# Final

1. When an ML estimator is applied to a model which is misspecified in ways that do not affect its consistency, it is said to be a quasi ML estimator.

2.

(i) LS estimator is unbiased and consistent, but not efficient. In addition, the test statistics based on conventional LS coefficient standard errors are invalid.

(ii) Breusch - Pagan Test

$$y_i = \beta_1 + \beta_2 X_{i2} + \cdots + \beta_K X_{iK} + u_i \quad (1)$$
$$\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{i2} + \cdots + \alpha_K X_{ik} + v_i \quad (2)$$

$H_0: \alpha_2 = \alpha_3 = \cdots = \alpha_K = 0$ (Homoscedasticity)

$H_1$: $H_0$ is not true (Heteroscedasticity)

Step 1: obtain $\hat{u}_i$ from (1)

Step 2: obtain $R^2$ from (2)

Step 3: construct LM statistic: $nR^2 \sim \chi^2_{(k-1)}$

OR

White Test

$$y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + u_i \quad (1)$$
$$\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{i2} + \alpha_3 X_{i3} + \alpha_4 X_{i2}^2 + \alpha_5 X_{i3}^2 + \alpha_6 X_{i2} X_{i3} + v_i \quad (2)$$

$H_0: \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0$ (Homoscedasticity)

$H_1$: $H_0$ is not true (Heteroscedasticity)

Step 1: obtain $\hat{u}_i$ from (1)

Step 2: obtain $R^2$ from (2)

Step 3: construct LM statistic: $nR^2 \sim \chi^2_{(5)}$

(iii) Heteroscedasticity - Consistent Standard Errors

$$\widehat{Var}(\hat{\beta}) = (X'X)^{-1} \sum_{i=1}^{n} \hat{u}_i^2 x_i x_i' (X'X)^{-1}, \quad X = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix}$$

AND

Generalized Least Squares / Weighted Least Squares

$$\begin{aligned} \hat{\beta} &= (X'\Omega^{-1}X)^{-1} X'\Omega^{-1} y & , \quad Var(u) = \sigma^2 \Omega \\ &= [(PX)'(PX)]^{-1} (PX)'(Py) & , \quad \Omega^{-1} = P'P \\ &= \left( \sum_{i=1}^{n} \frac{1}{\sqrt{z_i}} x_i \frac{1}{\sqrt{z_i}} x_i' \right)^{-1} \left( \sum_{i=1}^{n} \frac{1}{\sqrt{z_i}} x_i \frac{1}{\sqrt{z_i}} y_i \right) & , \quad \sigma_i^2 = \sigma^2 z_i \end{aligned}$$

3

(a) $H_0: \beta_2 = 0$
$H_1: \beta_2 \neq 0$

$$\frac{-0.45 - 0}{0.3} = -1.5 > -1.684 = t_{\frac{\alpha}{2}=0.05,\ df=40}$$

We fail to reject $H_0: \beta_2 = 0$ and the price elasticity is not significantly different from zero.

(b) $H_0: \beta_3 \leq 1$
$H_1: \beta_3 > 1$

$$\frac{1.25 - 1}{0.2} = 1.25 < 1.684 = t_{\alpha=0.05,\ df=40}$$

We fail to reject $H_0: \beta_3 \leq 1$ and the income elasticity is not significantly greater than one.

4

$\hat{u}'\hat{u} = (18-2)\,0.0358 = 0.5728$

$\hat{u}_1'\hat{u}_1 = (9-2)\,0.0199 = 0.1393$

$\hat{u}_2'\hat{u}_2 = (9-2)\,0.0276 = 0.1932$

$$F = \frac{[0.5728 - (0.1393 + 0.1932)]/2}{(0.1393 + 0.1932)/(18-4)} = 5.059 >$$

$F_{\alpha=0.05}(df_1 = 2,\ df_2 = 14) = 3.74$

We reject the null hypothesis that the saving functions are the same over the two subperiods.

5

$$X(X'X)^{-1}X'\underline{i} = [\underline{i}, \underline{X_2}]\left\{\begin{bmatrix}\underline{i}'\\ \underline{X_2}'\end{bmatrix}[\underline{i}, \underline{X_2}]\right\}^{-1}\begin{bmatrix}\underline{i}'\\ \underline{X_2}'\end{bmatrix}\underline{i}$$

$$= [\underline{i}, \underline{X_2}]\begin{bmatrix}\underline{i}'\underline{i} & \underline{i}'\underline{X_2}\\ \underline{X_2}'\underline{i} & \underline{X_2}'\underline{X_2}\end{bmatrix}^{-1}\begin{bmatrix}\underline{i}'\underline{i}\\ \underline{X_2}'\underline{i}\end{bmatrix}$$

$$= [\underline{i}, \underline{X_2}]\begin{bmatrix}n & \sum_i X_{i2}\\ \sum_i X_{i2} & \sum_i X_{i2}^2\end{bmatrix}^{-1}\begin{bmatrix}n\\ \sum_i X_{i2}\end{bmatrix}$$

$$= [\underline{i}, \underline{X_2}]\frac{1}{n\sum_i X_{i2}^2 - (\sum_i X_{i2})^2}\begin{bmatrix}\sum_i X_{i2}^2 & -\sum_i X_{i2}\\ -\sum_i X_{i2} & n\end{bmatrix}\begin{bmatrix}n\\ \sum_i X_{i2}\end{bmatrix}$$

$$= [\underline{i}, \underline{X_2}]\frac{1}{n\sum_i X_{i2}^2 - (\sum_i X_{i2})^2}\begin{bmatrix}n\sum_i X_{i2}^2 - (\sum_i X_{i2})^2\\ 0\end{bmatrix}$$

$$= [\underline{i}, \underline{X_2}]_{n\times 2}\begin{bmatrix}1\\ 0\end{bmatrix}_{2\times 1}$$

$$= \underline{i}$$

The above is not the only way to show the equivalence.
$X(X'X)^{-1}X'\underline{i}$ implies $y = \hat{y} + \hat{u}$ where $y = \underline{i}$ and $\hat{y} = X\hat{\beta}$ with $\hat{\beta} = (X'X)^{-1}X'\underline{i}$.