Homework 4

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$$l = \ln f(y;\beta,\delta) = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{i=1}^{n}\ln \delta_i^2 - \frac{1}{2}\sum_{i=1}^{n}\frac{(y_i-\beta x_i)^2}{\delta_i^2}$$

$$= -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{i=1}^{n}\ln \delta^2 w_i^2 - \frac{1}{2}\sum_{i=1}^{n}\frac{(y_i-\beta x_i)^2}{\delta^2 w_i^2}$$

$$= -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{i=1}^{n}\ln \delta^2 - \frac{1}{2}\sum_{i=1}^{n}\ln w_i^2 - \frac{1}{2\delta^2}\sum_{i=1}^{n}\frac{1}{w_i^2}(y_i-\beta x_i)^2$$

$$= -\frac{n}{2}\ln 2\pi - n\ln\delta - \sum_{i=1}^{n}\ln w_i - \frac{1}{2\delta^2}\sum_{i=1}^{n}\frac{1}{w_i^2}(y_i-\beta x_i)^2$$

$$\frac{\partial l}{\partial \beta} = \frac{1}{\delta^2}\sum_{i=1}^{n}\frac{1}{w_i^2}(y_i-\beta x_i)x_i = 0$$

$$\sum_{i=1}^{n}\left(\frac{x_i y_i - \beta x_i^2}{w_i^2}\right) = 0$$

$$\sum_{i=1}^{n}\left(\frac{x_i y_i}{w_i w_i} - \beta\frac{x_i^2}{w_i^2}\right) = 0$$

$$\beta\sum_{i=1}^{n}\frac{x_i^2}{w_i^2} = \sum_{i=1}^{n}\frac{x_i}{w_i}\frac{y_i}{w_i}$$

$$\hat{\beta} = \left(\sum_{i=1}^{n}\frac{x_i^2}{w_i^2}\right)^{-1}\sum_{i=1}^{n}\frac{x_i}{w_i}\frac{y_i}{w_i} \quad \text{or} \quad \frac{\sum_{i=1}^{n}x_i^* y_i^*}{\sum_{i=1}^{n}x_i^{*2}}$$

$$\frac{\partial l}{\partial \sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} \frac{1}{w_i^2} (y_i - \beta x_i)^2 = 0$$

$$\frac{n}{\sigma} = \frac{1}{\sigma^3} \sum_{i=1}^{n} \frac{1}{w_i^2} (y_i - \beta x_i)^2$$

$$\sigma^2 = \sum_{i=1}^{n} \frac{(y_i - \beta x_i)^2}{n w_i^2}$$

$$\hat{\sigma} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \frac{\hat{u}_i^2}{w_i^2}} \quad \text{or} \quad \sqrt{\sum_{i=1}^{n} \frac{\hat{u}_i^{*2}}{n}}$$