STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

K1

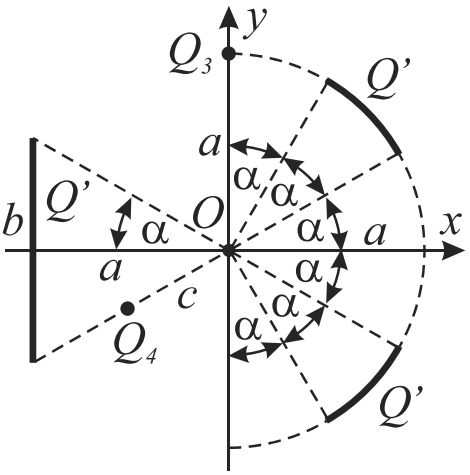
06.04.2021.

ZADACI

Zadatak 1. Tri tanka štapa, naelektrisana ravnomerno istom podužnom gustinom naelektrisanja Q', postavljena su kao što je prikazano na slici 1. Prvi štap, dužine b, je postavljen paralelno sa y osom Dekartovog koordinatnog sistema, simetrično u odnosu na x osu. Drugi i treći štap, savijeni u obliku luka poluprečnika a, su postavljeni simetrično u odnosu na x osu. Dva tačkasta naelektrisanja, naelektrisana količinama naelektrisanja Q_3 i Q_4, se nalaze na y osi i u trećem kvadrantu x-y ravni, respektivno.

a) Odrediti, u opštim brojevima, vektor jačine električnog polja koji u tački O (centar koordinatnog sistema) stvaraju štapovi i tačkasta naelektrisanja.
b) Odrediti količinu naelektrisanja tačkastog naelektrisanja, Q_4, tako da ukupan vektor jačine električnog polja u tački O nema y komponentu.
c) Izračunati količinu naelektrisanja na prvom štapu dužine b, (bonus 5p).

Brojni podaci: $a = 2\ cm$, $c = 1\ cm$, $Q' = 1\ nC/m$, $Q_3 = 10\ nC$, $\alpha = 30^o$.

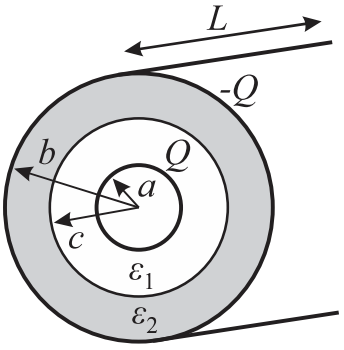


Slika 1.

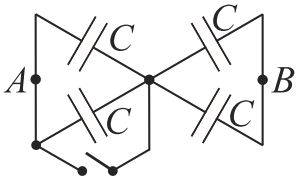
Zadatak 2. Na slici 2a je prikazan koaksijalni kabl dužine $L = 1\ m$, ispunjen sa dva sloja dielektrika permitivnosti $\varepsilon_1 = 6 \cdot \varepsilon_0$ i $\varepsilon_2 = 3 \cdot \varepsilon_0$. Poluprečnici elektroda kabla su $a = 1\ mm$ i $b = 7{,}39\ mm$. Poluprečnik razdvojne površi dva dielektrika je $c = 2{,}72\ mm$.

a) Izračunati kapacitivnost kabla.
b) Odrediti najveći napon na koji sme da se priključi kabl.
c) Izračunati promenu energije sadržanu u grupi kondenzatora prikazanoj na slici 2b, prilikom zatvaranja prekidača. Grupa je priključena na napon $U_{AB} = 20\ kV$, pri čemu kapacitivnost C predstavlja vrednost izračunatu pod *a)*.

Ostali brojni podaci: $Q = 1\ \mu C$, $E_{č1} = 60\ kV/cm$, $E_{č2} = 35\ kV/cm$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

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Osnovi elektrotehnike 1 (II kolokvijum)

K2

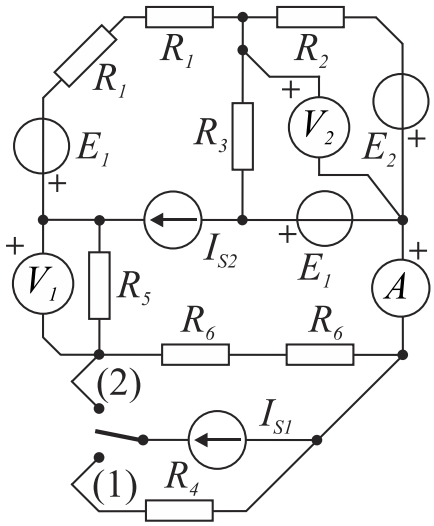
06.04.2021.

ZADACI

Zadatak 1. Kada se u kolu vremenski konstantnih struja sa slike 1 preklopnik prebaci iz položaja (1) u položaj (2), napon koji meri idealni voltmetar V_2 se poveća za 4 V.

a) Primenjujući teoremu superpozicije, izračunati jačinu struje strujnog generatora, I_{S1}.
b) Odrediti pokazivanja idealnih mernih instrumenata, kada je preklopnik u položaju (2). Kolo rešavati metodom konturnih struja.

Brojni podaci su: $R_1 = 2\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 2{,}5\ \Omega$, $R_5 = 1\ \Omega$, $R_6 = 3{,}5\ \Omega$, $E_1 = 5\ V$, $E_2 = 2{,}2\ V$, $I_{S2} = 1\ A$.

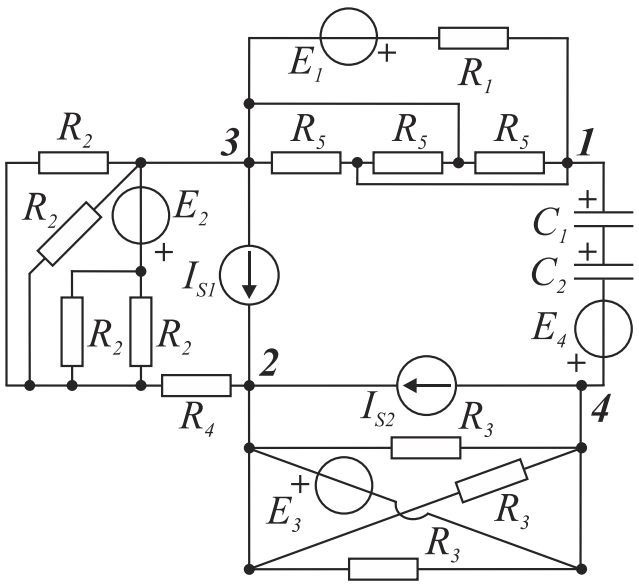


Slika 1.

Zadatak 2. Mrežu vremenski konstantnih struja, prikazanu na slici 2, transformisati na sledeći način:

a) Gornji deo mreže između tačaka **1** i **3** predstaviti ekvivalentnim Tevenenovim generatorom E_{T1}, R_{T1}.
b) Levi deo mreže između tačaka **2** i **3** predstaviti ekvivalentnim Tevenenovim generatorom E_{T2}, R_{T2}.
c) Donji deo mreže između tačaka **2** i **4** predstaviti ekvivalentnim Tevenenovim generatorom E_{T3}, R_{T3}.
d) Odrediti količine naelektrisanja, Q_1 i Q_2, na oblogama kondenzatora C_1 i C_2.
e) Odrediti napone na kondenzatorima C_1 i C_2, U_{C1} i U_{C2} (bonus 5p).

Brojni podaci su: $R_1 = 2\ k\Omega$, $R_2 = 5\ k\Omega$, $R_3 = 8\ k\Omega$, $R_4 = 3{,}75\ k\Omega$, $R_5 = 6\ k\Omega$, $E_1 = 2\ V$, $E_2 = E_4 = 11\ V$, $E_3 = 6{,}5\ V$, $I_{S1} = 2\ mA$, $I_{S2} = 1\ mA$, $C_1 = 20\ \mu F$, $C_2 = 30\ \mu F$.



Slika 2.

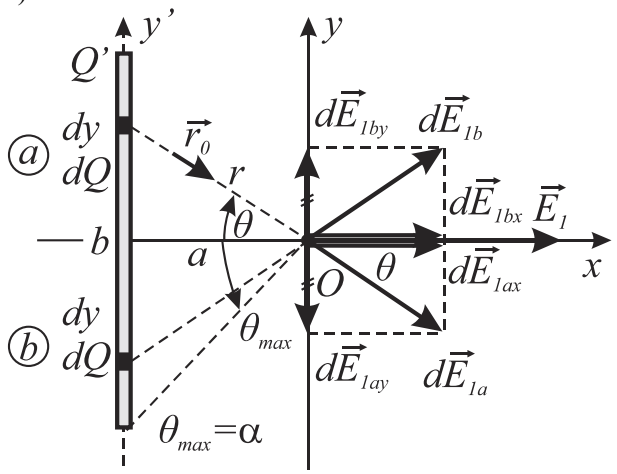
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

Ispit iz OET1, 06.04.2021.

I-1

a)



$$\overrightarrow{dE}_{1ay}+\overrightarrow{dE}_{1by}=0 \quad \Rightarrow \quad \boxed{\vec{E}_{1y}=0}$$

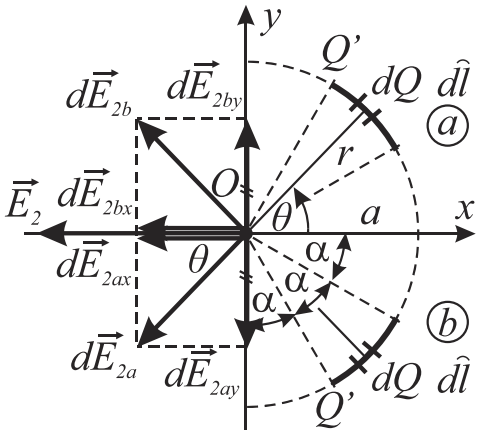
$$dE_{1ax}=dE_{1a}\cos\theta=\frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta=\frac{Q'dy}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$dE_{1ax}=\frac{Q'\dfrac{r\,d\theta}{\cos\theta}}{4\pi\varepsilon_0 r^2}\cos\theta=\frac{Q'}{4\pi\varepsilon_0\dfrac{a}{\cos\theta}}d\theta$$

$$E_1=2\int\limits_{\substack{po\ pola\\ štapa}} dE_{1ax}=2\frac{Q'}{4\pi\varepsilon_0 a}\int\limits_0^{\theta_{max}}\cos\theta\,d\theta$$

$$E_1=\frac{Q'}{2\pi\varepsilon_0 a}\left(\sin\theta_{max}-\sin 0\right)=\frac{Q'}{2\pi\varepsilon_0 a}\sin\alpha=\frac{Q'}{2\pi\varepsilon_0 a}\cdot\frac{1}{2}$$

$$\boxed{E_1=\frac{Q'}{4\pi\varepsilon_0 a}} \qquad \boxed{\vec{E}_1=E_1\cdot\vec{i}_x}$$



$$\overrightarrow{dE}_{2ay}+\overrightarrow{dE}_{2by}=0 \quad \Rightarrow \quad \boxed{\vec{E}_{2y}=0}$$

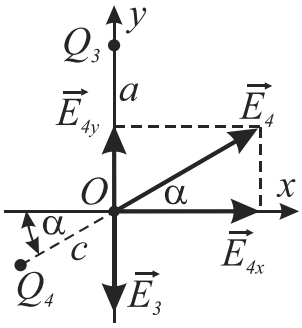
$$dE_{2ax}=dE_{2a}\cos\theta=\frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta=\frac{Q'dl}{4\pi\varepsilon_0 a^2}\cos\theta$$

$$dE_{2ax}=\frac{Q'a\,d\theta}{4\pi\varepsilon_0 a^2}\cos\theta=\frac{Q'}{4\pi\varepsilon_0 a}\cos\theta\,d\theta$$

$$E_2=2\int\limits_{\substack{po\ jednom\\ luku}} dE_{2ax}=2\frac{Q'}{4\pi\varepsilon_0 a}\int\limits_{\alpha}^{2\alpha}\cos\theta\,d\theta$$

$$E_2=\frac{Q'}{2\pi\varepsilon_0 a}\left(\sin 2\alpha-\sin\alpha\right)=\frac{Q'}{2\pi\varepsilon_0 a}\cdot\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$$

$$\boxed{E_2=\frac{Q'}{4\pi\varepsilon_0 a}\cdot\left(\sqrt{3}-1\right)} \qquad \boxed{\vec{E}_2=E_2\cdot\left(-\vec{i}_x\right)}$$



$$\boxed{E_3=\frac{Q_3}{4\pi\varepsilon_0 a^2}} \qquad \boxed{\vec{E}_3=E_3\cdot\left(-\vec{i}_y\right)}$$

$$E_4=\frac{Q_4}{4\pi\varepsilon_0 c^2}$$

$$E_{4x}=E_4\cos\alpha \qquad \boxed{E_{4x}=\frac{Q_4}{4\pi\varepsilon_0 c^2}\cdot\frac{\sqrt{3}}{2}} \qquad \boxed{\vec{E}_{4x}=E_{4x}\cdot\vec{i}_x}$$

$$E_{4y}=E_4\sin\alpha \qquad \boxed{E_{4y}=\frac{Q_4}{4\pi\varepsilon_0 c^2}\cdot\frac{1}{2}} \qquad \boxed{\vec{E}_{4y}=E_{4y}\cdot\vec{i}_y}$$

$$\boxed{\vec{E}_O=\vec{E}_1+\vec{E}_2+\vec{E}_3+\vec{E}_4=\left[\frac{Q'}{4\pi\varepsilon_0 a}-\frac{Q'}{4\pi\varepsilon_0 a}\cdot\left(\sqrt{3}-1\right)+\frac{Q_4}{4\pi\varepsilon_0 c^2}\cdot\frac{\sqrt{3}}{2}\right]\cdot\vec{i}_x+\left[\frac{Q_4}{4\pi\varepsilon_0 c^2}\cdot\frac{1}{2}-\frac{Q_3}{4\pi\varepsilon_0 a^2}\right]\cdot\vec{i}_y}$$

b)

$$\vec{E}_{Oy}=0 \quad \Rightarrow \quad \frac{Q_4}{4\pi\varepsilon_0 c^2}\cdot\frac{1}{2}=\frac{Q_3}{4\pi\varepsilon_0 a^2}, \qquad Q_4=\frac{2c^2}{a^2}Q_3=\frac{2\cdot 0{,}01^2}{0{,}02^2}\cdot Q_3=\frac{1}{2}\cdot Q_3 \qquad \boxed{Q_4=5\,nC}$$

c)

$$tg\alpha = \frac{b/2}{a}$$

$$b = 2a\, tg\alpha = 2 \cdot 0,02 \cdot tg30° = 2 \cdot 0,02 \cdot \frac{\sqrt{3}}{3} = 0,02309\ m = 2,309\ cm$$

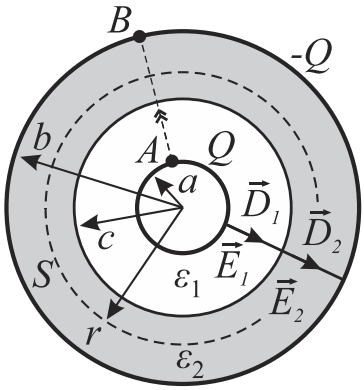
$$Q_{š1} = Q'b = 1 \cdot 10^{-9} \cdot 2,309 \cdot 10^{-2} = 2,309 \cdot 10^{-11} = 23,09 \cdot 10^{-12}\ C$$

$$\boxed{Q_{š1} = 23,09\ pC}$$ **(bonus 5p)**

Ispit iz OET1, 06.04.2021.

I-2

a)



Granični uslov:

$$D_{n1} = D_{n2}, \qquad D_1 = D_2 = D \qquad \left(E_{t1} = E_{t2} = 0\right)$$

$$\oint_S \vec{D}\cdot\vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_{S_{OM}} D\,ds = Q$$

$$D\,2r\pi\,L = Q \qquad \boxed{D = \frac{Q}{2\pi r\,L}}, \qquad a \le r \le b$$

$$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{2\pi\varepsilon_1 r\,L}}, \qquad a \le r \le c$$

$$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{2\pi\varepsilon_2 r\,L}}, \qquad c \le r \le b$$

$$U_{AB} = \int_A^B \vec{E}\cdot\vec{dl} = \int_a^b E\,dr = \int_a^c E_1\,dr + \int_c^b E_2\,dr = \int_a^c \frac{Q}{2\pi\varepsilon_1 r\,L}\,dr + \int_c^b \frac{Q}{2\pi\varepsilon_2 r\,L}\,dr = \frac{Q}{2\pi\varepsilon_1\,L}\ln\frac{c}{a} + \frac{Q}{2\pi\varepsilon_2\,L}\ln\frac{b}{c}$$

$$U_{AB} = \frac{Q}{2\pi L}\left(\frac{1}{\varepsilon_1}\ln\frac{c}{a} + \frac{1}{\varepsilon_2}\ln\frac{b}{c}\right)$$

$$C = \frac{Q}{U_{AB}} = \frac{2\pi L}{\frac{1}{\varepsilon_1}\ln\frac{c}{a} + \frac{1}{\varepsilon_2}\ln\frac{b}{c}} = \frac{2\pi\cdot 1}{\frac{1}{6\varepsilon_0}\cdot\ln 2{,}72 + \frac{1}{3\varepsilon_0}\cdot\ln 2{,}72} = 4\pi\cdot\varepsilon_0 \qquad \boxed{C = 111{,}2\ pF}$$

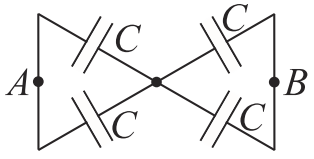
b)

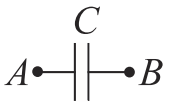
$$E_{1\max} = \frac{Q_{1\max}}{2\pi\varepsilon_1 a\,L} = \frac{C\,U_{1\max}}{2\pi\varepsilon_1 a\,L} \le E_{\check{C}1} \quad \Rightarrow \quad U_{1\max} = \frac{E_{\check{C}1}\,2\pi\varepsilon_1 a\,L}{C} = 18\ kV$$

$$E_{2\max} = \frac{Q_{2\max}}{2\pi\varepsilon_2 c\,L} = \frac{C\,U_{2\max}}{2\pi\varepsilon_2 c\,L} \le E_{\check{C}2} \quad \Rightarrow \quad U_{2\max} = \frac{E_{\check{C}2}\,2\pi\varepsilon_2 c\,L}{C} = 14{,}28\ kV$$

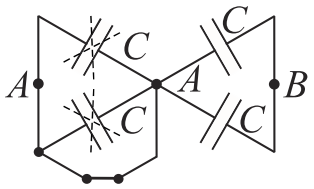
$$U_{\max} = \min\left\{U_{1\max}, U_{2\max}\right\} = U_{2\max} \qquad \boxed{U_{\max} = 14{,}28\ kV}$$

c)





$$C_{AB}^{O} = C$$



$$C_{AB}^{Z} = 2C$$

$$\Delta W_e = \frac{1}{2}\Delta C\,U_{AB}{}^2 = \frac{1}{2}\left(C_{AB}^{Z} - C_{AB}^{O}\right)U_{AB}{}^2 = \frac{1}{2}\left(2C - C\right)U_{AB}{}^2 = \frac{1}{2}C\,U_{AB}{}^2 = \frac{1}{2}\cdot 111{,}2\cdot 10^{-12}\cdot\left(20\cdot 10^3\right)^2$$

$$\boxed{\Delta W_e = 22{,}24\ mJ}$$

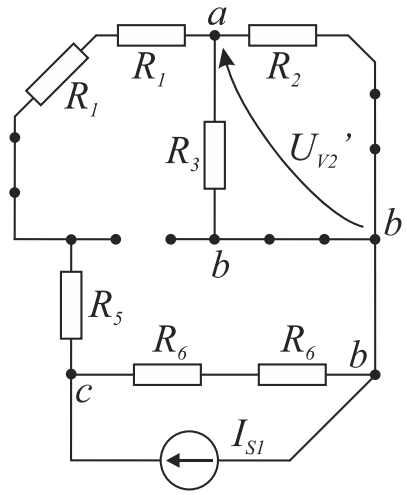
Ispit iz OET1, 06.04.2021.

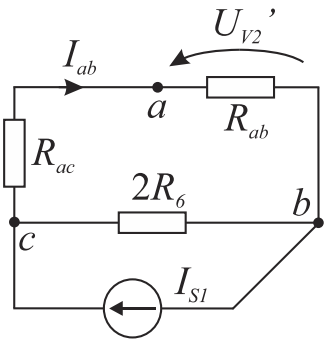
II-1

a)

$$\boxed{\begin{matrix}Svi\\ generatori\\ (2)\end{matrix}} = \boxed{\begin{matrix}Svi\\ sem\ I_{S1}\\ (1)\end{matrix}} + \boxed{\begin{matrix}Samo\\ I_{S1}\end{matrix}}$$

$$U_{V2}^{(2)} = U_{V2}^{(1)} + U_{V2}' \qquad \Rightarrow \qquad \Delta U_{V2} = U_{V2}^{(2)} - U_{V2}^{(1)} = U_{V2}' = 4\,V$$





$$R_{ac} = R_1 + R_1 + R_5 = 2 + 2 + 2 = 5\,\Omega$$
$$R_{ab} = R_2 \,||\, R_3 = 3 \,||\, 6 = 2\,\Omega$$

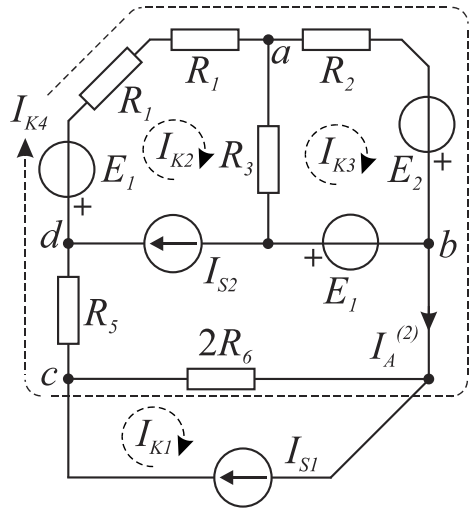
$$I_{ab} = \frac{U_{V2}'}{R_{ab}} = \frac{4}{2} = 2\ A$$

$$I_{ab} = \frac{2R_6}{2R_6 + R_{ac} + R_{ab}} I_{S1}$$

$$I_{S1} = \frac{2R_6 + R_{ac} + R_{ab}}{2R_6} I_{ab} = \frac{7+5+2}{7} \cdot 2$$

$$\boxed{I_{S1} = 4\ A}$$

b)



$$I_{K1} = I_{S1} = 4\ A$$
$$I_{K2} = I_{S2} = 1\ A$$
$$I_{K3}\left(R_2 + R_3\right) - I_{K2} R_3 + I_{K4} R_2 = E_1 + E_2$$
$$I_{K4}\left(2R_1 + R_2 + R_5 + 2R_6\right) - I_{K1} 2R_6 + I_{K2} 2R_1 + I_{K3} R_2 = -E_1 + E_2$$
$$I_{K3}\left(3+6\right) - 1\cdot 6 + I_{K4}\cdot 3 = 5 + 2{,}2$$
$$I_{K4}\left(4+3+1+7\right) - 4\cdot 7 + 1\cdot 4 + I_{K3}\cdot 3 = -5 + 2{,}2$$
$$9\,I_{K3} + 3\,I_{K4} = 13{,}2$$
$$3\,I_{K3} + 15\,I_{K4} = 21{,}2$$
$$I_{K3} = 1{,}07\ A$$
$$I_{K4} = 1{,}2\ A$$

$$\boxed{I_A^{(2)} = I_{K4} = 1{,}2\ A}$$

$$U_{V1}^{(2)} = U_{dc} = -I_{K4} R_5 = -1{,}2 \cdot 1$$

$$\boxed{U_{V1}^{(2)} = -1{,}2\,V}$$

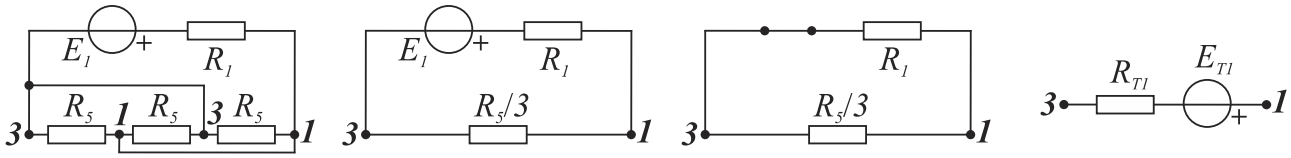
$$U_{V2}^{(2)} = U_{ab} = \left(I_{K2} - I_{K3}\right) R_3 + E_1 = \left(1 - 1{,}07\right) \cdot 6 + 5$$

$$\boxed{U_{V2}^{(2)} = 4{,}58\,V}$$

Ispit iz OET1, 06.04.2021.

II-2

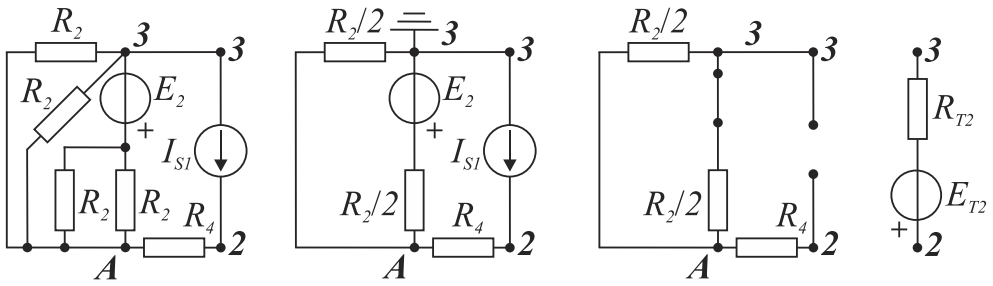
a)



$$E_{T1} = U_{13} = \frac{R_5/3}{R_5/3 + R_1} E_1 = \frac{2k}{2k + 2k} \cdot 2 \qquad \boxed{E_{T1} = 1\,V}$$

$$R_{T1} = R_{13} = \frac{R_5}{3} \| R_1 = 2k \| 2k \qquad \boxed{R_{T1} = 1\,k\Omega}$$

b)



$$V_3 = 0\,V$$

$$V_A\left(\frac{1}{R_2/2} + \frac{1}{R_2/2} + \frac{1}{R_2 + \infty}\right) = I_{S1} + \frac{E_2}{R_2/2}$$

$$V_A\left(\frac{1}{2{,}5k} + \frac{1}{2{,}5k}\right) = 2m + \frac{11}{2{,}5k} \qquad \Rightarrow \qquad V_A = 8\,V$$

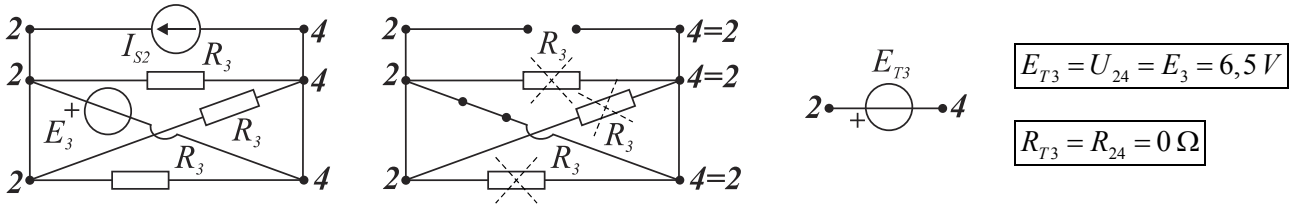
$$E_{T2} = U_{23} = R_4 I_{S1} + U_{A3} = 3{,}75k \cdot 2m + 8 \qquad \boxed{E_{T2} = 15{,}5\,V}$$

$$R_{T2} = R_{23} = \left(\frac{R_2}{2} \| \frac{R_2}{2}\right) + R_4$$

$$R_{T2} = \left(2{,}5k \| 2{,}5k\right) + 3{,}75k$$

$$\boxed{R_{T2} = 5\,k\Omega}$$

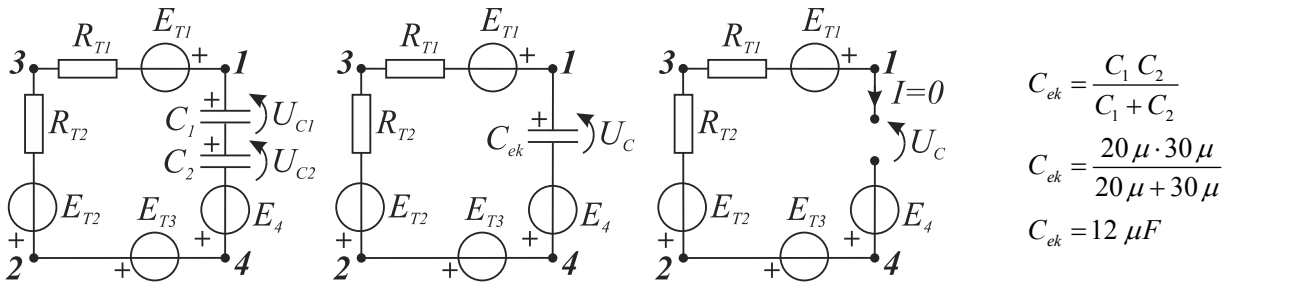
c)



$$\boxed{E_{T3} = U_{24} = E_3 = 6{,}5\,V}$$

$$\boxed{R_{T3} = R_{24} = 0\,\Omega}$$

d)



$$C_{ek} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{ek} = \frac{20\,\mu \cdot 30\,\mu}{20\,\mu + 30\,\mu}$$

$$C_{ek} = 12\,\mu F$$

$$Q_1 = Q_2 = Q = C_{ek} U_C = C_{ek}\left(E_{T1} - E_{T2} + E_{T3} + E_4\right) = 12\,\mu \cdot \left(1 - 15{,}5 + 6{,}5 + 11\right) \qquad \boxed{Q_1 = Q_2 = Q = 36\,\mu C}$$

e)

$$U_{C1} = \frac{Q_1}{C_1} = \frac{36\,\mu}{20\,\mu} \qquad \boxed{U_{C1} = 1{,}8\,V}$$

$$U_{C2} = \frac{Q_2}{C_2} = \frac{36\,\mu}{30\,\mu} \qquad \boxed{U_{C2} = 1{,}2\,V}$$

(bonus 5p)