STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

K1

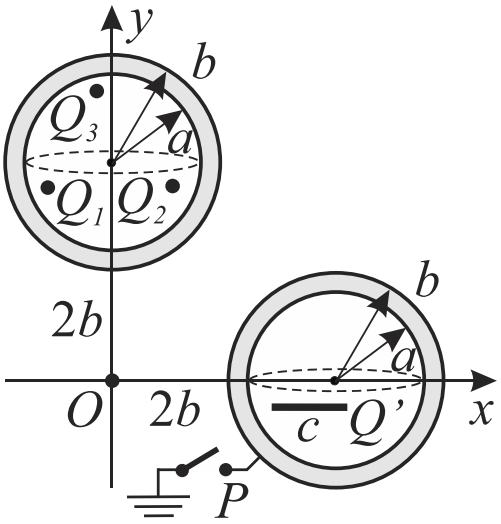
28.01.2022.

ZADACI

Zadatak 1. Dve identične nenaelektrisane provodne sferne ljuske, unutrašnjeg poluprečnika a i spoljašnjeg poluprečnika b, postavljene su kao što je prikazano na slici 1. Centar prve sferne ljuske se nalazi na y osi, na udaljenosti $3b$ od centra Dekartovog koordinatnog sistema (tačka O) i unutar nje se nalaze tri tačkasta naelektrisanja, Q_1, Q_2 i Q_3. Centar druge sferne ljuske se nalazi na x osi, na udaljenosti $3b$ od tačke O i unutar nje se nalazi tanak štap, dužine c, naelektrisan ravnomerno podužnom gustinom naelektrisanja Q' i postavljen paralelno sa x osom. Sferne ljuske se nalaze na dovoljno velikom međusobnom rastojanju, tako da je njihov međusobni uticaj na raspodelu naelektrisanja zanemarljiv. Sistem se nalazi u vazduhu.

a) Odrediti ukupne količine naelektrisanja na unutrašnjoj i spoljašnjoj površi obe provodne ljuske.
b) Odrediti vektor jačine električnog polja u tački O.
c) Odrediti potencijal tačke O u odnosu na referentnu tačku u beskonačnosti.
d) Odrediti potencijal tačke O u odnosu na referentnu tačku u beskonačnosti nakon zatvaranja prekidača P (bonus 5p).

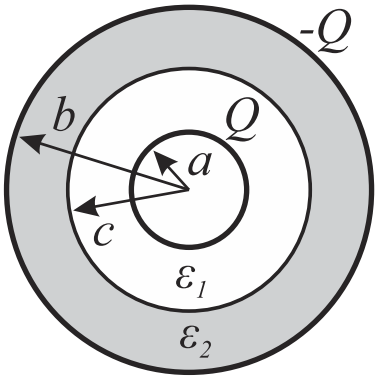
Brojni podaci su: $a = 1\ cm$, $b = 1{,}1\ cm$, $c = 0{,}7\ cm$, $Q' = 1\ nC/m$, $Q_1 = 2\ nC$, $Q_2 = 5\ nC$, $Q_3 = -7\ nC$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.



Slika 1.

Zadatak 2. Na slici 2 je prikazan sferni kondenzator, ispunjen sa dva sloja dielektrika relativnih permitivnosti $\varepsilon_{r1} = 8$ i ε_{r2}. Poluprečnici elektroda kondenzatora su $a = 1\ mm$ i $b = 3\ mm$, dok je poluprečnik razdvojne površi dva dielektrika $c = 2\ mm$.

a) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora.
b) Odrediti relativnu permitivnost drugog dielektrika, ε_{r2}, tako da kada je kondenzator priključen na maksimalno dozvoljeni napon, maksimalni vektor jačine električnog polja u oba sloja dielektrika bude iste vrednosti.
c) Izračunati količinu vezanog naelektrisanja uz unutrašnju elektrodu, kada je kondenzator priključen na napon $U = 8\ kV$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **28.01.2022.**

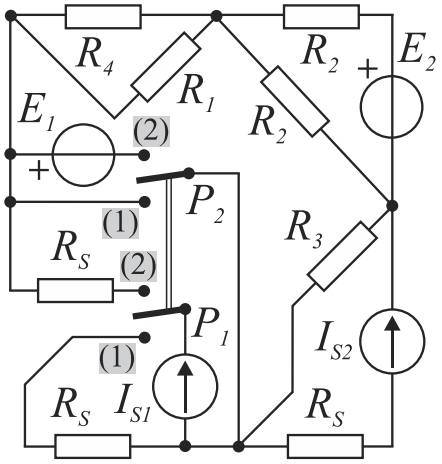
ZADACI

Zadatak 1. U kolu vremenski konstantnih struja, sa slike 1, posle prebacivanja paralelno vezanih preklopnika P_1 i P_2 iz položaja (1) u položaj (2), struja naponskog generatora elektromotorne sile E_1 se poveća za $\Delta I_{E1} = 1\ A$ (smer struje je usaglašen sa elektromotornom silom generatora).

a) Primenjujući teoremu superpozicije, odrediti jačinu struje strujnog generatora, I_{S1}.

b) Izračunati snage naponskog generator E_2 i strujnog generatora I_{S2} u stacionarnom stanju mreže, koje nastane kada je preklopnik u položaju (2). Kolo rešavati primenom metode potencijala čvorova.

Brojni podaci su: $E_1 = 24\ V$, $E_2 = 10\ V$, $I_{S2} = 1\ A$,
$R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 4\ \Omega$, $R_4 = 15\ \Omega$, $R_S = 5\ \Omega$.



Slika 1.

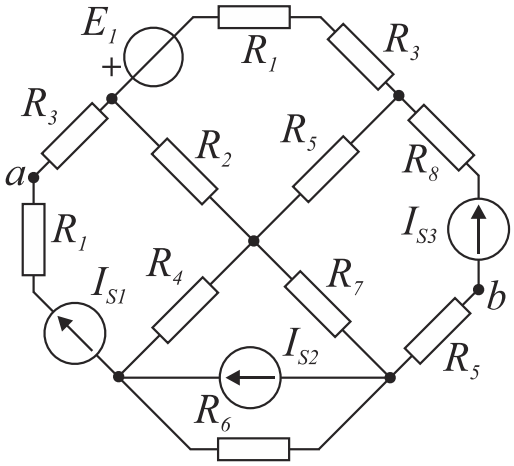
Zadatak 2.

a) Mrežu vremenski konstantnih struja sa slike 2a, predstaviti u vidu Tevenenovog generatora u odnosu na tačke *a* i *b*. Kolo rešavati primenom metode konturnih struja.

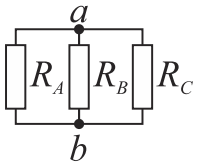
b) Ako se na Tevenenov generator određen pod *a)* priključi grupa od tri otpornika otpornosti R_A, R_B i R_C, prikazana na slici 2b, odrediti njihove otpornosti tako da se na čitavoj grupi razvija maksimalna moguća snaga.

c) Izračunati snage otpornika otpornosti R_A, R_B i R_C. (bonus 5p).

Brojni podaci su: $R_1 = R_4 = R_6 = 1\ k\Omega$, $R_2 = R_5 = 3\ k\Omega$,
$R_3 = R_7 = 2\ k\Omega$, $R_8 = 5\ k\Omega$, $R_B = 2 \cdot R_A$, $R_C = 4 \cdot R_A$,
$E_1 = 3\ V$, $I_{S1} = 1\ mA$, $I_{S2} = 2\ mA$, $I_{S3} = 3\ mA$.



Slika 2a.



Slika 2b.

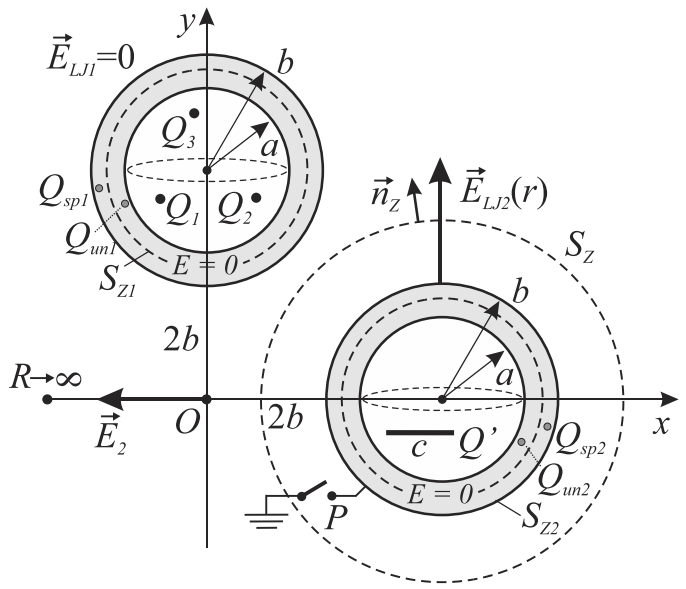
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

Ispit iz OET1, 28.01.2022.

I-1

a)



$$\oint_{S_{Z1}} \vec{E}\cdot d\vec{S} = \frac{Q_{unutar\,S_{Z1}}}{\varepsilon_0} \qquad E=0,\ \text{u provodniku}$$

$$0 = \frac{\cancel{(Q_1+Q_2+Q_3)}^{\,0} + Q_{un1}}{\varepsilon_0} \qquad \boxed{Q_{un1}=0}$$

$$Q_{LJ1} = Q_{un1} + Q_{sp1} = 0 \qquad \boxed{Q_{sp1}=0}$$

$$Q_{\check{S}} = Q'\,c$$

$$\oint_{S_{Z2}} \vec{E}\cdot d\vec{S} = \frac{Q_{unutar\,S_{Z2}}}{\varepsilon_0} \qquad E=0,\ \text{u provodniku}$$

$$0 = \frac{Q_{\check{S}} + Q_{un2}}{\varepsilon_0} \qquad \boxed{Q_{un2} = -Q_{\check{S}}}$$

$$Q_{LJ2} = Q_{un2} + Q_{sp2} = 0 \qquad \boxed{Q_{sp2} = -Q_{un2} = Q_{\check{S}}}$$

b)

$$Q_{sp1}=0 \quad \Rightarrow \quad \boxed{\vec{E}_{LJ1}=0}, \quad r>b$$

$$\oint_{S_Z} \vec{E}\cdot d\vec{S} = \frac{Q_{unutar\,S_Z}}{\varepsilon_0} \qquad \sphericalangle\left(\vec{E}_{LJ2},\vec{n}_Z\right)=0$$

$$E_{LJ2}(r)\,4\pi r^2 = \frac{Q_{\check{S}}+Q_{un}+Q_{sp}}{\varepsilon_0} = \frac{Q_{\check{S}}}{\varepsilon_0}$$

$$E_{LJ2}(r) = \frac{Q_{\check{S}}}{4\pi\varepsilon_0 r^2}\,, \qquad r>b$$

$$E_2 = E_{LJ2uO}(r=3b) = \frac{Q_{\check{S}}}{4\pi\varepsilon_0(3b)^2} = \frac{Q'\,c}{36\pi\varepsilon_0 b^2} = 57{,}83\,\frac{V}{m}$$

$$\boxed{\vec{E}_O = \vec{E}_2 = \frac{Q'\,c}{36\pi\varepsilon_0 b^2}\cdot\left(-\vec{i}_x\right) = 57{,}83\,\frac{V}{m}\cdot\left(-\vec{i}_x\right)}$$

c)

$$V_O = \int_O^{R\to\infty} \vec{E}_{LJ2}\cdot d\vec{l} = \int_{3b}^{\infty} E_{LJ2}(r)\,dr = \int_{3b}^{\infty} \frac{Q_{\check{S}}}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q_{\check{S}}}{4\pi\varepsilon_0}\left(\frac{1}{3b} + \cancel{\frac{1}{\infty}}^{\,0}\right) = \frac{Q_{\check{S}}}{4\pi\varepsilon_0}\,\frac{1}{3b}$$

$$V_O = \frac{Q_{\check{S}}}{12\pi\varepsilon_0 b} = \frac{Q'\,c}{12\pi\varepsilon_0 b} \qquad \boxed{V_O = 1{,}91\,V}$$

d)

Nakon zatvaranja prekidača *P*, spoljašnja površ druge provodne sferne ljuske se uzemlji.

$$Q_{sp2}=0 \quad \Rightarrow \quad \boxed{\vec{E}_{LJ2}=0}, \quad r>b$$

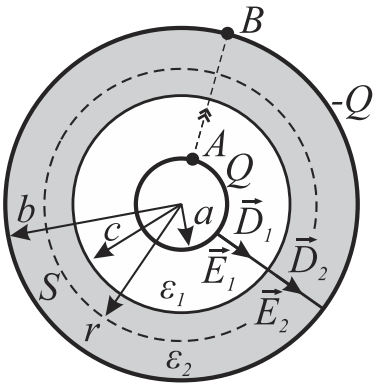
$$V_O' = \int_O^{R\to\infty} \cancel{\vec{E}_{LJ2}}^{\,0}\cdot d\vec{l}$$

$$\boxed{V_O' = 0\,V} \qquad \textbf{(bonus 5p)}$$

Ispit iz OET1, 28.01.2022.

I-2

a)



Granični uslov:

$D_{n1} = D_{n2} \qquad D_1 = D_2 = D$

$E_{t1} = E_{t2} = 0$

$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$

$\int_S D\, ds = Q$

$D\, 4r^2\pi = Q \qquad \boxed{D = \frac{Q}{4\pi r^2}}, \qquad a \le r \le b$

$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4\pi\varepsilon_1 r^2}}, \qquad a \le r \le c$

$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4\pi\varepsilon_2 r^2}}, \qquad c \le r \le b$

$$U_{AB} = \int_A^B \vec{E}\cdot\vec{dl} = \int_a^b E\, dr = \int_a^c E_1\, dr + \int_c^b E_2\, dr = \int_a^c \frac{Q}{4\pi\varepsilon_1 r^2}\, dr + \int_c^b \frac{Q}{4\pi\varepsilon_2 r^2}\, dr = \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a}-\frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{c}-\frac{1}{b}\right)$$

$$U_{AB} = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_1}\frac{c-a}{ac} + \frac{1}{\varepsilon_2}\frac{b-c}{bc}\right)$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{4\pi}{\frac{1}{\varepsilon_1}\frac{c-a}{ac} + \frac{1}{\varepsilon_2}\frac{b-c}{bc}}}$$

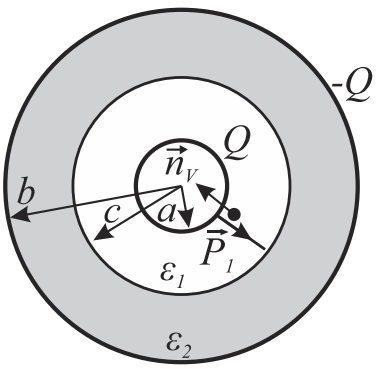
b)

$U = U_{\max} \quad \rightarrow \quad Q = Q_{\max}$

$$\left.\begin{aligned} E_{1\max} &= \frac{Q_{\max}}{4\pi\varepsilon_1 a^2} \\ E_{2\max} &= \frac{Q_{\max}}{4\pi\varepsilon_2 c^2} \\ E_{1\max} &= E_{2\max} \end{aligned}\right\} \quad \Rightarrow \quad \varepsilon_1 a^2 = \varepsilon_2 c^2, \qquad \varepsilon_{r1}\varepsilon_0 a^2 = \varepsilon_{r2}\varepsilon_0 c^2, \qquad \varepsilon_{r2} = \varepsilon_{r1}\frac{a^2}{c^2} = 8\cdot\frac{\left(1\cdot10^{-3}\right)^2}{\left(2\cdot10^{-3}\right)^2}$$

$\boxed{\varepsilon_{r2} = 2}$

c)



$$\left.\begin{aligned} C &= 0{,}76\ pF \\ U &= 8\ kV \end{aligned}\right\} \quad \Rightarrow \quad Q = CU = 0{,}76\ pF \cdot 8\ kV = 6{,}08\ nC$$

$$P_1 = D - \varepsilon_0 E_1 = D - \varepsilon_0\frac{D}{\varepsilon_1} = \left(1-\frac{1}{\varepsilon_{r1}}\right)D = \left(1-\frac{1}{8}\right)\frac{Q}{4\pi r^2} = \frac{7}{8}\frac{Q}{4\pi r^2}$$

$$\sigma_V = \vec{P}_1\cdot\vec{n}_V = -P_1(a) = -\frac{7}{8}\frac{Q}{4\pi a^2}$$

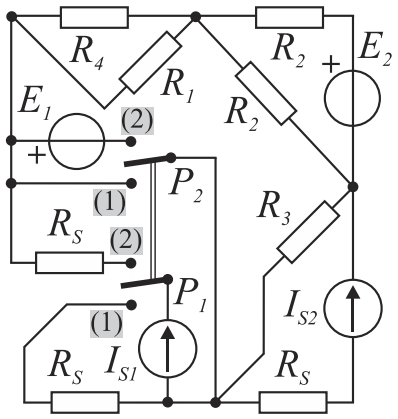
$$Q_V = \sigma_V\, 4\pi a^2 = -\frac{7}{8}\frac{Q}{4\pi a^2}\, 4\pi a^2 = -\frac{7}{8}Q = -\frac{7}{8}\cdot 6{,}08\ nC$$

$\boxed{Q_V = -5{,}32\ nC}$

Ispit iz OET1, 28.01.2022.

II-1

a)

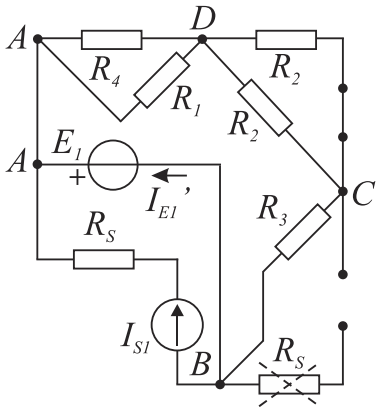


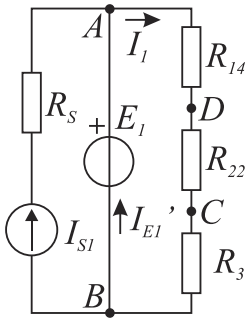
$$\boxed{\text{Svi generatori}} = \boxed{\text{Svi sem } I_{S1} \text{ i } E_1} + \boxed{\text{Samo } I_{S1} \text{ i } E_1}$$

(2) (1)

$$I_{E1}^{(2)} = I_{E1}^{(1)} + I_{E1}'$$

$$\Delta I_{E1} = I_{E1}^{(2)} - I_{E1}^{(1)} = I_{E1}' = 1\ A$$





$$R_{14} = \frac{R_1 R_4}{R_1 + R_4} = \frac{10 \cdot 15}{10 + 15} = 6\ \Omega$$

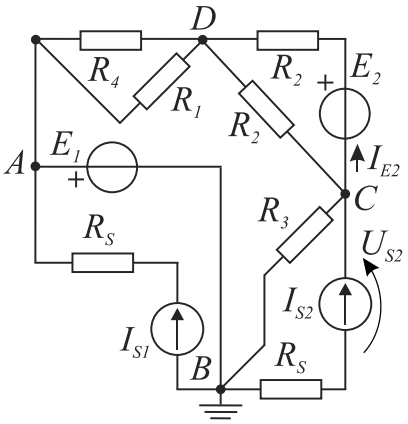
$$R_{22} = \frac{R_2 R_2}{R_2 + R_2} = \frac{20 \cdot 20}{20 + 20} = 10\ \Omega$$

$$U_{AB} = E_1 = 24\ V$$

$$I_1 = \frac{U_{AB}}{R_{14} + R_{22} + R_3} = \frac{24}{6 + 10 + 4} = \frac{24}{20} = 1{,}2\ A$$

$$I_{S1} = I_1 - I_{E1} = 1{,}2 - 1 \qquad \boxed{I_{S1} = 0{,}2\ A}$$

b)



$n_č = 4, \quad n_{i.n.g.} = 1$

$MPČ: \quad n_č - 1 - n_{i.n.g.} = 4 - 1 - 1 = 2$

$V_B = 0\ V, \qquad V_A = E_1 = 24\ V$

$$V_C\left(\frac{1}{R_2} + \frac{1}{R_2 + 0} + \frac{1}{R_3} + \frac{1}{R_S + \infty}\right) - V_D\left(\frac{1}{R_2} + \frac{1}{R_2 + 0}\right) = I_{S2} - \frac{E_2}{R_2}$$

$$V_D\left(\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_2} + \frac{1}{R_2 + 0}\right) - V_A\left(\frac{1}{R_1} + \frac{1}{R_4}\right) - V_C\left(\frac{1}{R_2} + \frac{1}{R_2 + 0}\right) = \frac{E_2}{R_2}$$

$$V_C\left(\frac{1}{20} + \frac{1}{20} + \frac{1}{4}\right) - V_D\left(\frac{1}{20} + \frac{1}{20}\right) = 1 - \frac{10}{20} \qquad / \cdot 20$$

$$V_D\left(\frac{1}{10} + \frac{1}{15} + \frac{1}{20} + \frac{1}{20}\right) - 24 \cdot \left(\frac{1}{10} + \frac{1}{15}\right) - V_C\left(\frac{1}{20} + \frac{1}{20}\right) = \frac{10}{20} \qquad / \cdot 60$$

$$\left.\begin{aligned} 7\,V_C - 2\,V_D &= 10 \\ -6\,V_C + 16\,V_D &= 270 \end{aligned}\right\} \qquad V_C = 7\ V \qquad V_D = 19{,}5\ V$$

$$I_{E2} = \frac{V_C - V_D + E_2}{R_2} = \frac{7 - 19{,}5 + 10}{20} = \frac{-2{,}5}{20} = -0{,}125\ A$$

$$P_{E2} = E_2\, I_{E2} = 10 \cdot (-0{,}125) \qquad \boxed{P_{E2} = -1{,}25\ W}$$

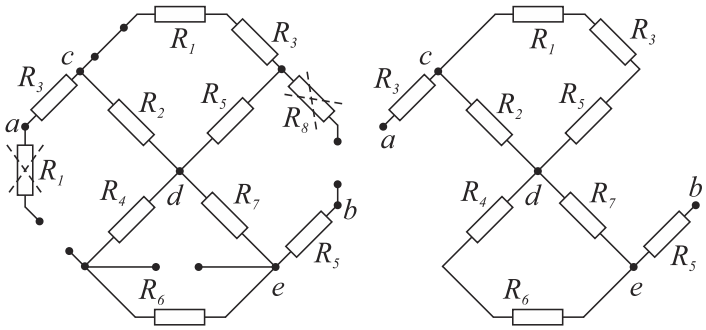
$$U_{S2} = (V_C - V_B) + R_S I_{S2} = (7 - 0) + 5 \cdot 1 = 12\ V$$

$$P_{S2} = U_{S2}\, I_{S2} = 12 \cdot 1 \qquad \boxed{P_{S2} = 12\ W}$$

Ispit iz OET1, 28.01.2022.

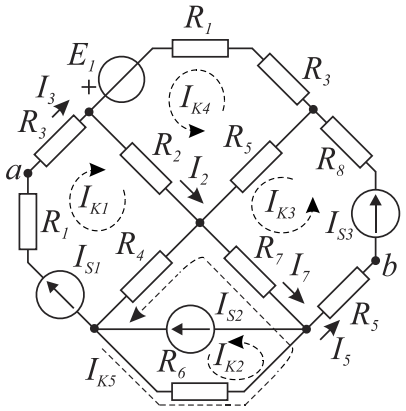
II-2

a)



$$R_T = R_3 + R_2 \,||\, (R_1 + R_3 + R_5) + R_7 \,||\, (R_4 + R_6) + R_5$$
$$R_T = 2k + 3k \,||\, (1k + 2k + 3k) + 2k \,||\, (1k + 1k) + 3k$$
$$R_T = 2k + 3k \,||\, 6k + 2k \,||\, 2k + 3k$$
$$R_T = 2k + 2k + 1k + 3k = 8k$$

$$\boxed{R_T = 8\,k\Omega}$$



$$n_g = 9, \quad n_č = 5, \quad n_{s.g.} = 3$$
$$MKS: \quad n_g - (n_č - 1) - n_{s.g.} = 9 - (5-1) - 3 = 5 - 3 = 2$$

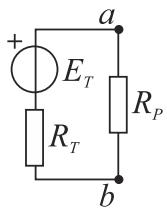
$$K1: \quad I_{K1} = I_{S1} = 1\ mA$$
$$K2: \quad I_{K2} = I_{S2} = 2\ mA$$
$$K3: \quad I_{K3} = I_{S3} = 3\ mA$$
$$K4: \quad (R_1 + R_2 + R_3 + R_5) I_{K4} + R_2 I_{K1} - R_5 I_{K3} = E_1$$
$$K5: \quad (R_4 + R_6 + R_7) I_{K5} + R_4 I_{K1} + R_6 I_{K2} - R_7 I_{K3} = 0$$

$$9k\ I_{K4} = E_1 - 3k\ I_{K1} + 3k\ I_{K3} = 3 - 3k \cdot 1m + 3k \cdot 3m = 9$$
$$I_{K4} = 1\ mA$$
$$4k\ I_{K5} = -1k\ I_{K1} - 1k\ I_{K2} + 2k\ I_{K3} = -1k \cdot 1m - 1k \cdot 2m + 2k \cdot 3m = 3$$
$$I_{K5} = 0,75\ mA$$

$$U_{ab} = R_3 I_3 + R_2 I_2 + R_7 I_7 + R_5 I_5 = R_3 I_{K1} + R_2 (I_{K1} + I_{K4}) + R_7 (I_{K3} - I_{K5}) + R_5 I_{K3}$$
$$U_{ab} = 2k \cdot 1m + 3k \cdot 2m + 2k \cdot 2,25m + 3k \cdot 3m = 2 + 6 + 4,5 + 9 = 21,5\,V$$

$$\boxed{E_T = 21,5\,V}$$

b)



$$\frac{1}{R_P} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} = \frac{1}{R_A} + \frac{1}{2R_A} + \frac{1}{4R_A} = \frac{4}{4R_A} + \frac{2}{4R_A} + \frac{1}{4R_A} = \frac{7}{4R_A} \qquad R_P = \frac{4}{7} R_A$$

$$R_P = R_T = 8\,k\Omega$$

$$\boxed{R_A = \frac{7}{4} R_P = 14\,k\Omega} \qquad \boxed{R_B = 2R_A = 28\,k\Omega} \qquad \boxed{R_C = 4R_A = 56\,k\Omega}$$

c)

$$U_{ab} = \frac{R_P}{R_T + R_P} E_T = \frac{E_T}{2} = 10,75\,V$$

$$P_A = \frac{U_{ab}^2}{R_A} = \frac{10,75^2}{14k} \qquad \boxed{P_A = 8,25\,mW}$$

$$P_B = \frac{U_{ab}^2}{R_B} = \frac{10,75^2}{28k} \qquad \boxed{P_B = 4,13\,mW}$$

$$P_C = \frac{U_{ab}^2}{R_C} = \frac{10,75^2}{56k} \qquad \boxed{P_C = 2,06\,mW} \qquad \textbf{(bonus 5p)}$$