**STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE**

K T E T
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

# Osnovi elektrotehnike 1 (I kolokvijum)

**K1**

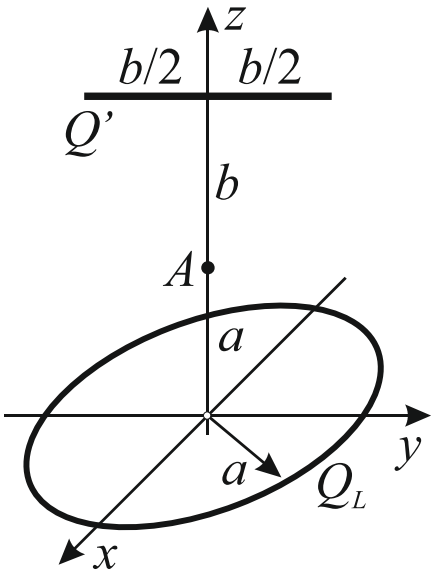
**13.07.2022.**

## ZADACI

**Zadatak 1.** Tanak prsten, poluprečnika $a$, naelektrisan je ravnomerno količinom naelektrisanja $Q_L$, i postavljen je u $x$-$y$ ravni Dekartovog koordinatnog sistema, kao što je prikazano na slici 1. Tanak štap, dužine $b$, naelektrisan ravnomerno podužnom gustinom naelektrisanja $Q'$, postavljen je u $y$-$z$ ravni, paralelno sa $y$ osom. Sredina je vazduh.

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja koji u tački $A$ stvara prsten. Tačka $A$ se nalazi na $z$ osi, na udaljenosti $a$ od centra koordinatnog sistema.
b) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja koji u tački $A$ stvara štap. Štap se nalazi na udaljenosti $b$ od tačke $A$.
c) Odrediti dužinu štapa, $b$, tako da rezultantni vektor jačine električnog polja u tački $A$ bude jednak nuli.

Brojni podaci su: $a = 1\ cm$, $Q_L = 10\ nC$, $Q' = 1\ \mu C/m$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.

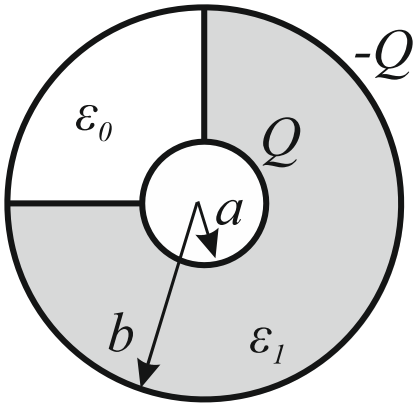



Slika 1.

**Zadatak 2.** Na slici 2 je prikazan sferni kondenzator, poluprečnika elektroda $a$ i $b$. Kondenzator je ispunjen čvrstim dielektrikom, relativne permitivnosti $\varepsilon_{r1}$, pri čemu dielektrik zauzima 3/4 zapremine kondenzatora.

a) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora.
b) U deo kondenzatora gde je vazduh naspe se tečni dielektrik, relativne permitivnosti $\varepsilon_{r2} = 6$. Odrediti relativnu permitivnost čvrstog dielektrika, $\varepsilon_{r1}$, ako se zna da se nakon dodavanja tečnog dielektrika kapacitivnost kondenzatora poveća za 20%.
c) Izračunati količinu vezanog naelektrisanja uz obe razdvojne površi, nakon dodavanja tečnog dielektrika.

Brojni podaci su: $a = 3\ mm$, $b = 6\ mm$.




Slika 2.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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# Osnovi elektrotehnike 1 (II kolokvijum)

**K2**

**13.07.2022.**

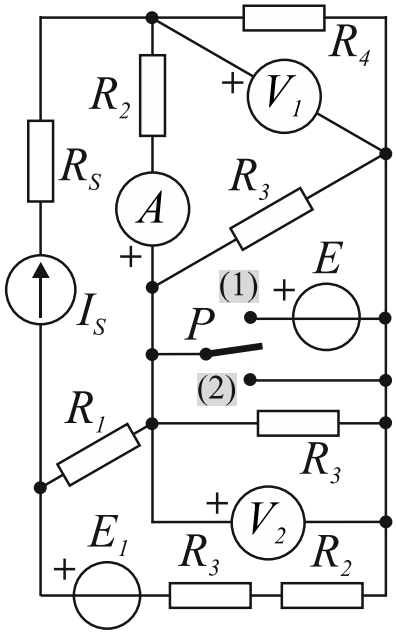
## ZADACI

**Zadatak 1.** U kolu vremenski konstantnih struja, sa slike 1, posle prebacivanja preklopnik $P$ iz položaja (2) u položaj (1), napon koji meri voltmetar $V_1$ se poveća za $\Delta U_{V1} = 2\ V$.

a) Primenjujući teoremu superpozicije, odrediti vrednost *ems* naponskog generatora $E$.

b) Odrediti pokazivanja idealnih mernih instrumenata, kada je preklopnik u položaju (1). Kolo rešavati primenom metode potencijala čvorova.

Brojni podaci su: $E_1 = 2\ V$, $I_S = 0{,}9\ A$, $R_1 = 10\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = R_4 = 5\ \Omega$, $R_S = 20\ \Omega$.




Slika 1.

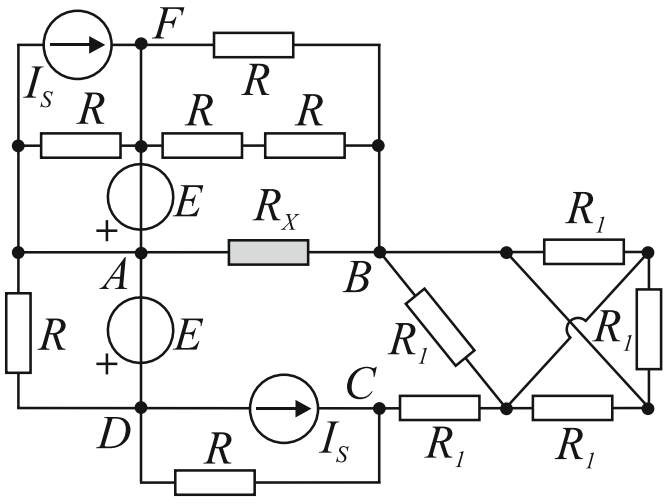
**Zadatak 2.** U električnom kolu, prikazanom na slici 2, na otporniku otpornosti $R_X$ razvija se maksimalno moguća snaga.

a) Zameniti grupu od 5 otpornika otpornosti $R_1$, između tačaka $B$ i $C$, jednim otpornikom. Izračunati vrednost otpornosti tog otpornika, $R_{BC}$.

b) Primenom Tevenenove teoreme, izračunati vrednost otpornosti otpornika $R_X$.

c) Izračunati snagu koja se razvija na otporniku otpornosti $R_X$.

Brojni podaci su: $E = 5\ V$, $I_S = 1\ A$, $R = 10\ \Omega$, $R_1 = 40\ \Omega$.




Slika 2.

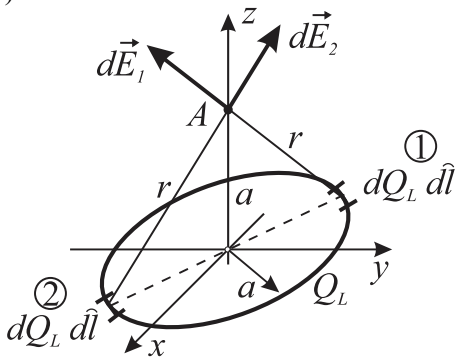
## PRAVILA POLAGANJA

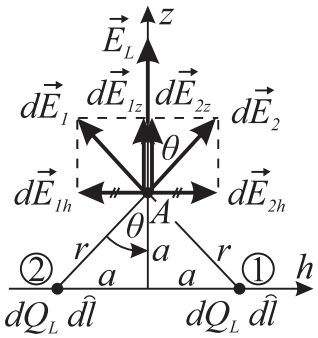
Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET1, 13.07.2022.

**I-1**

a)




Zbog simetrije je:

$d\vec{E}_{1h} + d\vec{E}_{2h} = 0 \quad \Rightarrow \quad \boxed{E_h = 0}$

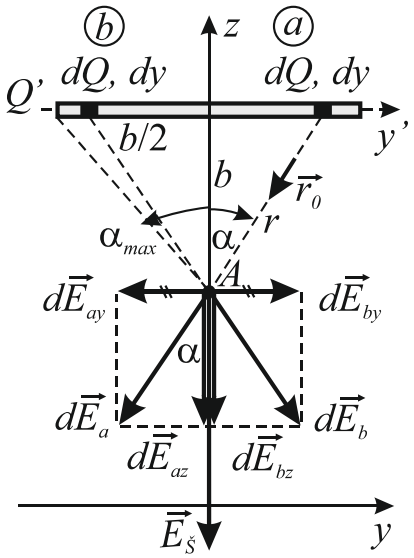
$\boxed{E_x = 0}$ $\qquad$ $\boxed{E_y = 0}$

$$dE_{1z} = dE_{2z} = dE_1 \cos\theta = \frac{dQ_L}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q_L'\,dl}{4\pi\varepsilon_0 r^2}\frac{a}{r} = \frac{Q_L'\,a}{4\pi\varepsilon_0 r^3}dl$$

$$E_L = \int_{\substack{po\\luku}} dE_{1z} = \frac{Q_L'\,a}{4\pi\varepsilon_0 r^3}\int_0^{2a\pi} dl = \frac{Q_L'\,a}{4\pi\varepsilon_0 r^3}2a\pi = \frac{Q_L\,a}{4\pi\varepsilon_0\left(a^2+a^2\right)^{\frac{3}{2}}} = \frac{Q_L}{8\sqrt{2}\pi\varepsilon_0 a^2}$$

$$\boxed{\vec{E}_L = \frac{Q_L}{8\sqrt{2}\pi\varepsilon_0 a^2}\cdot\vec{i}_z}$$

b)




$\overrightarrow{dE}_{ay} + \overrightarrow{dE}_{by} = 0 \quad \Rightarrow \quad \boxed{\vec{E}_y = 0}$

$$dE_{az} = dE_a\cos\alpha = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\alpha = \frac{Q'dy}{4\pi\varepsilon_0 r^2}\cos\alpha \qquad \left(dy = \frac{r\,d\alpha}{\cos\alpha}\right)$$

$$dE_{az} = \frac{Q'\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\cos\alpha = \frac{Q'}{4\pi\varepsilon_0\dfrac{b}{\cos\alpha}}d\alpha \qquad \left(r = \frac{b}{\cos\alpha}\right)$$

$$E_Š = 2\int_{\substack{po\ pola\\štapa}} dE_{az} = 2\frac{Q'}{4\pi\varepsilon_0 b}\int_0^{\alpha_{max}}\cos\alpha\,d\alpha = \frac{Q'}{2\pi\varepsilon_0 b}\left(\sin\alpha_{max} - \sin 0\right)$$

$$E_Š = \frac{Q'}{2\pi\varepsilon_0 b}\cdot\frac{\dfrac{b}{2}}{\sqrt{b^2+\left(\dfrac{b}{2}\right)^2}} = \frac{Q'}{2\pi\varepsilon_0 b}\cdot\frac{\sqrt{5}}{5} = \frac{\sqrt{5}\,Q'}{10\pi\varepsilon_0 b}$$

$$\boxed{\vec{E}_Š = \frac{\sqrt{5}\,Q'}{10\pi\varepsilon_0 b}\cdot\left(-\vec{i}_z\right)}$$

c)

$$\vec{E}_A = \vec{E}_L + \vec{E}_Š = \left[\frac{Q_L}{8\sqrt{2}\pi\varepsilon_0 a^2} - \frac{\sqrt{5}\,Q'}{10\pi\varepsilon_0 b}\right]\cdot\vec{i}_z$$

$$\vec{E}_A = 0 \qquad \Rightarrow \qquad \frac{Q_L}{8\sqrt{2}\pi\varepsilon_0 a^2} = \frac{\sqrt{5}\,Q'}{10\pi\varepsilon_0 b}$$

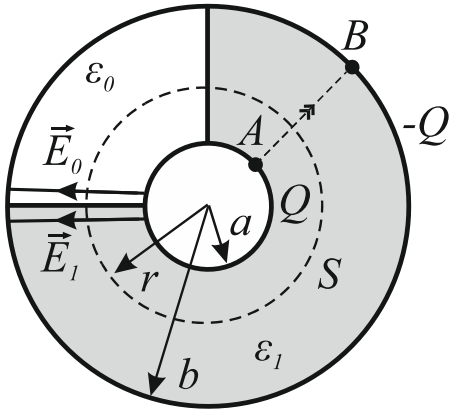
$$b = \frac{8\sqrt{2}\,a^2\sqrt{5}\,Q'}{10\,Q_L} = \frac{8\sqrt{2}\cdot 0{,}01^2\cdot\sqrt{5}\cdot 1\cdot 10^{-6}}{10\cdot 10\cdot 10^{-9}} = 0{,}0253\,m$$

$\boxed{b = 2{,}53\,cm}$

# Ispit iz OET1, 13.07.2022.

**I-2**

a)




Granični uslov:

$E_{t1} = E_{t0} \qquad E_1 = E_0 = E$

$D_{n1} = D_{n0} = 0$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{u\,S}$$

$$\int_{S_1} D_1 \, ds + \int_{S_0} D_0 \, ds = Q$$

$$D_1 \frac{3}{4} \cdot 4r^2\pi + D_0 \frac{1}{4} \cdot 4r^2\pi = Q \qquad D_1 = \varepsilon_1 E \qquad D_0 = \varepsilon_0 E$$

$$\boxed{E = \frac{Q}{\left(3\varepsilon_1 + \varepsilon_0\right) r^2 \pi}}, \qquad a \le r \le b$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E \, dr = \int_a^b \frac{Q}{\left(3\varepsilon_1 + \varepsilon_0\right) r^2 \pi} \, dr = \frac{Q}{\left(3\varepsilon_1 + \varepsilon_0\right)\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{\left(3\varepsilon_1 + \varepsilon_0\right)\pi}{\frac{1}{a} - \frac{1}{b}}}$$

b)

$$\varepsilon_0 \to \varepsilon_2 \qquad \Rightarrow \qquad C_{NOVO} = \frac{\left(3\varepsilon_1 + \varepsilon_2\right)\pi}{\frac{1}{a} - \frac{1}{b}} = 1{,}2\,C$$

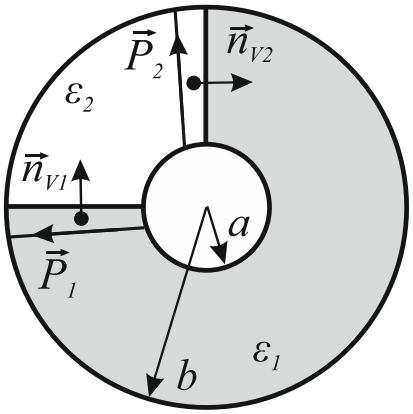
$$\frac{C_{NOVO}}{C} = \frac{\dfrac{\left(3\varepsilon_1 + \varepsilon_2\right)\pi}{\frac{1}{a} - \frac{1}{b}}}{\dfrac{\left(3\varepsilon_1 + \varepsilon_0\right)\pi}{\frac{1}{a} - \frac{1}{b}}} = \frac{3\varepsilon_1 + \varepsilon_2}{3\varepsilon_1 + \varepsilon_0} = \frac{\left(3\varepsilon_{r1} + \varepsilon_{r2}\right)\varepsilon_0}{\left(3\varepsilon_{r1} + 1\right)\varepsilon_0} = \frac{3\varepsilon_{r1} + \varepsilon_{r2}}{3\varepsilon_{r1} + 1} = 1{,}2$$

$3\varepsilon_{r1} + \varepsilon_{r2} = 3{,}6\varepsilon_{r1} + 1{,}2$

$0{,}6\varepsilon_{r1} = \varepsilon_{r2} - 1{,}2 = 6 - 1{,}2 = 4{,}8$

$$\boxed{\varepsilon_{r1} = 8}$$

c)




Na obe razdvojne površi:

$\sigma_{V1} = \vec{P}_1 \cdot \vec{n}_{V1} = 0 \qquad \sphericalangle\left(\vec{P}_1, \vec{n}_{V1}\right) = 90°$

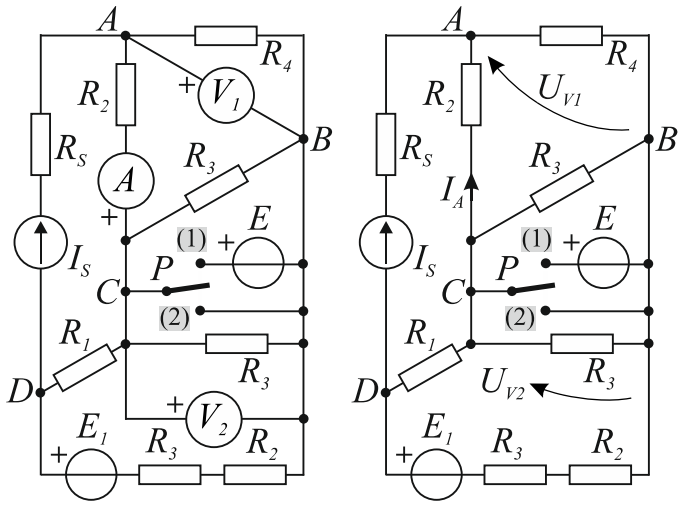
$\sigma_{V2} = \vec{P}_2 \cdot \vec{n}_{V2} = 0 \qquad \sphericalangle\left(\vec{P}_2, \vec{n}_{V2}\right) = 90°$

$$\boxed{Q_{V\,uk} = 0\,C}$$

Ispit iz OET1, 13.07.2022.

**II-1**

a)





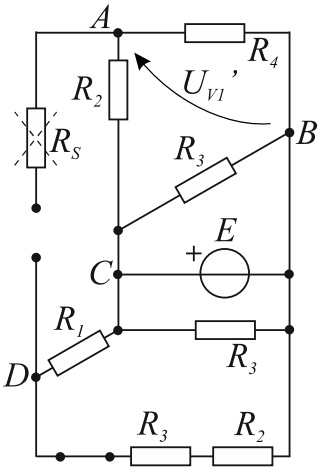


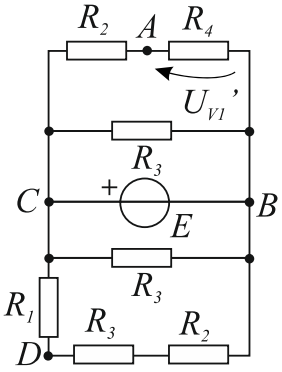
$$\boxed{\text{Svi generatori}} = \boxed{\text{Svi sem } E} + \boxed{\text{Samo } E}$$

(1) (2)

$$U_{V1}^{(1)} = U_{V1}^{(2)} + U_{V1}'$$

$$\Delta U_{V1} = U_{V1}^{(1)} - U_{V1}^{(2)} = U_{V1}' = 2\,V$$



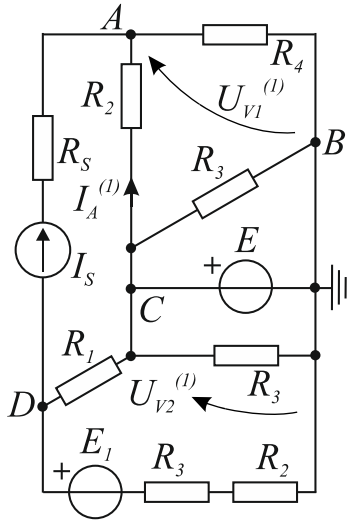




$$U_{V1}' = \frac{R_4}{R_2 + R_4} E$$

$$E = \frac{R_2 + R_4}{R_4} U_{V1}' = \frac{15+5}{5} \cdot 2$$

$$\boxed{E = 8\,V}$$

b)




$$V_B = 0\,V, \qquad V_C = E = 8\,V$$

$$V_A\left(\frac{1}{\infty} + \frac{1}{R_2} + \frac{1}{R_4}\right) - V_C\left(\frac{1}{R_2}\right) - V_D\left(\frac{1}{\infty}\right) = I_S$$

$$V_D\left(\frac{1}{\infty} + \frac{1}{R_1} + \frac{1}{R_2 + R_3}\right) - V_A\left(\frac{1}{\infty}\right) - V_C\left(\frac{1}{R_1}\right) = -I_S + \frac{E_1}{R_2 + R_3}$$

$$V_A\left(\frac{1}{15} + \frac{1}{5}\right) - 8 \cdot \left(\frac{1}{15}\right) = 0,9 \qquad / \cdot 15$$

$$V_D\left(\frac{1}{10} + \frac{1}{15+5}\right) - 8 \cdot \left(\frac{1}{10}\right) = -0,9 + \frac{2}{15+5} \qquad / \cdot 20$$

$$\left.\begin{aligned} 4V_A &= 21,5 \\ 3V_D &= 0 \end{aligned}\right\} \qquad V_A = 5,375\,V \qquad V_D = 0\,V$$

$$I_A^{(1)} = \frac{V_C - V_A}{R_2} = \frac{8 - 5,375}{15} = \frac{2,625}{15} \qquad \boxed{I_A^{(1)} = 0,175\,A}$$

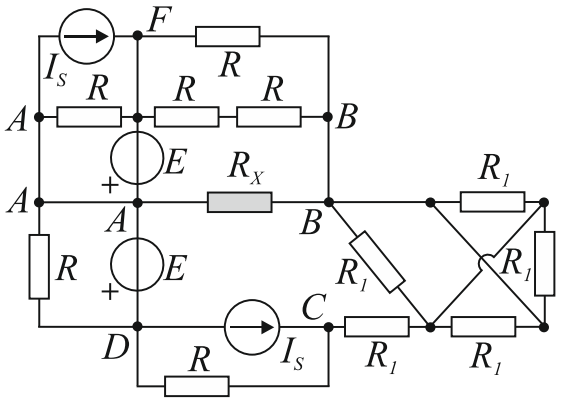
$$U_{V1}^{(1)} = V_A - V_B = V_A \qquad \boxed{U_{V1}^{(1)} = 5,375\,V}$$

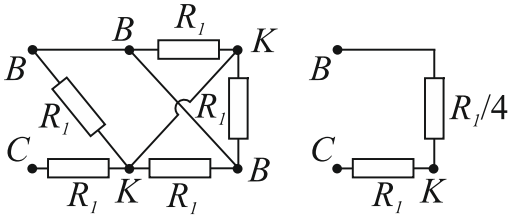
$$U_{V2}^{(1)} = V_C - V_B = V_C = E \qquad \boxed{U_{V2}^{(1)} = 8\,V}$$

Ispit iz OET1, 13.07.2022.

**II-2**

a)



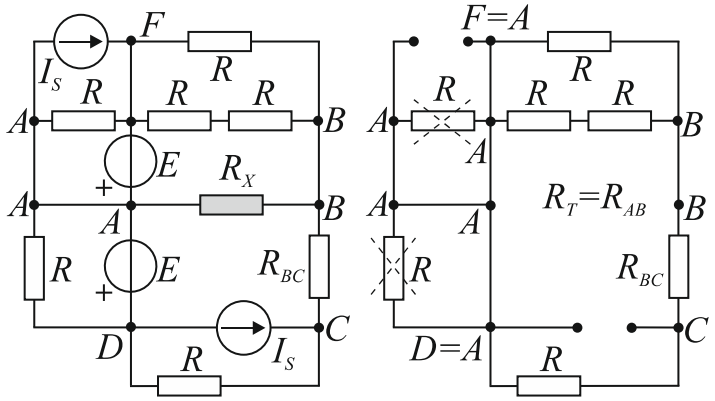



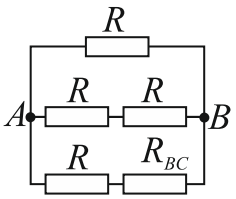

$$R_{BC} = R_1 + \frac{R_1}{4} = \frac{5R_1}{4} = \frac{5 \cdot 40}{4}$$

$$\boxed{R_{BC} = 50\,\Omega}$$

b)

$R_T$:




$$R_T = R_{AB} = R \,\|\, 2R \,\|\, \left(R + R_{BC}\right)$$

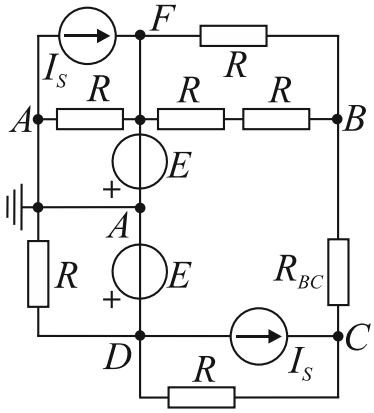
$$R_T = 10 \,\|\, 20 \,\|\, \left(10 + 50\right)$$

$$R_T = 10 \,\|\, 20 \,\|\, 60 = 6\,\Omega$$

$$\boxed{R_X = R_T = 6\,\Omega}$$

c)

$E_T$:




$$V_A = 0\,V, \qquad V_D = E = 5\,V, \qquad V_F = -E = -5\,V$$

$$V_B\left(\frac{1}{R} + \frac{1}{2R} + \frac{1}{R_{BC}}\right) - V_C\left(\frac{1}{R_{BC}}\right) - V_F\left(\frac{1}{R} + \frac{1}{2R}\right) = 0$$

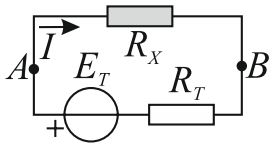
$$V_C\left(\frac{1}{R_{BC}} + \frac{1}{\infty} + \frac{1}{R}\right) - V_B\left(\frac{1}{R_{BC}}\right) - V_D\left(\frac{1}{\infty} + \frac{1}{R}\right) = I_S$$

$$V_B\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{50}\right) - V_C\left(\frac{1}{50}\right) - (-5)\cdot\left(\frac{1}{10} + \frac{1}{20}\right) = 0 \qquad /\cdot 100$$

$$V_C\left(\frac{1}{50} + \frac{1}{10}\right) - V_B\left(\frac{1}{50}\right) - 5\cdot\left(\frac{1}{10}\right) = 1 \qquad /\cdot 100$$

$$\left.\begin{aligned} 17\,V_B - 2\,V_C &= -75 \\ -2\,V_B + 12\,V_C &= 150 \end{aligned}\right\} \qquad V_B = -3\,V \qquad V_C = 12\,V$$

$$E_T = U_{AB} = V_A - V_B = -V_B = -(-3) \qquad \boxed{E_T = 3\,V}$$




$$I = \frac{E_T}{R_T + R_X} = \frac{3}{6+6} = 0{,}25\ A$$

$$P_{RX} = R_X I^2 = 6 \cdot 0{,}25^2 \qquad \boxed{P_{RX} = 0{,}375\,W}$$