Taller #1 - 2 corte

Autor
Oscar David Poblador Parra
20211005116

Profesor
Henry Borrego Guerrero

Universidad Distrital Francisco José de Caldas

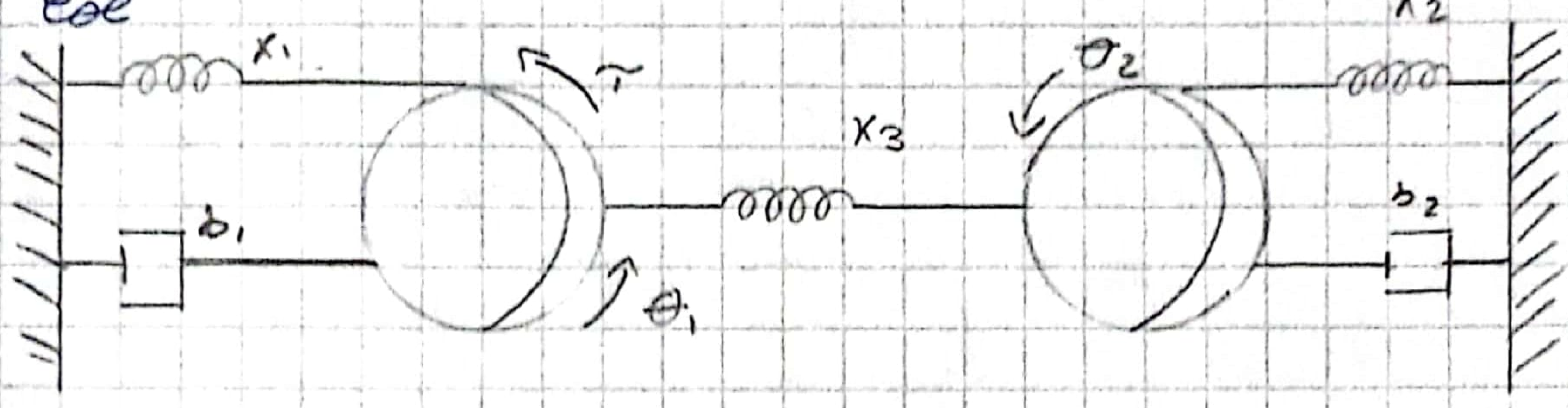
Facultad Ingeniería

Sistemas Dinámicos

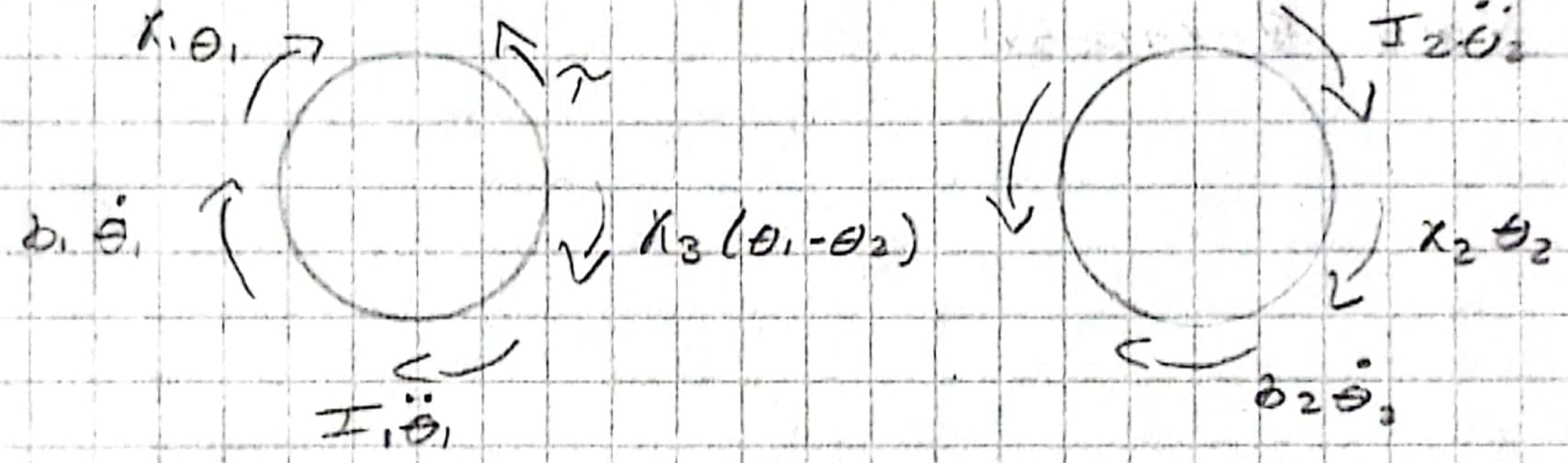
Bogotá - Colombia

Abril 2024

Eje



✓ Al realizar el diagrama de cuerpo libre



✓ Para la masa rotacional 1

$$\tau = K_1\theta_1 + b_1\dot{\theta}_1 + I_1\ddot{\theta}_1 + K_3(\theta_1-\theta_2)$$
$$\tau = K_1\theta_1 + K_3\theta_1 - K_3\theta_2 + b_1\dot{\theta}_1 + I\ddot{\theta}_1$$

$$\ddot{\theta}_1 = -\frac{\tau}{I} - \frac{b_1}{I}\dot{\theta}_1 + \frac{K_3}{I}\theta_2 - \frac{(K_1+K_3)}{I}\theta_1 \qquad (1)$$

✓ Para la masa rotacional 2

$$0 = K_3(\theta_1-\theta_2) - b_2\dot{\theta}_2 - K_2\theta_2 - I_2\ddot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{K_3}{I}(\theta_1-\theta_2) - \frac{b_2}{I}\dot{\theta}_2 - \frac{K_2}{I}\theta_2$$

$$\ddot{\theta}_2 = \frac{K_3}{I}\theta_1 - \frac{K_3}{I}\theta_2 - \frac{b_2}{I}\dot{\theta}_2 - \frac{K_2}{I}\theta_2 \qquad (2)$$

$$\ddot{\theta}_2 = \frac{K_3}{I}\theta_1 - \frac{(K_3+K_2)}{I}\theta_2 - \frac{b_2}{I}\dot{\theta}_2$$

✓ si se consideran las variables de estado así

$$q_1 = \theta_1 \; ; \quad q_2 = \dot{\theta}_1 \; ; \quad \dot{q}_2 = \ddot{\theta}_1$$

$$q_3 = \theta_2 \; ; \quad q_4 = \dot{\theta}_2 \; ; \quad \dot{q}_4 = \ddot{\theta}_2$$

✓ Reemplazando en (1) y (2)

$$\dot{q}_2 = -\frac{b_1}{I} q_2 + \frac{K_3}{I} q_3 - \frac{(K_1+K_3)}{I} q_1 - \frac{\tau}{I} \qquad (3)$$

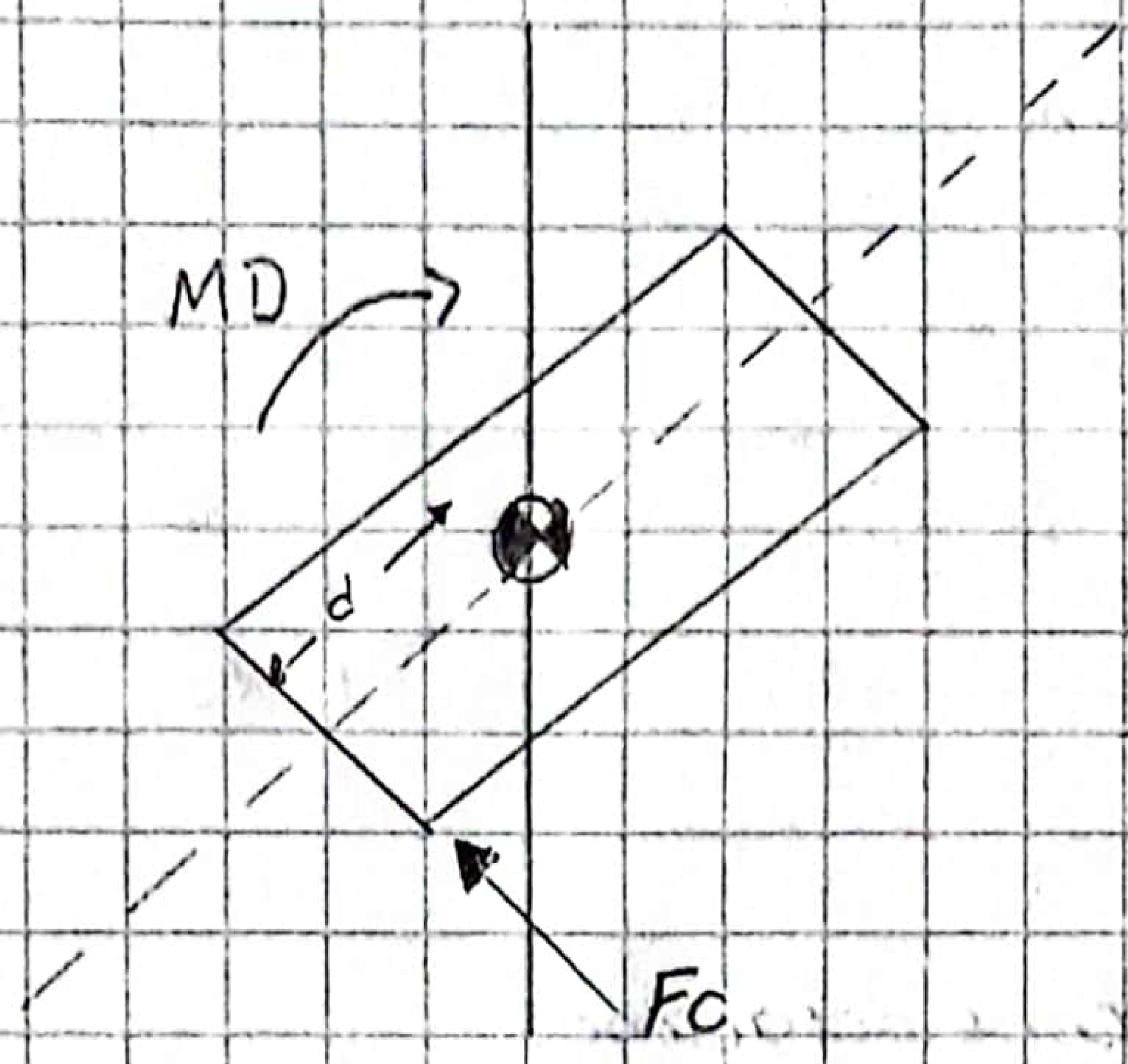
$$\dot{q}_4 = \frac{K_3}{I} q_1 - \frac{(K_3+K_2)}{I} q_3 - \frac{b_2}{I} q_4 \qquad (4)$$

✓ Para la matriz de estados

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -(K_1+K_3)/I & -b_1/I & K_3/I & 0 \\ 0 & 0 & 1 & 0 \\ K_3/I & 0 & -(K_3+K_2)/I & -b_2/I \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -1/I \\ 0 \\ 0 \end{bmatrix} \tau$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Eje Un satelite Presenta una Perturbación



$$F_c d + M_D = I\ddot{\theta}$$

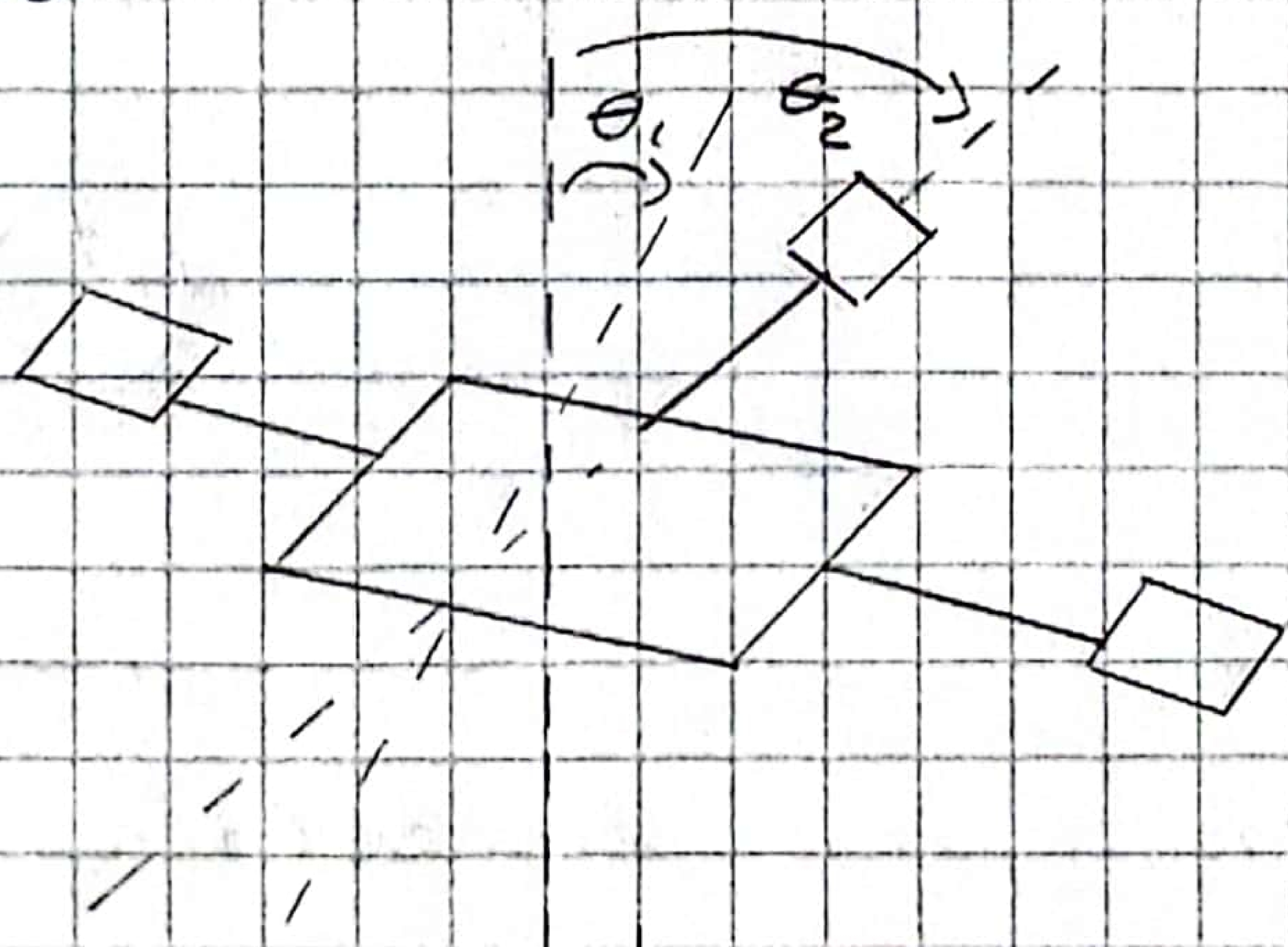
$$u = I\ddot{\theta}$$

$$u(s) = Is^2\,\theta(s)$$

$$\frac{\theta(s)}{u(s)} = \frac{1}{Is^2}$$

Ejercicio

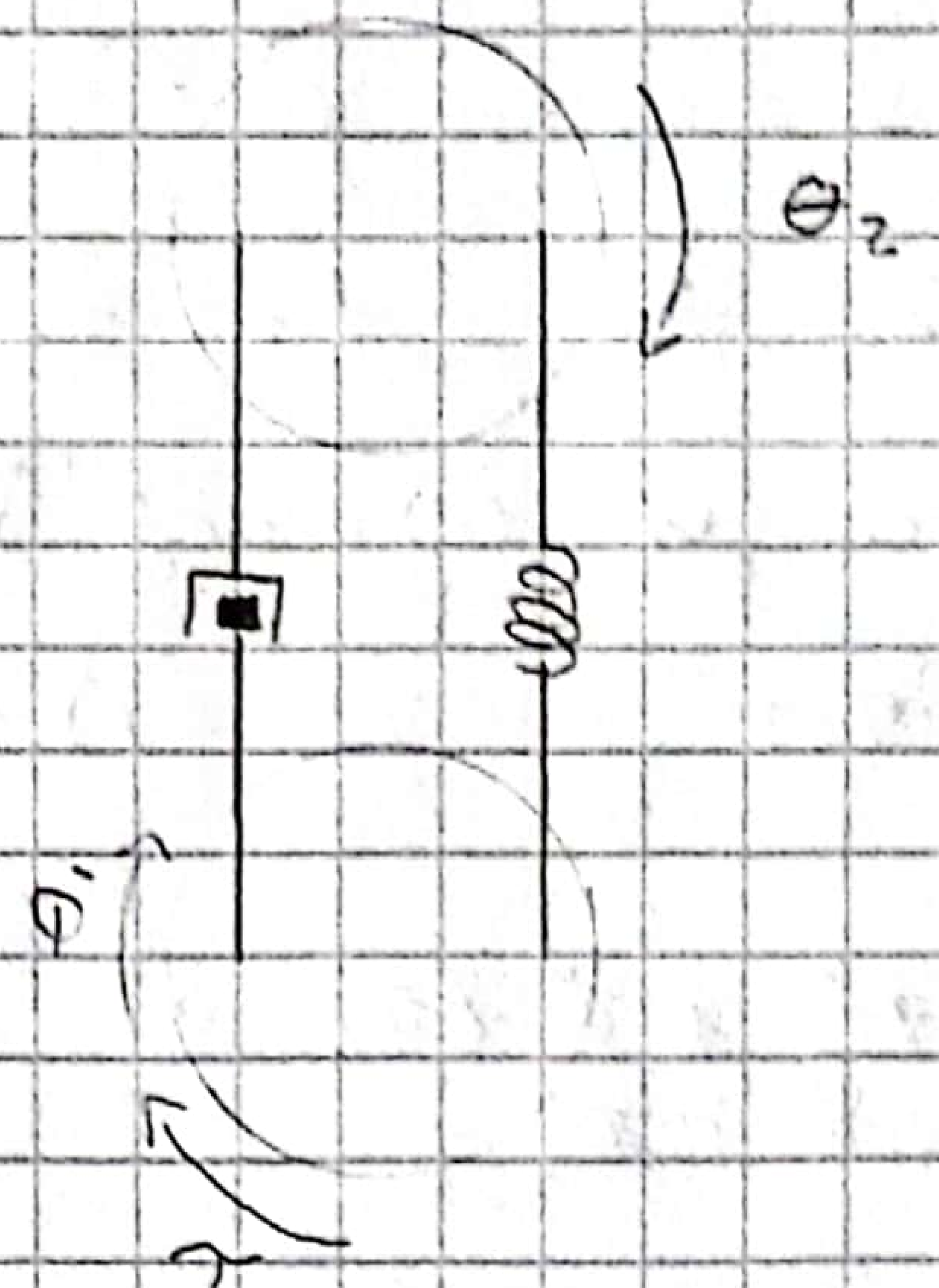
1)

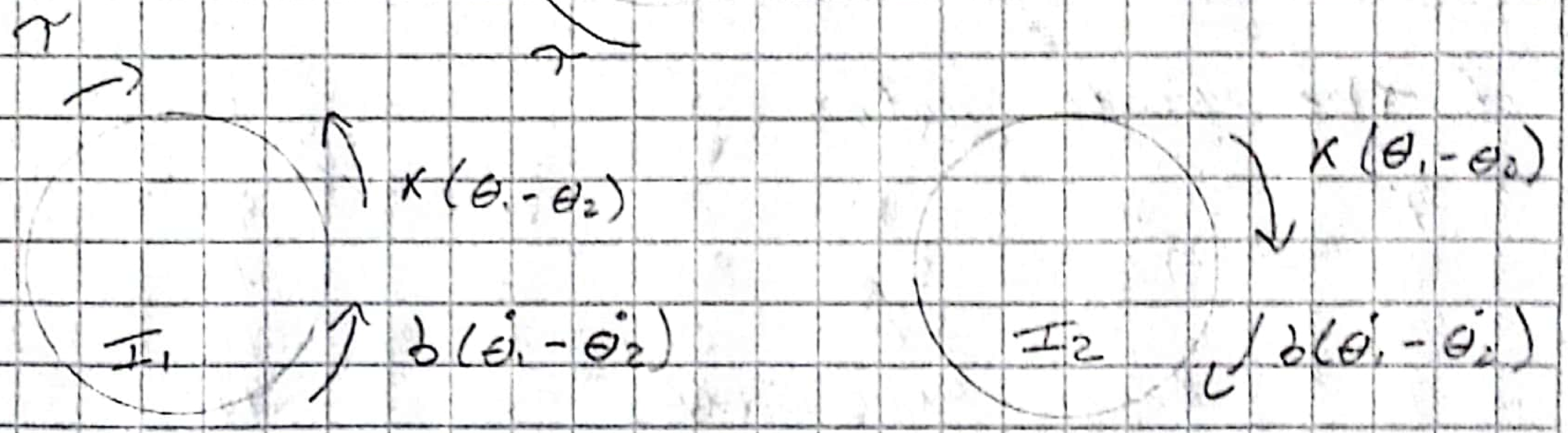


θ_1 : body ; θ_2 = sensor

- Find the state matrix

✓ The Free-Body diagram





Body

Sensor

✓ The equation for the body

$$\tau_c = \kappa(\theta_1-\theta_2) + b(\dot\theta_1-\dot\theta_2) + I\ddot\theta_1$$

$$\ddot\theta_1 = \frac{\tau_c}{I_1} - \frac{\kappa}{I_1}(\theta_1-\theta_2) - \frac{b}{I_1}(\dot\theta_1-\dot\theta_2) \qquad (1)$$

✓ The equation for the sensor

$$0 = \kappa(\theta_1-\theta_2) + b(\dot\theta_1-\dot\theta_2) - I_2\ddot\theta_2$$

$$\ddot\theta_2 = \frac{\kappa}{I_2}(\theta_1-\theta_2) + \frac{b}{I_2}(\dot\theta_1-\dot\theta_2) \qquad (2)$$

✓ The state variable

$q_1 = \theta_1 \; ; \; q_2 = \dot{q}_1 = \dot{\theta}_1 \; ; \; \dot{q}_2 = \ddot{\theta}_1$

$q_3 = \theta_2 \; ; \; q_4 = \dot{q}_3 = \dot{\theta}_2 \; ; \; \dot{q}_4 = \ddot{\theta}_2$

✓ if we replace in (1) and (2)

$$\dot{q}_2 = -\frac{K}{I_1}(q_1 - q_3) - \frac{b}{I_1}(q_2 - q_4) + \frac{\tau_c}{I_1} \qquad (3)$$

$$\dot{q}_4 = \frac{K}{I_2}(q_1 - q_3) + \frac{b}{I_2}(q_2 - q_4) \qquad (4)$$

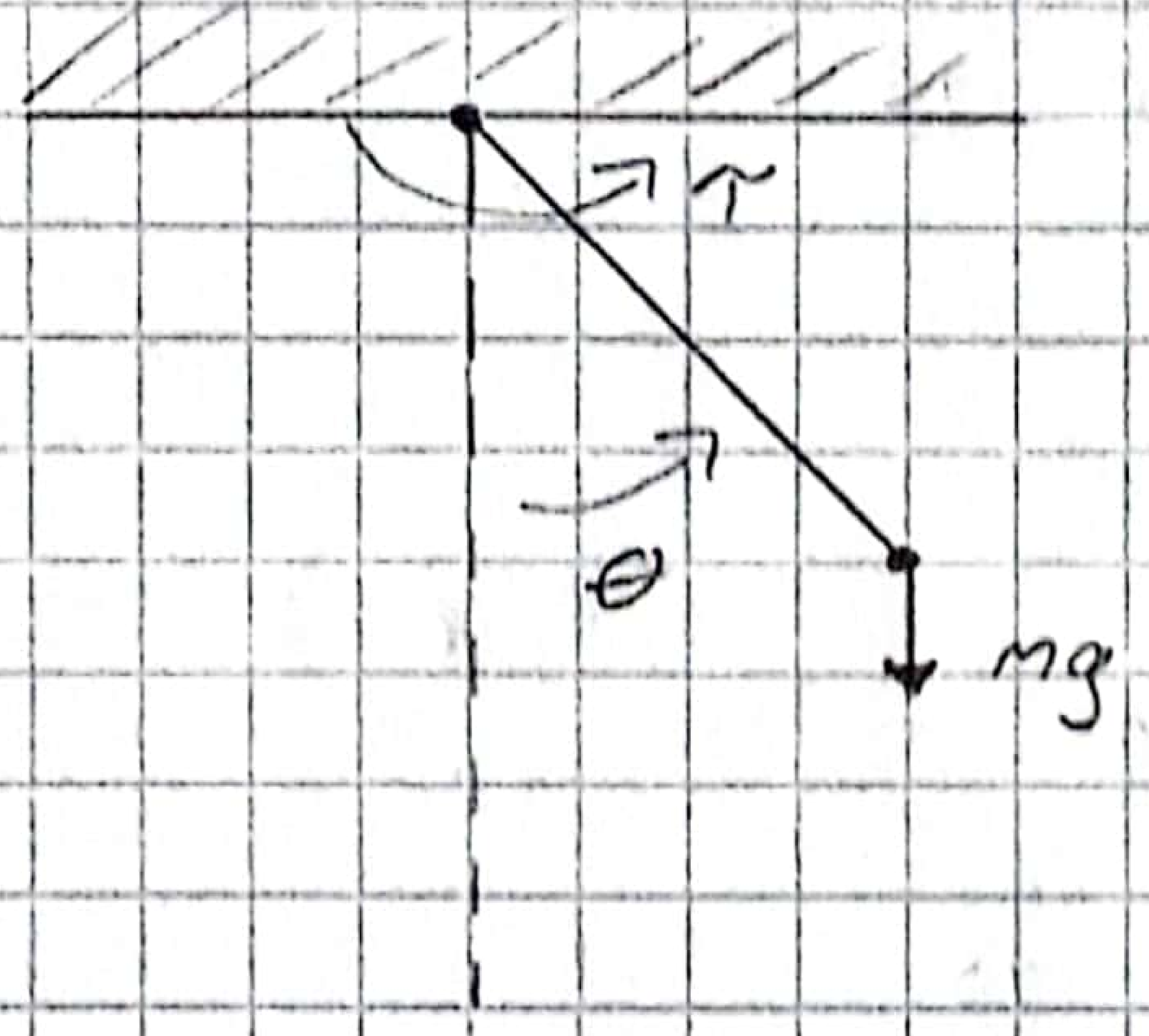
✓ The state matrix is

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/I_1 & -b/I_1 & K/I_1 & b/I_1 \\ 0 & 0 & 0 & 1 \\ K/I_2 & b/I_2 & -K/I_2 & -b/I_2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 1/I_1 \\ 0 \\ 0 \end{bmatrix} \tau c$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

2)



• Find the state matrix

✓ The state equation for the mass

$\tau = mgl\sin(\theta) + I\ddot{\theta}$

✓ The moment of inertia about the pivot is

$I = ml^2 \rightarrow \tau = mgl\sin(\theta) + ml^2\ddot{\theta}$

✓ The state equation

$$\ddot{\theta} = \frac{\tau}{ml^2} - \frac{g}{l}\sin(\theta)$$

✓ if we consider the angular motion too small we can affirm

$\sin(\theta) \simeq \theta \quad \text{if} \quad \theta \ll 1$

$$\ddot{\theta} = \frac{\tau}{ml^2} - \frac{g}{l}\theta \qquad (1)$$

✓ The state variable is

$q_1 = \theta_1$; $\dot{q}_1 = \dot{\theta}_1$

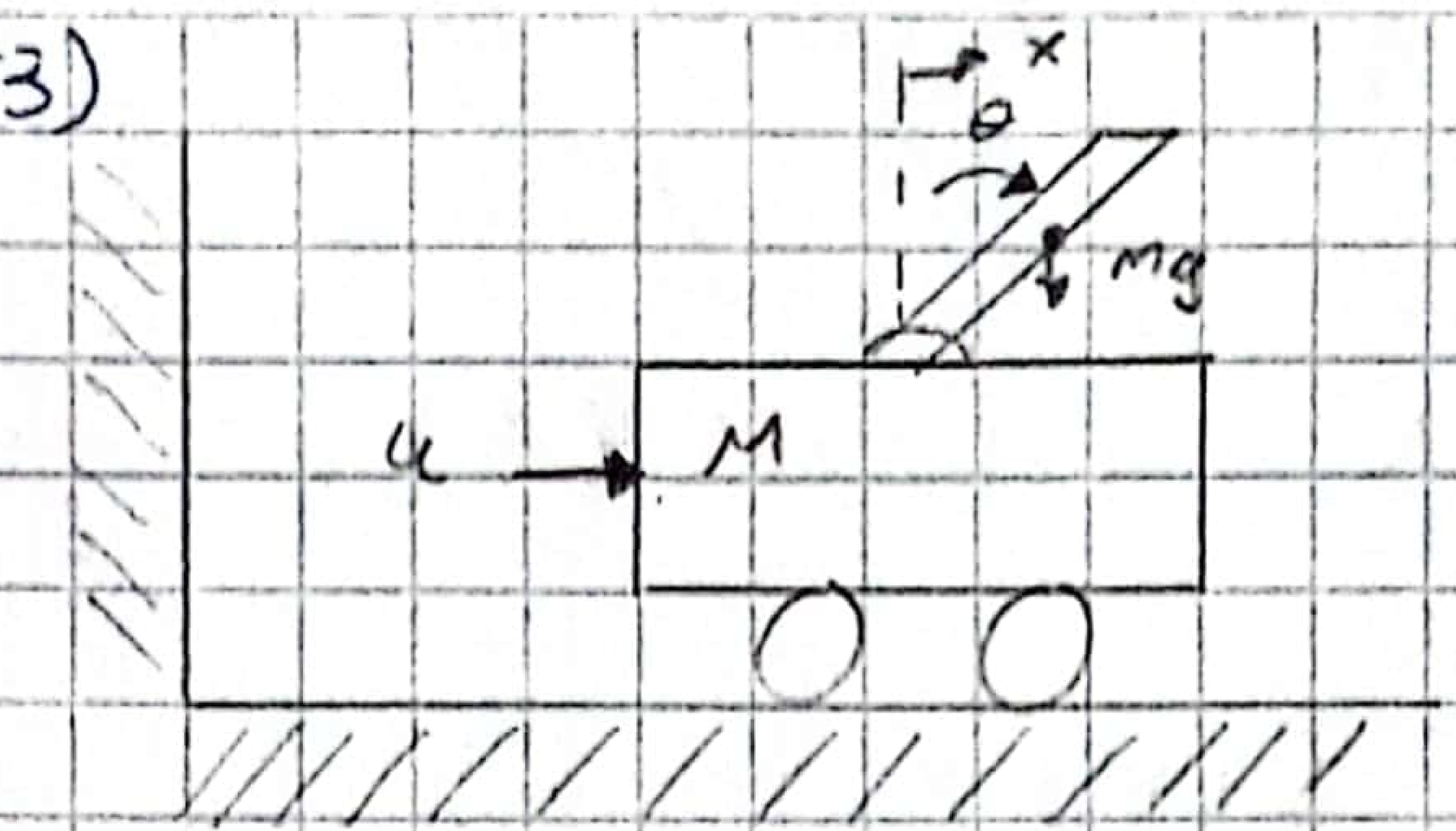
$$\dot{q}_1 = -\frac{g}{l} q_1 + \frac{\tau}{ml^2} \qquad (2)$$

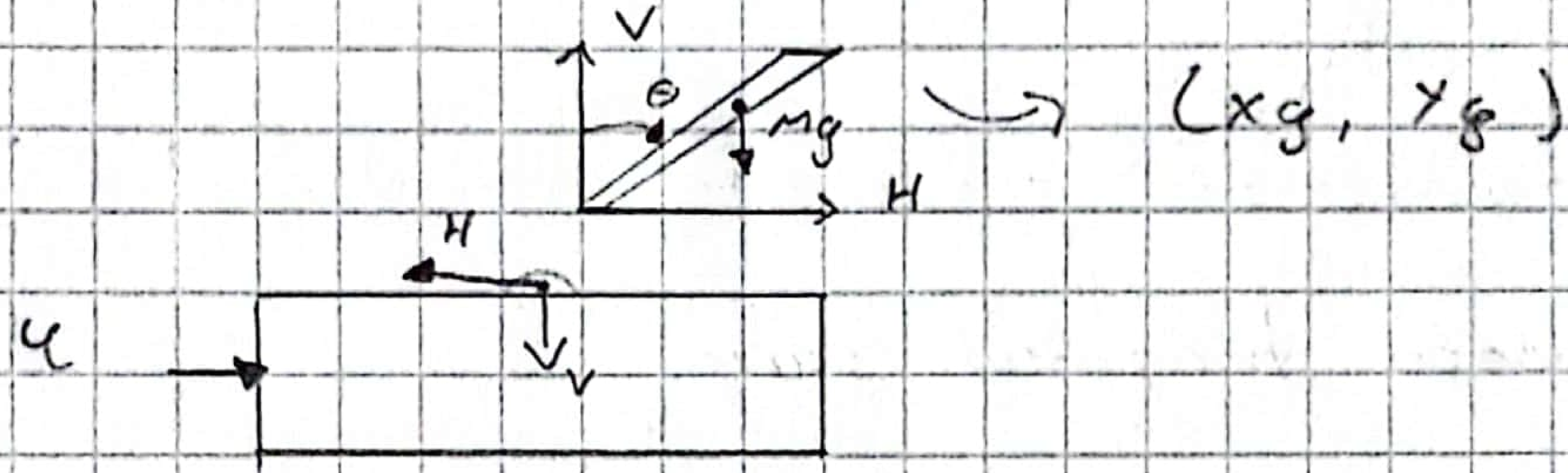
✓ The state matrix is

$$\begin{bmatrix} \dot{q}_1 \end{bmatrix} = \begin{bmatrix} -\frac{g}{l} \end{bmatrix} \begin{bmatrix} q_1 \end{bmatrix} + \begin{bmatrix} \frac{1}{ml^2} \end{bmatrix} \tau$$

$$\begin{bmatrix} \theta \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} q_1 \end{bmatrix}$$

3)





$$x_g = x + l\sin(\theta)$$
$$y_g = l\cos(\theta)$$

✓ Movimiento Rotacional

$$I\ddot{\theta} = Vl\sin(\theta) - Hl\cos(\theta) \qquad (1)$$

✓ Movimiento horizontal

$$H = m\frac{d^2(x + l\sin(\theta))}{dt^2}$$

$$H = m\ddot{x} + \frac{m\, d^2(l\sin(\theta))}{dt^2}$$

$$H = m\ddot{x} + \frac{m\, d(l\cos(\theta)\dot{\theta})}{dt}$$

$$H = m\ddot{x} + ml(-\sin(\theta)\dot{\theta}\dot{\theta} + \cos(\theta)\ddot{\theta})$$

$$H = m\ddot{x} - ml\sin(\theta)\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta} \qquad (2)$$

✓ Movimiento Vertical

$$m \frac{d^2(\ell\cos(\theta))}{dt} = V - mg \qquad (3)$$

$$m \frac{d(\ell - \sin(\theta)\dot{\theta})}{dt} = V - mg$$

$$m\ell(-\cos(\theta)\dot{\theta}^2 - \sin(\theta)\ddot{\theta}) = V - mg$$

✓ Movimiento horizontal carro

$$M\ddot{x} = u - H \qquad (4)$$

→ si θ muy pequeño

$$\sin(\theta) \to \theta \ ; \ \cos(\theta) \to 1 \ ; \ \theta\dot{\theta} \to 0$$

✓ De (1)

$$I\ddot{\theta} = V\ell\theta - H\ell \qquad (5)$$

✓ De (2)

$$m\ddot{x} - m\ell\theta\dot{\theta}^2 + m\ell\ddot{\theta} = H$$

$$m\ddot{x} + m\ell\ddot{\theta} = H \to m(\ddot{x} + \ell\ddot{\theta}) = H \qquad (6)$$

✓ De (3)

$$0 = V - mg \to V = mg \qquad (7)$$

✓ Reemplazando H de (4) y (6)

$$M\ddot{x} = u - m(\ddot{x} + \ell\ddot{\theta})$$

$$\ddot{x}(M+m) + m\ell\ddot{\theta} = u \qquad (8)$$

Reemplazando (5) y (6) y (7)

$$I\ddot{\theta} = Vl\theta - Hl$$

$$I\ddot{\theta} = (mg)l\theta - (m(\ddot{x} + l\ddot{\theta}))l$$

$$I\ddot{\theta} = mgl\theta - ml\ddot{x} - ml^2\ddot{\theta}$$

$$\ddot{\theta}(I + ml^2) + ml\ddot{x} = mgl\theta \qquad (9)$$

Al despejar $\ddot{x}$ de (8) y reemplazar en (9)

$$\ddot{x}(M+m) + ml\ddot{\theta} = u$$

$$\ddot{x} = \frac{u - ml\ddot{\theta}}{M+m}$$

$$\ddot{\theta}(I + ml^2) + ml\ddot{x} = mgl\theta$$

$$\ddot{\theta}(I + ml^2) + ml\left(\frac{u - ml\ddot{\theta}}{M+m}\right) = mgl\theta$$

$$\ddot{\theta}(I + ml^2) + \frac{mlu}{M+m} - \frac{m^2l^2\ddot{\theta}}{M+m} = mgl\theta$$

$$\ddot{\theta}\left(\frac{I(M+m) + Mml^2}{M+m}\right) + \frac{mlu}{M+m} = mgl\theta$$

$$\ddot{\theta} = \left(\frac{(M+m)}{I(M+m) + Mml^2}\right) mgl\theta - \frac{mlu}{I(M+m) + Mml^2} \qquad (10)$$

✓ Al despejar $\ddot{\theta}$ de (8) y reemplazar en (9)

$$\ddot{x}(M+m) + ml\ddot{\theta} = u$$

$$\ddot{\theta} = \frac{u - \ddot{x}(M+m)}{ml}$$

$$\ddot{\theta}(I + ml^2) + ml\ddot{x} = mgl\theta$$

$$\left(\frac{u - \ddot{x}(M+m)}{ml}\right)(I + ml^2) + ml\ddot{x} = mgl\theta$$

$$u\left(\frac{I+ml^2}{ml}\right) - \ddot{x}\left(\frac{(M+m)(I+ml^2)}{ml}\right) + ml\ddot{x} = mgl\theta$$

$$-\ddot{x}\left(\frac{I(M+m) + Mml^2}{ml}\right) + u\left(\frac{I+ml^2}{ml}\right) = mgl\theta$$

$$-\ddot{x} = m^2l^2g\theta\left(\frac{1}{I(M+m)+Mml^2}\right) - u\left(\frac{I+ml^2}{I(M+m)+Mml^2}\right)$$

$$\ddot{x} = -m^2l^2g\theta\left(\frac{1}{I(M+m)+Mml^2}\right) + u\left(\frac{I+ml^2}{I(M+m)+Mml^2}\right) \quad (11)$$

✓ Al considerar las variables de estado así:

$q_1 = x$; $q_2 = \dot{x}$; $\dot{q}_2 = \ddot{x}$

$q_3 = \theta$; $q_4 = \dot{\theta}$; $\dot{q}_4 = \ddot{\theta}$

✓ Al reemplazar las variables de estado de (10) y (11)

$$\dot{q}_4 = \left(\frac{(M+m)}{I(M+m)+Mml^2}\right) mgl\, q_3 - \frac{ml}{I(M+m)+Mml^2} u$$

$$\dot{q}_2 = \frac{-m^2 l^2 g}{I(M+m)+Mml^2} q_3 + \frac{I+ml^2}{I(M+m)+Mml^2} u$$

✓ La Matriz de variables de estado es así:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & (-m^2l^2g)/(I(M+m)+Mml^2) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & ((M+m)mgl)/(I(M+m)+Mml^2) & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ (I+ml^2)/(I(M+m)+Mml^2) \\ 0 \\ -ml/(I(M+m)+Mml^2) \end{bmatrix} u$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$