{(1-Y)},$$

where $0 \leq \theta \leq 1$ and $Y = 0$ or 1. Thus the probability of a success ($Y = 1$) is given by $f(1) = \theta$ and the probability of failure ($Y = 0$) is given by $f(0) = 1-\theta$.

   (a) Show $E(Y) = \theta$ and $Var(Y) = \theta(1-\theta)$.

   (b) If a random sample of n observation is drawn from this distribution, find the maximum likelihood estimator of θ.

   (c) Find the variance of the maximum likelihood estimator of θ. Hint: $E(Y_i, Y_j) = E(Y_i)E(Y_j)$ if $Cov(Y_i, Y_j) = 0$ with $i \neq j$.

   (d) Is the maximum likelihood estimator of θ unbiased and/or consistent?

   (e) Find the asymptotic variance of the maximum likelihood estimator of θ using the Hessian matrix.

   (f) Is the asymptotic variance the same as the exact finite sample variance?

2. Write down the probability density functions of the joint normal distribution for X and Y ($f(X, Y)$) and the marginal distribution for X ($f(X)$), and take the ratio of $f(X, Y)$ to $f(X)$ to show that the conditional mean is a linear function of X and the conidtional variance is invariant with X. Specifically, show that $\mu_{Y|X} = \alpha + \beta X$ and $\sigma^2_{Y|X} = \sigma^2_Y(1-\rho^2)$, where $\alpha = \mu_Y - \beta\mu_X$ and $\beta = \rho\frac{\sigma_Y}{\sigma_X}$. Note: μ is mean, σ^2 is variance and ρ is correlation coefficient.

3. Consider a multiple linear regression model ($y = X\beta + u$) under the classical assumptions and answer the following questions about the F statistic, $F = \frac{1}{q}(R\widehat{\beta} - \underline{r})'[s^2R(X'X)^{-1}R']^{-1}(R\widehat{\beta} - \underline{r})$. The notations here are defined as the same as the lecture notes.

   (a) Suppose that $k = 3$ ($\beta' = (\beta_1, \beta_2, \beta_3)$) and $H_0 : \beta_2 + \beta_3 = 1$. What is R, $\underline{r}$, q and $(R\widehat{\beta} - \underline{r})$?

   (b) Suppose that $(X'X)^{-1}$ is represented by

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}.$$

   What is $s^2R(X'X)^{-1}R'$?

   (c) Plug the results from the above two sub-questions in the F statistic and also show the t statistic.