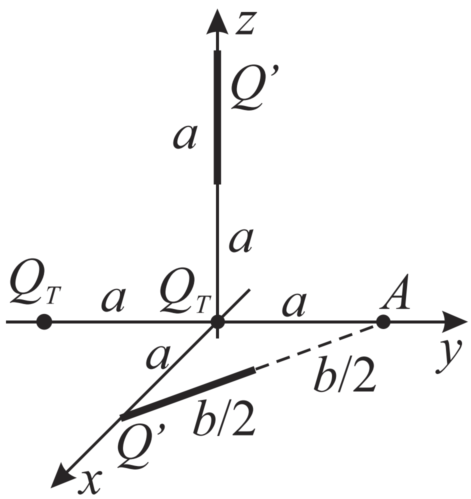
STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE		
K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **08.02.2021.**

ZADACI

Zadatak 1. Dva tanka štapa, naelektrisana ravnomerno istom podužnom gustinom naelektrisanja Q', postavljena su kao što je prikazano na slici 1. Prvi štap, dužine a, se nalazi na z osi Dekartovog koordinatnog sistema, pri čemu je jedan njegov kraja na udaljenosti a od centra sistema. Drugi štap, dužine $b/2$, je postavljen u prvom kvadrantu x-y ravni. Dva tačkasta naelektrisanja, naelektrisana količinom naelektrisanja Q_T, nalaze se na y osi, jedan u centru sistema, a drugi na udaljenosti a od centra sistema.

a) Odrediti, u opštim brojevima, vektor jačine električnog polja koji u tački A (koja se nalazi na y osi) stvaraju štapovi.
b) Količinu naelektrisanja tačkastih naelektrisanja, Q_T, tako da ukupan vektor jačine električnog polja u tački A nema y komponentu.
c) **Izračunati potencijal u tački u centru sistema, koji potiče od štapa dužine a, postavljenog na z osi, u odnosu na referentnu tačku u beskonačnosti (bonus 5p).**

Brojni podaci su: $a = 1\ cm$, $b = a\sqrt{2}$, $Q' = 1\ nC/m$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.

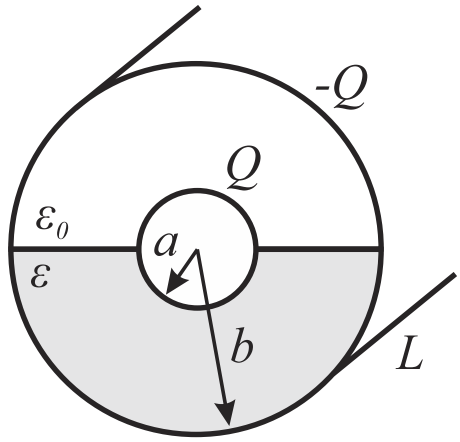


Slika 1.

Zadatak 2. Vazdušni koaksijalni kabl, dužine L, ispunjen je do pola tečnim dielektrikom, relativne permitivnosti $\varepsilon_r = 4$, i postavljen u horizontalni položaj, kao što je prikazano na slici 2. Poluprečnici elektroda ovog kabla su a i b. Kabl se priključi na izvor napona U, a zatim odvoji od izvora. Nakon toga, kondenzator se uspravi i lagano dopuni dielektrikom do kraja.

a) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora u horizontalnom položaju.
b) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora u vertikalnom položaju.
c) Odrediti i skicirati funkciju zavisnosti napona između obloga kondenzatora od visine tečnog dielektrika u kablu, kada je on u vertikalnom položaju.

Brojni podaci su: $a = 1\ mm$, $b = 7{,}5\ mm$, $L = 10\ cm$, $U = 1\ kV$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

KTET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	K2 08.02.2021.

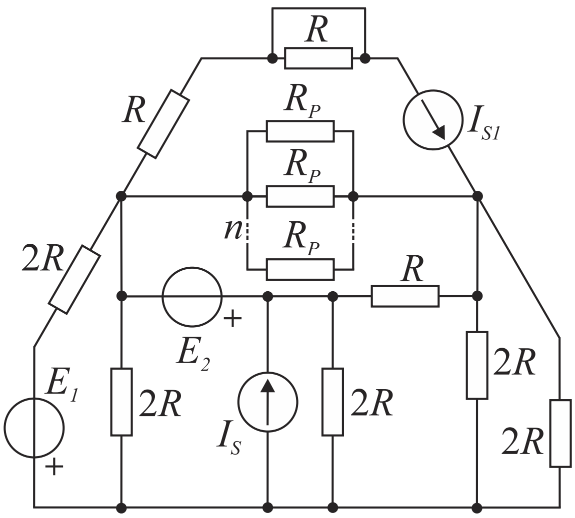
ZADACI

Zadatak 1. U kolu vremenski konstantnih struja, sa slike 1, na grupi od n paralelno vezanih otpornika, otpornosti $R_P = 50\ k\Omega$, razvija se maksimalna moguća snaga.

a) Primenom Tevenenove teoreme, izračunati broj paralelno vezanih otpornika, n. Pri određivanju *ems* Tevenenovog generatora, kolo rešavati primenom metode sa minimalnim brojem jednačina.

b) Izračunati snagu grupe od n paralelno vezanih otpornika, kao i snagu svakog od otpornika otpornosti R_P.

Brojni podaci su: $R = 10\ k\Omega$, $E_1 = 6\ V$, $E_2 = 2\ V$, $I_S = 1\ mA$, $I_{S1} = 3\ mA$.



Slika 1.

Zadatak 2. U kolu vremenski konstantnih struja, sa slike 2, poznate su brojne vrednosti svih elemenata, osim otpornosti otpornika R_X.

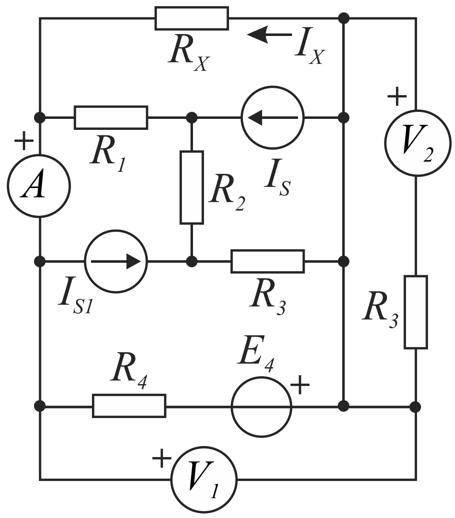
a) Primenjujući teoremu o kompenzaciji i metodu konturnih struja, izračunati otpornost otpornika R_X, tako da jačina struje kroz njegove priključke ima vrednost $I_X = 100\ mA$, u naznačenom referentnom smeru.

b) Izračunati snagu strujnog generatora I_S, kada otpornik R_X ima otpornost izračunatu pod *a*).

c) Odrediti pokazivanja idealnih mernih instrumenata, kada otpornik R_X ima otpornost izračunatu pod *a*).

d) Za koliko će se promeniti pokazivanje idealnog voltmetra V_2, kada se otpornost otpornika R_3 povećа duplo <u>(bonus 5p)</u>?

Brojni podaci su: $R_1 = 30\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = R_4 = 20\ \Omega$, $I_S = 150\ mA$, $I_{S1} = 62{,}5\ mA$, $E_4 = 4\ V$.



Slika 2.

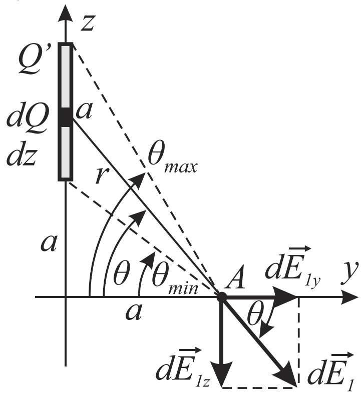
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

Ispit iz OET1, 08.02.2021.

I-1

a)



$$dE_1 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dz}{4\pi\varepsilon_0 r^2} \qquad dE_{1y} = dE_1\cos\theta = \frac{Q'\,dz}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$dE_{1z} = dE_1\sin\theta = \frac{Q'\,dz}{4\pi\varepsilon_0 r^2}\sin\theta$$

$$dE_{1y} = \frac{Q'\,dz}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'\dfrac{r\,d\theta}{\cos\theta}}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'\,d\theta}{4\pi\varepsilon_0\dfrac{a}{\cos\theta}} = \frac{Q'}{4\pi\varepsilon_0 a}\cos\theta\,d\theta$$

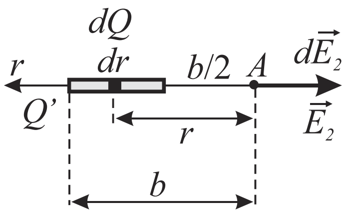
$$E_{1y} = \int\limits_{\text{duž štapa 1}} dE_{1y} = \frac{Q'}{4\pi\varepsilon_0 a}\int\limits_{\theta_{\min}}^{\theta_{\max}}\cos\theta\,d\theta = \frac{Q'}{4\pi\varepsilon_0 a}\left(\sin\theta_{\max} - \sin\theta_{\min}\right)$$

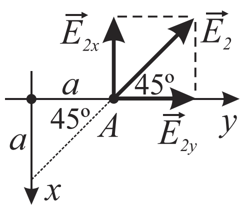
$$E_{1y} = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{2a}{\sqrt{a^2+(2a)^2}} - \frac{a}{\sqrt{a^2+a^2}}\right) \qquad \boxed{\vec{E}_{1y} = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{2\sqrt{5}}{5} - \frac{\sqrt{2}}{2}\right)\cdot\vec{i}_y}$$

$$dE_{1z} = \frac{Q'\,dz}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{Q'\dfrac{r\,d\theta}{\cos\theta}}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{Q'\dfrac{d\theta}{\cos\theta}}{4\pi\varepsilon_0\dfrac{a}{\cos\theta}}\sin\theta = \frac{Q'}{4\pi\varepsilon_0 a}\sin\theta\,d\theta$$

$$E_{1z} = \int\limits_{\text{duž štapa 1}} dE_{1z} = \frac{Q'}{4\pi\varepsilon_0 a}\int\limits_{\theta_{\min}}^{\theta_{\max}}\sin\theta\,d\theta = \frac{Q'}{4\pi\varepsilon_0 a}\left(\cos\theta_{\min} - \cos\theta_{\max}\right) = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{a}{\sqrt{a^2+a^2}} - \frac{a}{\sqrt{a^2+(2a)^2}}\right)$$

$$\boxed{\vec{E}_{1z} = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{5}}{5}\right)\cdot\left(-\vec{i}_z\right)}$$





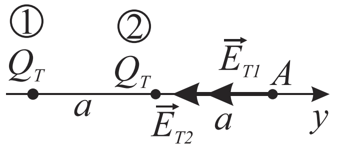
$$dE_2 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dr}{4\pi\varepsilon_0 r^2}$$

$$E_2 = \int\limits_{\substack{\text{po}\\ \text{štapu 2}}} dE_2 = \frac{Q'}{4\pi\varepsilon_0}\int\limits_{b/2}^{b}\frac{dr}{r^2} = \frac{Q'}{4\pi\varepsilon_0}\left(\frac{1}{b/2} - \frac{1}{b}\right) = \frac{Q'}{4\pi\varepsilon_0 b}$$

$$E_{2x} = E_2\sin 45^\circ \qquad \boxed{\vec{E}_{2x} = \frac{Q'}{4\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\cdot\left(-\vec{i}_x\right)} \qquad E_{2y} = E_2\cos 45^\circ \qquad \boxed{\vec{E}_{2y} = \frac{Q'}{4\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\cdot\vec{i}_y}$$

$$\boxed{\vec{E}_A = \vec{E}_{1y} + \vec{E}_{1z} + \vec{E}_{2x} + \vec{E}_{2y} = \frac{Q'}{4\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\cdot\left(-\vec{i}_x\right) + \left(\frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{2\sqrt{5}}{5} - \frac{\sqrt{2}}{2}\right) + \frac{Q'}{4\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\right)\cdot\vec{i}_y + \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{5}}{5}\right)\cdot\left(-\vec{i}_z\right)}$$

b)



$$Q_T < 0 \qquad E_{T1} = \frac{|Q_T|}{4\pi\varepsilon_0 (2a)^2} = \frac{|Q_T|}{16\pi\varepsilon_0 a^2} \qquad \boxed{\vec{E}_{T1} = \frac{|Q_T|}{16\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right)}$$

$$E_{T2} = \frac{|Q_T|}{4\pi\varepsilon_0 a^2} \qquad \boxed{\vec{E}_{T2} = \frac{|Q_T|}{4\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right)}$$

$$\vec{E}_{Ay} = 0 \qquad \Rightarrow \qquad \vec{E}_{1y} + \vec{E}_{2y} + \vec{E}_{T1} + \vec{E}_{T2} = 0$$

$$\left(\frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{2\sqrt{5}}{5} - \frac{\sqrt{2}}{2}\right) + \frac{Q'}{4\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\right)\cdot\vec{i}_y + \frac{|Q_T|}{16\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right) + \frac{|Q_T|}{4\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right) = 0$$

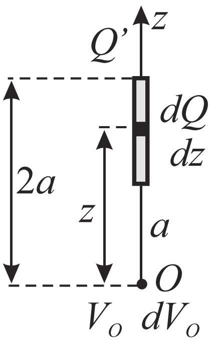
$$\frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{2\sqrt{5}}{5}-\frac{\sqrt{2}}{2}\right)+\frac{Q'}{4\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}=\frac{|Q_T|}{16\pi\varepsilon_0 a^2}+\frac{|Q_T|}{4\pi\varepsilon_0 a^2}$$

$$|Q_T| = 0,55\cdot 10^{-11} = 5,5\cdot 10^{-12}\ C$$

$$Q_T = -5,5\cdot 10^{-12}\ C$$

$$\boxed{Q_T = -5,5\ pC}$$

c)



$$dV_O = \frac{dQ}{4\pi\varepsilon_0 r} = \frac{Q'\,dz}{4\pi\varepsilon_0 z}$$

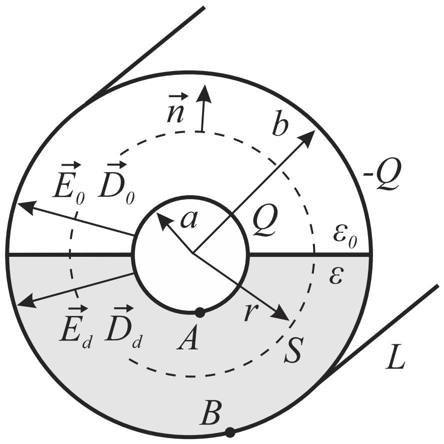
$$V_O = \int\limits_{\substack{po\\ štapu\ 2}} dE_2 = \frac{Q'}{4\pi\varepsilon_0}\int_a^{2a}\frac{dz}{z} = \frac{Q'}{4\pi\varepsilon_0}\ln\frac{2a}{2} = \frac{Q'}{4\pi\varepsilon_0}\ln 2 = \frac{1\cdot 10^{-9}}{4\pi\cdot 8,85\cdot 10^{-12}}\cdot\ln 2$$

$\boxed{V_O = 6,24\,V}$ **<u>(bonus 5p)</u>**

Ispit iz OET1, 08.02.2021.

I-2

a)



Granični uslov:

$$E_{t0} = E_{td}, \qquad E_0 = E_d = E \qquad \left(D_{n0} = D_{nd} = 0\right)$$

$$\oint_S \vec{D}\cdot\vec{ds} = Q_{u\,S}$$

$$\int_{S_{B1}} \cancel{\vec{D}\cdot\vec{ds}}^{\,0} + \int_{S_{B2}} \cancel{\vec{D}\cdot\vec{ds}}^{\,0} + \int_{S_{OM}} \vec{D}\cdot\vec{ds} = Q_{u\,S}$$

$$\int_{OM_0} D_0\, ds + \int_{OM_d} D_d\, ds = Q$$

$$D_0 \cdot \frac{1}{2}\cdot 2r\pi L + D_d \cdot \frac{1}{2}\cdot 2r\pi L = Q$$

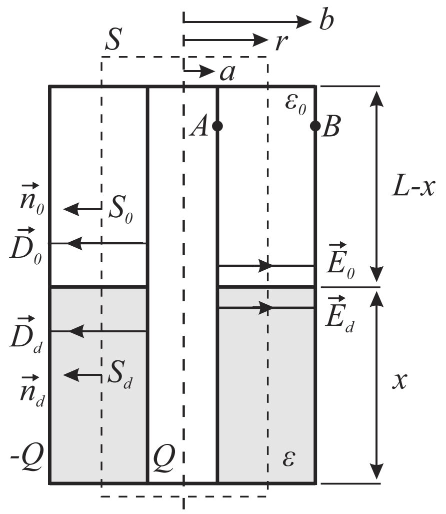
$$\varepsilon_0 E \cdot \frac{1}{2}\cdot 2r\pi L + \varepsilon E \cdot \frac{1}{2}\cdot 2r\pi L = Q \qquad D_0 = \varepsilon_0 E \qquad D_d = \varepsilon E$$

$$\boxed{E = \frac{Q}{\varepsilon_0\left(1+\varepsilon_r\right) r\pi L}}, \qquad a \le r \le b$$

$$U_{AB}^H = \int_A^B \vec{E}\cdot\vec{dl} = \int_a^b E\, dr = \int_a^b \frac{Q}{\varepsilon_0\left(1+\varepsilon_r\right) r\pi L}\, dr = \frac{Q}{\varepsilon_0\left(1+\varepsilon_r\right)\pi L}\ln\frac{b}{a} \qquad \boxed{C^H = \frac{Q}{U_{AB}^H} = \frac{\varepsilon_0\left(1+\varepsilon_r\right)\pi L}{\ln\frac{b}{a}}}$$

$$(*) \qquad Q = C^H\, U_{AB}^H = \frac{\varepsilon_0\left(1+\varepsilon_r\right)\pi L}{\ln\frac{b}{a}} U \qquad \left(U_{AB}^H = U = 1\,kV\right)$$

b)



Granični uslov:

$$E_{t0} = E_{td}, \qquad E_0 = E_d = E \qquad \left(D_{n0} = D_{nd} = 0\right)$$

$$\oint_S \vec{D}\cdot\vec{ds} = Q_{u\,S}$$

$$\int_{S_{B1}} \cancel{\vec{D}\cdot\vec{ds}}^{\,0} + \int_{S_{B2}} \cancel{\vec{D}\cdot\vec{ds}}^{\,0} + \int_{S_{OM}} \vec{D}\cdot\vec{ds} = Q_{u\,S}$$

$$\int_{OM_0} D_0\, ds + \int_{OM_d} D_d\, ds = Q$$

$$D_0\, 2r\pi\left(L-x\right) + D_d\, 2r\pi\, x = Q$$

$$\varepsilon_0 E\, 2r\pi\left(L-x\right) + \varepsilon E\, 2r\pi\, x = Q \qquad D_0 = \varepsilon_0 E \qquad D_d = \varepsilon E$$

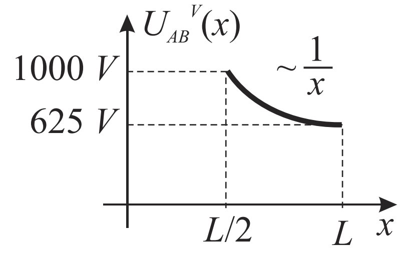
$$\boxed{E = \frac{Q}{2r\pi\varepsilon_0\left(L-x+\varepsilon_r x\right)}}, \qquad a \le r \le b$$

$$U_{AB}^V = \int_A^B \vec{E}\cdot\vec{dl} = \int_a^b E\, dr = \int_a^b \frac{Q}{2r\pi\varepsilon_0\left(L-x+\varepsilon_r x\right)}\, dr = \frac{Q}{2\pi\varepsilon_0\left(L-x+\varepsilon_r x\right)}\ln\frac{b}{a} \qquad \boxed{C^V = \frac{Q}{U_{AB}^V} = \frac{2\pi\varepsilon_0\left(L-x+\varepsilon_r x\right)}{\ln\frac{b}{a}}}$$

c)

$$U_{AB}^V = \frac{Q^{(*)}}{2\pi\varepsilon_0\left(L-x+\varepsilon_r x\right)}\ln\frac{b}{a} = \frac{\dfrac{\varepsilon_0\left(1+\varepsilon_r\right)\pi L}{\ln\frac{b}{a}} U}{2\pi\varepsilon_0\left(L-x+\varepsilon_r x\right)}\ln\frac{b}{a}$$

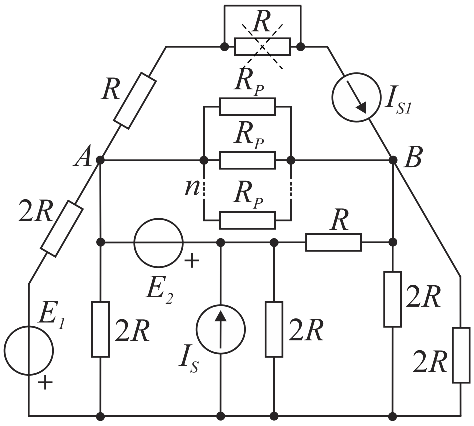
$$U_{AB}^V = \frac{\left(1+\varepsilon_r\right)L}{2\cdot\left(L-x+\varepsilon_r x\right)} U \qquad \boxed{U_{AB}^V(x) = \frac{500}{2\cdot\left(0{,}1+3\cdot x\right)}}$$

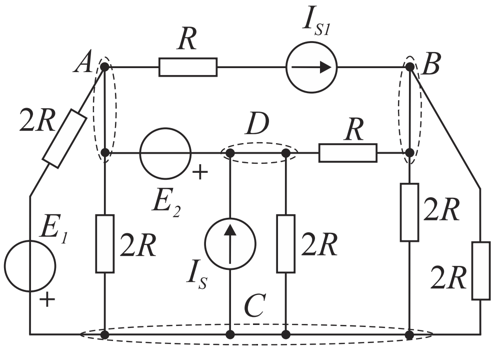


Ispit iz OET1, 08.02.2021.

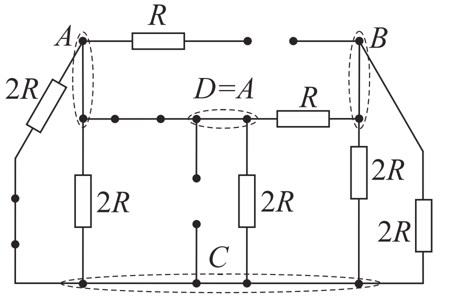
II-1

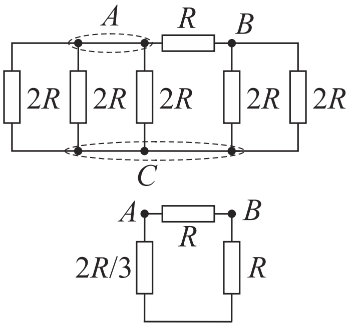
a)





R_T:



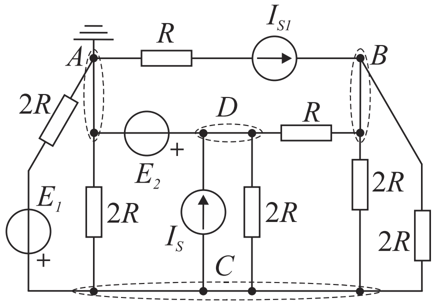


$$R_T = R_{AB} = R \parallel \left(\frac{2R}{3} + R \right)$$

$$R_T = \frac{R \cdot \frac{5}{3}R}{R + \frac{5}{3}R} = \frac{\frac{5}{3}R^2}{\frac{8}{3}R} = \frac{5}{8}R = \frac{5}{8} \cdot 10\,k$$

$$\boxed{R_T = 6,25\,k\Omega}$$

E_T:



$$V_A = 0\,V, \qquad V_D = E_2 = 2\,V$$

$$V_B\left(\cancel{\frac{1}{R+\infty}}^{\,0} + \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}\right) - V_C\left(\frac{1}{2R} + \frac{1}{2R}\right) - V_D\left(\frac{1}{R}\right) = I_{S1} \qquad /\cdot 2R$$

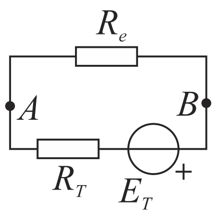
$$V_C\left(\frac{1}{2R} + \frac{1}{2R} + \cancel{\frac{1}{\infty}}^{\,0} + \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R}\right) - V_B\left(\frac{1}{2R} + \frac{1}{2R}\right) - V_D\left(\cancel{\frac{1}{\infty}}^{\,0} + \frac{1}{2R}\right) =$$

$$= -I_S + \frac{E_1}{2R} \qquad /\cdot 2R$$

$$4\,V_B - 2\,V_C = 64 \qquad\qquad V_B = 18,5\,V$$

$$-2\,V_B + 5\,V_C = -12 \qquad\qquad V_C = 5\,V$$

$$E_T = U_{BA} = V_B - V_A \qquad \boxed{E_T = 18,5\,V}$$



$$\left.\begin{array}{l} R_e = \dfrac{R_P}{n} \\ R_e = R_T \end{array}\right\} \Rightarrow \quad \frac{R_P}{n} = R_T \quad \Rightarrow \quad n = \frac{R_P}{R_T} = \frac{50\,k}{6,25\,k} \qquad \boxed{n = 8}$$

b)

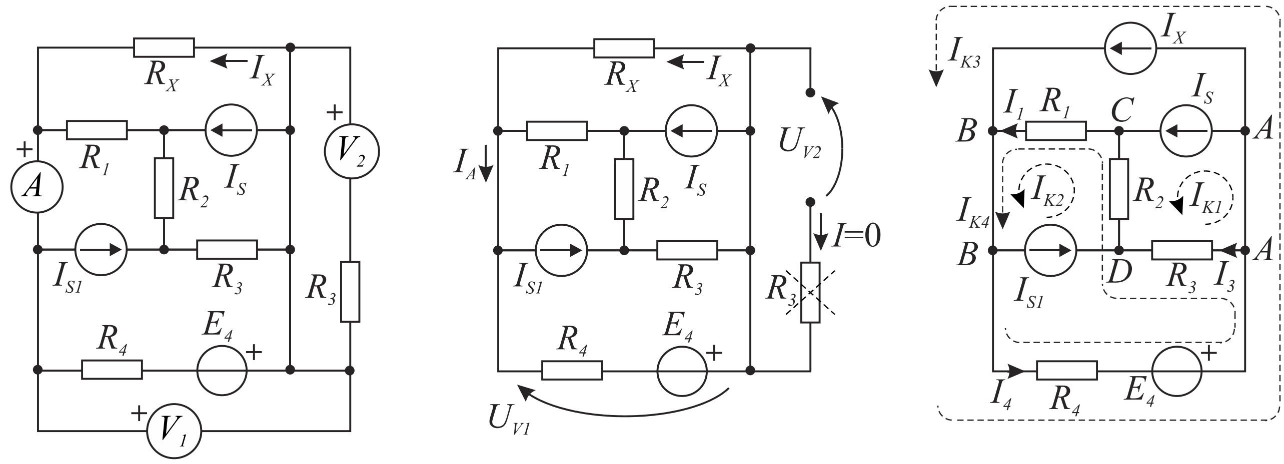
$$U_e = \frac{E_T}{2} = 9,25\,V \qquad P_e = \frac{U_e^{\,2}}{R_e} = \frac{9,25^2}{6,25\,k} \qquad \boxed{P_e = 13,69\,mW}$$

$$P_P = \frac{U_e^{\,2}}{R_P} = \frac{9,25^2}{50\,k} \qquad \boxed{P_P = 1,71\,mW}$$

Ispit iz OET1, 08.02.2021.

II-2

a)



$$MKS: \quad \left[n_g-(n_č-1)\right]-n_{s.g.}=\left[7-(4-1)\right]-3=\left[7-3\right]-3=4-3=1$$

$$I_{K1}=I_S=150\ mA$$

$$I_{K2}=I_{S1}=62,5\ mA$$

$$I_{K3}=I_X=100\ mA$$

$$\underline{I_{K4}(R_1+R_2+R_3+R_4)-I_{K1}(R_2+R_3)+I_{K2}(R_1+R_2)+I_{K3}R_4=E_4}$$

$$I_{K4}(30+10+20+20)-150m\cdot(10+20)+62,5m\cdot(30+10)+100m\cdot 20=4$$

$$80\,I_{K4}=4+4,5-2,5-2=4$$

$$\boxed{I_{K4}=50\ mA}$$

$$I_1=I_{K2}+I_{K4}=62,5m+50m=112,5\ mA$$

$$I_3=I_{K4}-I_{K1}=50m-150m=-100\ mA$$

$$I_4=I_{K3}+I_{K4}=100m+50m=150\ mA$$

$$U_{AB}=E_4-I_4R_4=4-150m\cdot 20=1V$$

$$R_X=\frac{U_{AB}}{I_X}=\frac{1}{0,1} \qquad \boxed{R_X=10\,\Omega}$$

b)

$$U_S=U_{CA}=U_{CB}+U_{BA}=I_1R_1+I_4R_4-E_4=112,5m\cdot 30+150m\cdot 20-4=2,375\,V$$

$$P_S=U_SI_S=2,375\cdot 150m$$

$$\boxed{P_S=356,25\ mW}$$

c)

$$I_A=I_{K2}+I_{K3}+I_{K4}=62,5m+100m+50m \qquad \boxed{I_A=212,5\ mA}$$

$$U_{V1}=U_{BA}=I_4R_4-E_4=150m\cdot 20-4 \qquad \boxed{U_{V1}=-1V}$$

$$\boxed{U_{V2}=0\,V}$$

d)

$\boxed{\Delta U_{V2}=0\,V}$ $U_{V2}=0\ V$, bez obzira ne vrednost otpornosti otpornika R_3. **(bonus 5p)**