MATH/COSC 303

Assignment 5

Due: Mar 25 in LAB, assignments are due at the end of lab.

Hand Written Questions:

1. Consider $f(x) = x$ and $\int_a^b f(x)dx$ where $a < b$.
   a) Show that the Trapezoid rule yields the exact solution.
   b) Show that Simpson's rule yields the exact solution.
2. Consider $f(x) = x^2$ and $\int_a^b f(x)dx$ where $a < b$.
   a) Show that the Trapezoid rule does not yield the exact solution.
   b) Show that Simpson's rule yields the exact solution.
3. Consider $f(x) = x^3$ and $\int_a^b f(x)dx$ where $a < b$.
   a) Show that the Trapezoid rule does not yield the exact solution.
   b) Show that Simpson's rule yields the exact solution.

BONUS Consider $f(x) = x^4$ and $\int_a^b f(x)dx$ where $a < b$.
   a) Show that Simpson's rule does not yield the exact solution.
   b) Show that Boole's rule yields the exact solution.

Computer Assisted Questions:

4. Consider
$$\int_1^2 \frac{1}{x}dx.$$
   a) Use Riemann's Left Hand Rule, Riemann's Right Hand Rule, Riemann's Midpoint Rule, the Trapezoid Rule, Simpson's Rule, and Boole's rule to approximate the integral.
   b) Compute the relative error in each case.
5. Consider
$$\int_0^\pi \sin(x) + \cos(x)dx.$$
   a) Use the composite Trapezoid rule with $n = 2, 10, 100$, and 1000 intervals to approximate the integral. Compute the relative error in each case.
   b) Use the composite Simpson's rule with $n = 1, 5, 50, 500$ and 5000 intervals to approximate the integral. Compute the relative error in each case.
6. Use adaptive quadrature to approximate
$$\frac{1}{\sqrt{2\pi}} \int_{-1}^1 e^{-\frac{x^2}{2}} dx$$
   to 4 significant digits.