STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

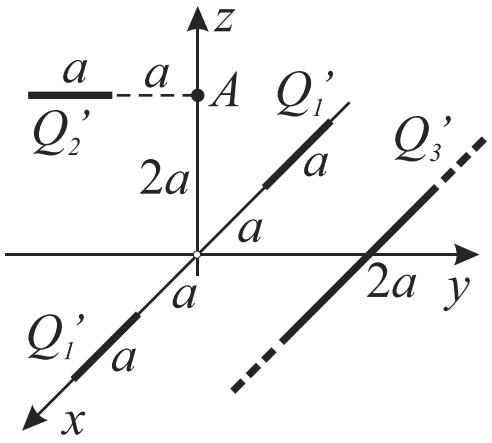
K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **12.02.2022.**

ZADACI

Zadatak 1. Dva tanka štapa, dužine a, naelektrisana istim podužnim gustinama naelektrisanja Q_1', postavljena su na x osi Dekartovog koordinatnog sistema, kao što je prikazano na slici 1. Početak oba štapa je na rastojanju a od koordinatnog početka. Tanak štap, dužine a, naelektrisan podužnom gustinom naelektrisanja Q_2', postavljen je paralelno sa y osom, na rastojanju a od tačke A, koja se nalazi na z osi na visini $2a$. Sistem se nalazi u vazduhu.

a) Odrediti, u opštim brojevima, vektor jačine električnog polja koji u tački A stvaraju štapovi naelektrisani sa Q_1'.
b) Odrediti, u opštim brojevima, vektor jačine električnog polja koji u tački A stvara štap naelektrisan sa Q_2'.
c) Odrediti podužnu gustinu naelektrisanja beskonačno dugačkog štapa, Q_3', koji leži u x-y ravni, paralelno sa x osom, tako da rezultantni vektor jačine električnog polja u tački A ima samo z komponentu.

Brojni podaci su: $a = 1\ cm$, $Q_1' = 1\ nC/m$, $Q_2' = 2\ nC/m$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.

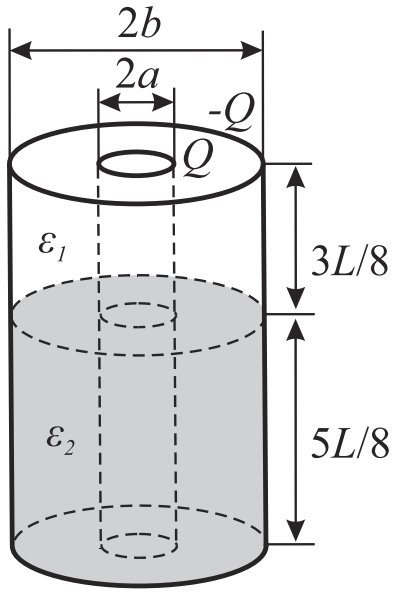


Slika 1.

Zadatak 2. Na slici 2 je prikazan koaksijalni kabl dužine $L = 8\ m$, ispunjen sa dva sloja dielektrika: tečni – relativne permitivnosti $\varepsilon_{r1} = 7$ i čvrsti relativne permitivnosti $\varepsilon_{r2} = 5$. Poluprečnici elektroda kabla su a i $b = 2{,}7a$. Elektrode kondenzatora su naelektrisane naelektrisanjem $+Q$ i $-Q$.

a) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora.
b) Odrediti nepoznati poluprečnik unutrašnje elektrode a, ako je maksimalni dozvoljeni napon na koji sme da se priključi kondenzator $U_{max} = 20\ kV$.
c) Izračunati kapacitivnost kondenzatora, nakon potpunog ispuštanja tečnog dielektrika relativne permitivnosti ε_{r1}.
d) Izračunati količinu vezanog naelektrisanja na razdvojnoj površi dva dielektrika, kada je kondenzator priključen na napon $U = 7\ kV$ (bonus 5p).

Ostali brojni podaci: $E_{Č1} = 80\ kV/cm$, $E_{Č2} = 100\ kV/cm$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **12.02.2022.**

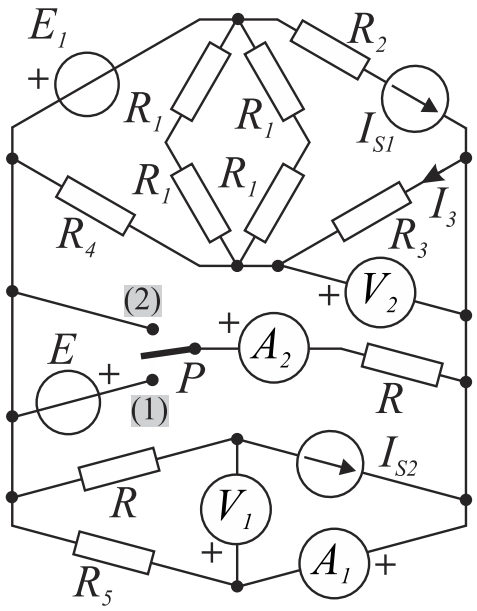
ZADACI

Zadatak 1. U kolu vremenski konstantnih struja, sa slike 1, posle prebacivanja preklopnik P iz položaja (2) u položaj (1), jačina struja kroz otpornik otpornosti R_3 se poveća za $\Delta I_3 = 30\ mA$, u naznačenom referentnom smeru.

a) Primenjujući teoremu superpozicije, odrediti vrednost *ems* naponskog generatora E.

b) Odrediti pokazivanja idealnih mernih instrumenata kada je preklopnik u položaju (1). Kolo rešavati primenom metode potencijala čvorova.

Brojni podaci su: $E_1 = 12\ V$, $I_{S1} = 100\ mA$, $I_{S2} = 200\ mA$, $R_1 = 80\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 80\ \Omega$, $R_5 = 20\ \Omega$, $R = 10\ \Omega$.



Slika 1.

Zadatak 2. U kolu vremenski konstantnih struja, sa slike 2, klizač promenljivog otpornika R_P se nalazi u položaju u kome se na ovom otporniku razvija maksimalno moguća snaga.

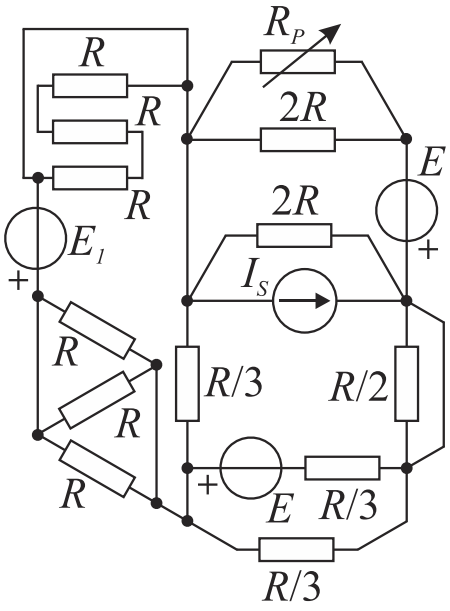
a) Izračunati vrednost otpornosti otpornika R_P.

b) Izračunati snagu strujnog generatora I_S.

c) Izračunati snagu koja se razvija na otporniku R_P.

d) Skicirati grafički kako se menja snaga promenljivog otpornika R_P, ako se vrednost njegove otpornosti menja od vrednosti 0 do vrednosti $5R_P$. (bonus 5p).

Brojni podaci su: $R = 12\ \Omega$, $E_1 = 12\ V$, $E = 8\ V$, $I_S = 2\ A$.



Slika 2.

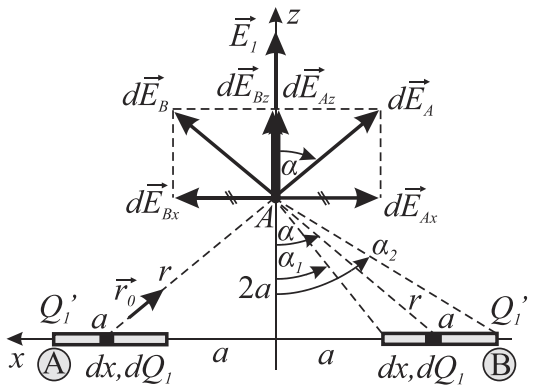
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET1, 12.02.2022.

I-1

a)



$$dE_A = dE_B = \frac{dQ_1}{4\pi\varepsilon_0 r^2} = \frac{Q_1'\,dx}{4\pi\varepsilon_0 r^2}$$

$$dE_{Ax} = dE_A \sin\alpha$$

$$dE_{Az} = dE_A \cos\alpha$$

$$\overrightarrow{dE}_{Ax} + \overrightarrow{dE}_{Bx} = 0$$

$$dE_z = dE_{Az} + dE_{Bz} = 2\,dE_{Az} = 2\,dE_A \cos\alpha = 2\,\frac{Q_1'\,dx}{4\pi\varepsilon_0 r^2}\cos\alpha$$

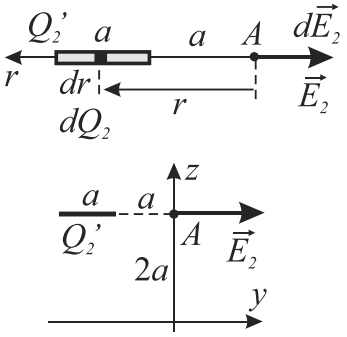
$$dE_z = 2\,\frac{Q_1'\,\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\cos\alpha = 2\,\frac{Q_1'\,d\alpha}{4\pi\varepsilon_0\,\dfrac{2a}{\cos\alpha}} = 2\,\frac{Q_1'}{4\pi\varepsilon_0 2a}\cos\alpha\,d\alpha$$

$$E_1 = \int dE_z = 2\,\frac{Q_1'}{4\pi\varepsilon_0 2a}\int_{\alpha_1}^{\alpha_2}\cos\alpha\,d\alpha = \frac{Q_1'}{4\pi\varepsilon_0 a}\left(\sin\alpha_2 - \sin\alpha_1\right) = \frac{Q_1'}{4\pi\varepsilon_0 a}\left(\frac{2a}{\sqrt{(2a)^2+(2a)^2}} - \frac{a}{\sqrt{a^2+(2a)^2}}\right)$$

$$E_1 = \frac{Q_1'}{4\pi\varepsilon_0 a}\left(\frac{2}{2\sqrt{2}} - \frac{1}{\sqrt{5}}\right) = \frac{Q_1'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{5}}{5}\right)$$

$$\boxed{\vec{E}_1 = \frac{Q_1'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{5}}{5}\right)\cdot\vec{i}_z}$$

b)

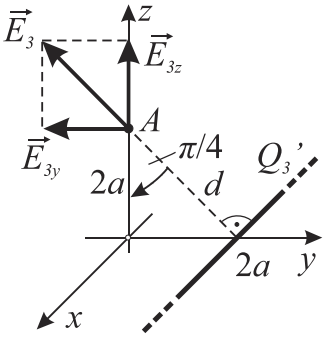


$$dE_2 = \frac{dQ_2}{4\pi\varepsilon_0 r^2} = \frac{Q_2'\,dr}{4\pi\varepsilon_0 r^2}$$

$$E_2 = \int_{\substack{po\\štapu}} dE_2 = \frac{Q_2'}{4\pi\varepsilon_0}\int_a^{2a}\frac{dr}{r^2} = \frac{Q_2'}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{Q_2'}{4\pi\varepsilon_0}\,\frac{1}{2a} = \frac{Q_2'}{8\pi\varepsilon_0 a}$$

$$\boxed{\vec{E}_2 = \frac{Q_2'}{8\pi\varepsilon_0 a}\cdot\vec{i}_y}$$

c)



Pretpostavka: $Q_3' > 0$

$$E_3 = \frac{Q_3'}{2\pi\varepsilon_0 d} = \frac{Q_3'}{4\pi\varepsilon_0 a\sqrt{2}} \qquad d = \sqrt{(2a)^2+(2a)^2} = 2a\sqrt{2}$$

$$E_{3y} = E_3 \sin\frac{\pi}{4} = \frac{Q_3'}{4\pi\varepsilon_0 a\sqrt{2}}\,\frac{\sqrt{2}}{2} = \frac{Q_3'}{8\pi\varepsilon_0 a} \qquad \boxed{\vec{E}_{3y} = \frac{Q_3'}{8\pi\varepsilon_0 a}\cdot\left(-\vec{i}_y\right)}$$

$$E_{3z} = E_3 \cos\frac{\pi}{4} = \frac{Q_3'}{4\pi\varepsilon_0 a\sqrt{2}}\,\frac{\sqrt{2}}{2} = \frac{Q_3'}{8\pi\varepsilon_0 a} \qquad \boxed{\vec{E}_{3z} = \frac{Q_3'}{8\pi\varepsilon_0 a}\cdot\vec{i}_z}$$

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 + \vec{E}_{3y} + \vec{E}_{3z} = \frac{Q_1'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{5}}{5}\right)\cdot\vec{i}_z + \frac{Q_2'}{8\pi\varepsilon_0 a}\cdot\vec{i}_y + \frac{Q_3'}{8\pi\varepsilon_0 a}\cdot\left(-\vec{i}_y\right) + \frac{Q_3'}{8\pi\varepsilon_0 a}\cdot\vec{i}_z$$

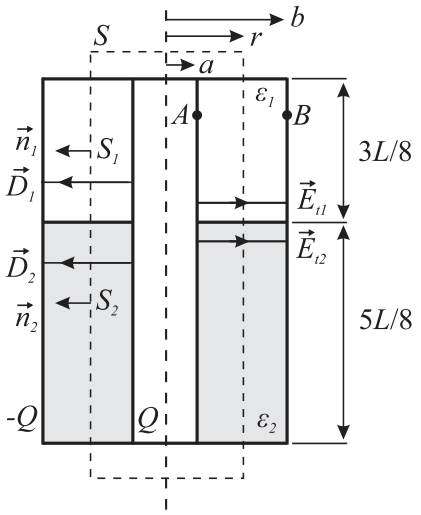
$$\vec{E}_{Ay} = 0 \quad \Rightarrow \quad \left|\vec{E}_2\right| = \left|\vec{E}_{3y}\right|, \qquad \frac{Q_2'}{8\pi\varepsilon_0 a} = \frac{Q_3'}{8\pi\varepsilon_0 a}$$

$Q_3' = Q_2'$ $\qquad \boxed{Q_3' = 2\ nC/m}$ Pretpostavka ok.

Ispit iz OET1, 12.02.2022.

I-2

a)



Granični uslov:

$E_{t1} = E_{t2} \qquad E_1 = E_2 = E$

$D_{n1} = D_{n2} = 0$

$\oint_S \vec{D} \cdot \vec{ds} = Q_{u\,S}$

$\int_{S_{OM}} D\, ds = Q \qquad \int_{OM_1} D_1\, ds + \int_{OM_2} D_2\, ds = Q$

$D_1\, 2\pi r \frac{3L}{8} + D_2\, 2\pi r \frac{5L}{8} = Q \qquad (D_1 = \varepsilon_1 E \qquad D_2 = \varepsilon_2 E)$

$$\boxed{E = \frac{Q}{\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi r}}, \qquad a \le r \le b$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^b \frac{Q}{\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi r} dr = \frac{Q}{\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi} \ln\frac{b}{a}$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi}{\ln\frac{b}{a}}}$$

b)

$$E_{\max}(r=a) = \frac{Q_{\max}}{\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi a} \le \min\left\{E_{Č1},\ E_{Č2}\right\} = E_{Č1} \qquad \Rightarrow \qquad Q_{\max} = E_{Č1}\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi a$$

$$U_{\max} = \frac{Q_{\max}}{\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi} \ln\frac{b}{a} = \frac{E_{Č1}\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi a}{\left(\varepsilon_1 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi} \ln\frac{b}{a} = E_{Č1}\, a \ln\frac{b}{a}$$

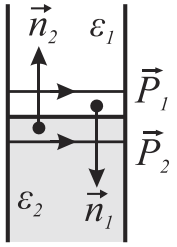
$$a = \frac{U_{\max}}{E_{Č1} \ln\frac{b}{a}} = \frac{20 \cdot 10^3}{80 \cdot 10^5 \cdot \ln 2{,}7} = 0{,}25 \cdot 10^{-2}\, m = 2{,}5 \cdot 10^{-3}\, m \qquad \boxed{a = 2{,}5\, mm}$$

c)

Posle ispuštanja tečnog dielektrika: $\varepsilon_1 \to \varepsilon_0$ $\qquad C^{POSLE} = \dfrac{\left(\varepsilon_0 \frac{3L}{8} + \varepsilon_2 \frac{5L}{8}\right) 2\pi}{\ln\frac{b}{a}}$

$\boxed{C^{POSLE} = 1{,}6\, nF}$

d)



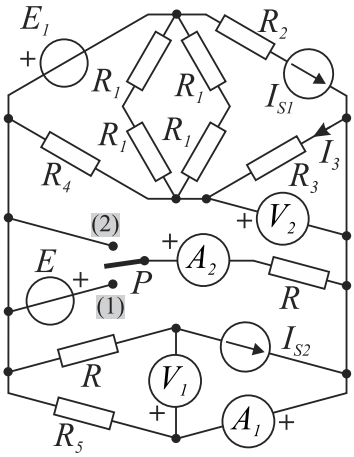
$\sigma_{V1} = \vec{P}_1 \cdot \vec{n}_1 = 0$ $\qquad \boxed{Q_{V1} = 0\, C}$

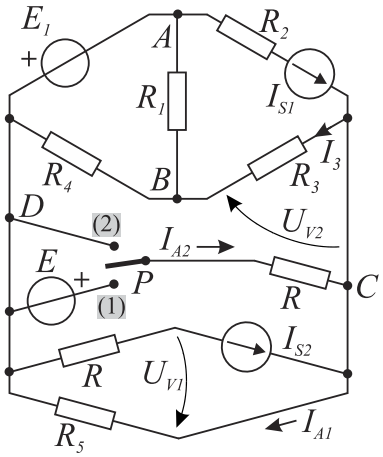
$\sigma_{V2} = \vec{P}_2 \cdot \vec{n}_2 = 0$ $\qquad \boxed{Q_{V2} = 0\, C}$ $\qquad$ **(bonus 5p)**

Ispit iz OET1, 12.02.2022.

II-1

a)



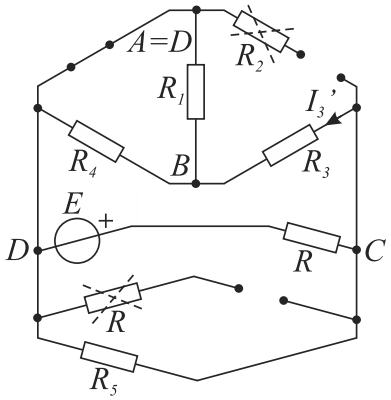


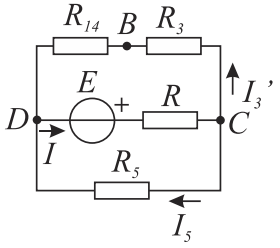
$$\boxed{\text{Svi generatori}} = \boxed{\text{Svi sem } E} + \boxed{\text{Samo } E}$$

(1) (2)

$$I_3^{(1)} = I_3^{(2)} + I_3'$$

$$\Delta I_3 = I_3^{(1)} - I_3^{(2)} = I_3' = 30\ mA$$





$$R_{14} = \frac{R_1 R_4}{R_1 + R_4} = \frac{80 \cdot 80}{80 + 80} = 40\ \Omega$$

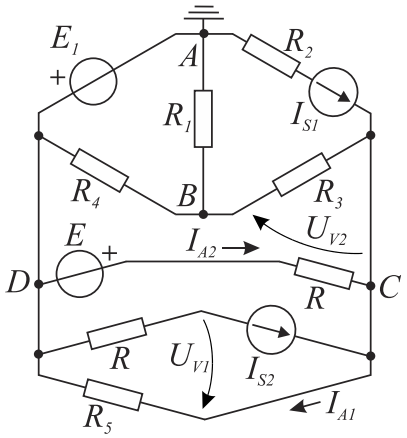
$$U_{CD} = I_3'\left(R_{14} + R_3\right) = 30\,m \cdot \left(40 + 20\right) = 1{,}8\ V$$

$$I_5 = \frac{U_{CD}}{R_5} = \frac{1{,}8}{20} = 90\ mA$$

$$I = I_3' + I_5 = 30\,m + 90\,m = 120\ mA$$

$$E = R\,I + U_{CD} = 10 \cdot 120\,m + 1{,}8 = \qquad \boxed{E = 3\,V}$$

b)



$n_č = 4, \quad n_{i.n.g.} = 1 \qquad MPČ: \quad n_č - 1 - n_{i.n.g.} = 4 - 1 - 1 = 2$

$V_A = 0\,V, \qquad V_D = E_1 = 12\,V$

$$V_B\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) - V_C\left(\frac{1}{R_3}\right) - V_D\left(\frac{1}{R_4}\right) = 0$$

$$V_C\left(\frac{1}{\infty} + \frac{1}{R_3} + \frac{1}{R} + \frac{1}{\infty} + \frac{1}{R_5}\right) - V_B\left(\frac{1}{R_3}\right) - V_D\left(\frac{1}{R} + \frac{1}{\infty} + \frac{1}{R_5}\right) = I_{S1} + I_{S2} + \frac{E}{R}$$

$$V_B\left(\frac{1}{80} + \frac{1}{20} + \frac{1}{80}\right) - V_C\left(\frac{1}{20}\right) - 12 \cdot \left(\frac{1}{80}\right) = 0 \qquad / \cdot 80$$

$$V_C\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{20}\right) - V_B\left(\frac{1}{20}\right) - 12 \cdot \left(\frac{1}{10} + \frac{1}{20}\right) = 0{,}1 + 0{,}2 + \frac{3}{10} \qquad / \cdot 20$$

$$\left.\begin{aligned} 6\,V_B - 4\,V_C &= 12 \\ -V_B + 4\,V_C &= 48 \end{aligned}\right\} \qquad V_B = 12\,V \qquad V_C = 15\,V$$

$$I_{A1} = \frac{V_C - V_D}{R_5} = \frac{15 - 12}{20} = \frac{3}{20} \qquad \boxed{I_{A1} = 0{,}15\ A}$$

$$I_{A2} = \frac{V_D - V_C + E}{R} = \frac{12 - 15 + 3}{10} = \frac{0}{20} \qquad \boxed{I_{A2} = 0\ A}$$

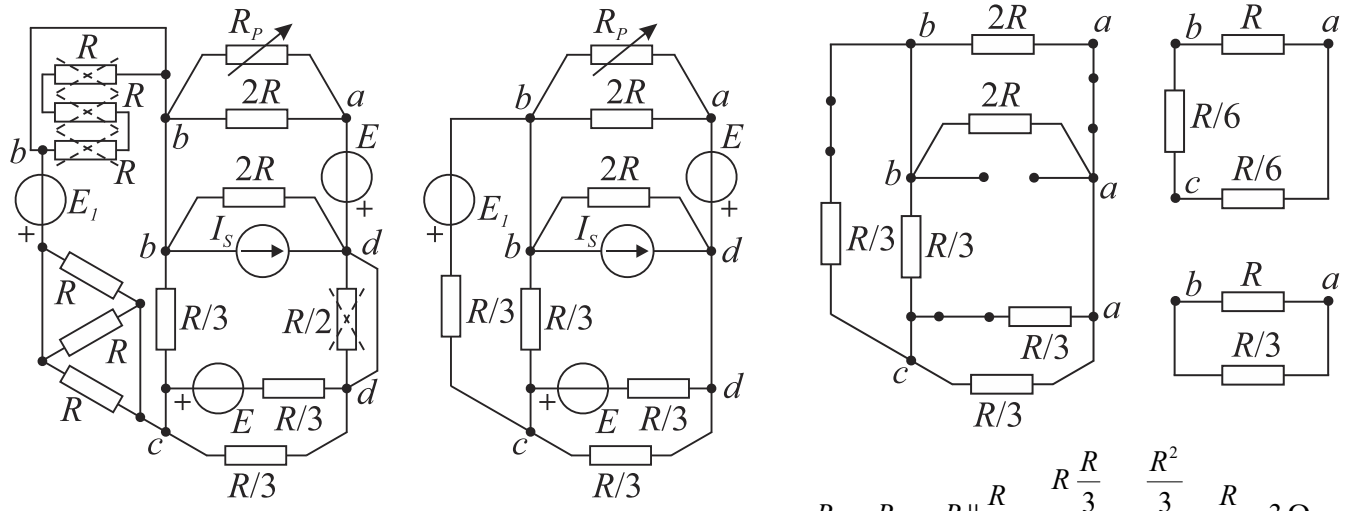
$$U_{V1} = R_5\,I_{A1} + R\,I_{S2} = 20 \cdot 0{,}15 + 10 \cdot 0{,}2 = 3 + 2 \qquad \boxed{U_{V1} = 5\,V}$$

$$U_{V2} = V_B - V_C = 12 - 15 \qquad \boxed{U_{V2} = -3\,V}$$

Ispit iz OET1, 12.02.2022.

II-2

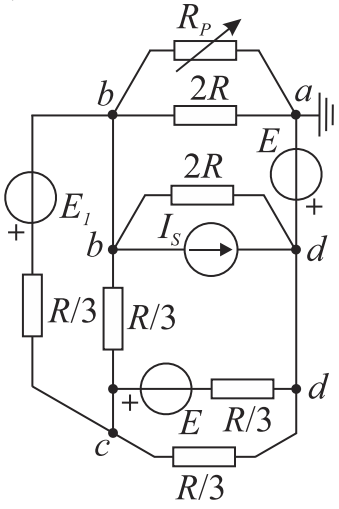
a)



$$R_T = R_{ab} = R \,||\, \frac{R}{3} = \frac{R\,\frac{R}{3}}{R+\frac{R}{3}} = \frac{\frac{R^2}{3}}{\frac{4R}{3}} = \frac{R}{4} = 3\,\Omega$$

$$\boxed{R_P = R_T = 3\,\Omega}$$

b)



$n_č = 4, \quad n_{i.n.g.} = 1 \qquad MPČ: \quad n_č - 1 - n_{i.n.g.} = 4 - 1 - 1 = 2$

$V_a = 0\,V, \qquad V_d = E = 8\,V$

$$V_b\left(\frac{1}{R_P}+\frac{1}{2R}+\frac{1}{2R}+\frac{1}{\infty}+\frac{1}{\frac{R}{3}}+\frac{1}{\frac{R}{3}}\right) - V_c\left(\frac{1}{\frac{R}{3}}+\frac{1}{\frac{R}{3}}\right) - V_d\left(\frac{1}{2R}+\frac{1}{\infty}\right) = -I_S - \frac{E_1}{\frac{R}{3}}$$

$$V_c\left(\frac{1}{\frac{R}{3}}+\frac{1}{\frac{R}{3}}+\frac{1}{\frac{R}{3}}+\frac{1}{\frac{R}{3}}\right) - V_b\left(\frac{1}{\frac{R}{3}}+\frac{1}{\frac{R}{3}}\right) - V_d\left(\frac{1}{\frac{R}{3}}+\frac{1}{\frac{R}{3}}\right) = \frac{E}{\frac{R}{3}} + \frac{E_1}{\frac{R}{3}}$$

$$V_b\left(\frac{1}{3}+\frac{1}{24}+\frac{1}{24}+\frac{1}{4}+\frac{1}{4}\right) - V_c\left(\frac{1}{4}+\frac{1}{4}\right) - 8\cdot\left(\frac{1}{24}\right) = -2 - \frac{12}{4} \qquad /\cdot 24$$

$$V_c\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right) - V_b\left(\frac{1}{4}+\frac{1}{4}\right) - 8\cdot\left(\frac{1}{4}+\frac{1}{4}\right) = \frac{8}{4} + \frac{12}{4} \qquad /\cdot 4$$

$$\left.\begin{aligned} 22\,V_b - 12\,V_c &= -112 \\ -2\,V_b + 4\,V_c &= 36 \end{aligned}\right\} \qquad V_b = -0,25\,V \qquad V_c = 8,875\,V$$

$$P_S = U_S\,I_S = (V_d - V_b)\,I_S = \left[8-(-0,25)\right]\cdot 2 = 8,25\cdot 2 \qquad \boxed{P_S = 16,5\,W}$$

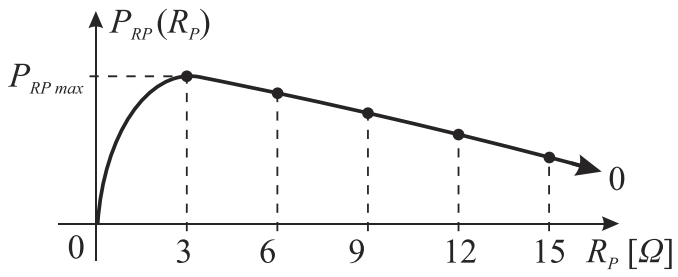
c)

$$P_{RP\max} = \frac{U_P{}^2}{R_P} = \frac{U_{ab}{}^2}{R_P} = \frac{(V_a - V_b)^2}{R_P}$$

$$P_{RP\max} = \frac{\left[0-(-0,25)\right]^2}{3} = \frac{0,25^2}{3}$$

$$\boxed{P_{RP\max} = 20,8\,mW}$$

d)



(bonus 5p)