STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K	TET	Osnovi elektrotehnike 1 (I kolokvijum)	K1
Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs			16.06.2021.

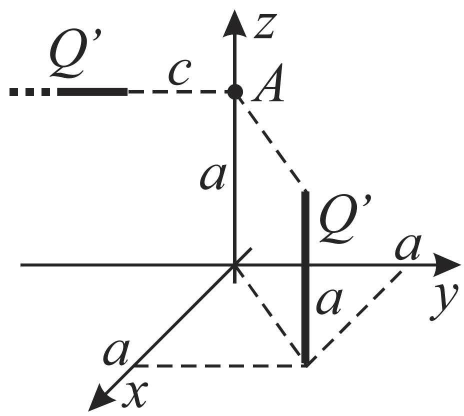
Osnovi elektrotehnike 1
(I kolokvijum)

ZADACI

Zadatak 1. Dva tanka štapa, naelektrisana ravnomerno istom podužnom gustinom naelektrisanja Q', postavljena su kao što je prikazano na slici 1. Prvi štap, dužine a, je postavljen paralelno sa z osom Dekartovog koordinatnog sistema, pri čemu njegov donji kraj leži u prvom kvadrantu x-y ravni. Drugi štap je polubeskonačan, postavljen paralelno sa y osom, na udaljenosti c od tačke A, koja se nalazi na z osi, na visini a.

a) Odrediti, u opštim brojevima, ukupni vektor jačine električnog polja koji u tački A stvaraju štapovi.
b) Odrediti udaljenost c polubeskonačnog štapa od tačke A, tako da ukupan vektor jačine električnog polja u tački A nema y komponentu.

Brojni podaci: $a = 5\ cm$, $Q' = 10\ nC/m$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.

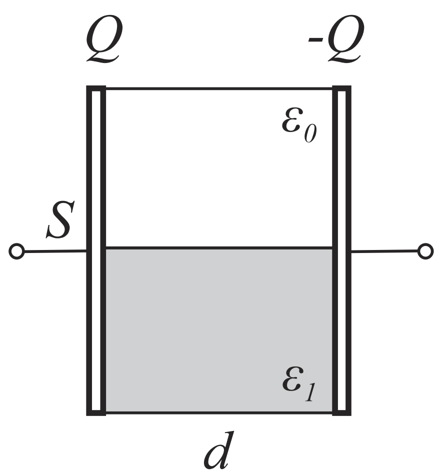


Slika 1.

Zadatak 2. Na slici 2 je prikazan vazdušni pločasti kondenzator ispunjen do pola sa tečnim dielektrikom, relativne permitivnosti ε_{r1}. Rastojanje između elektroda kondenzatora je $d = 1\ cm$, a ukupna površina elektroda je $S = 5\ cm^2$.

a) Izvesti u opštim brojevima izraz za kapacitivnost kondenzatora.
b) Odrediti relativnu permitivnost tečnog dielektrika, ε_{r1}, ako se zna da nakon ispuštanja 50% njegove zapremine, količina vezanog naelektrisanja uz desnu elektrodu opadne za jednu trećinu.
c) Izračunati maksimalni napon na koji sme da se priključi kondenzator, pre ispuštanja 50% zapremine tečnog dielektrika.

Brojni podaci: $E_{\check{c}0} = 30\ kV/cm$, $E_{\check{c}1} = 75\ kV/cm$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (II kolokvijum)

K2

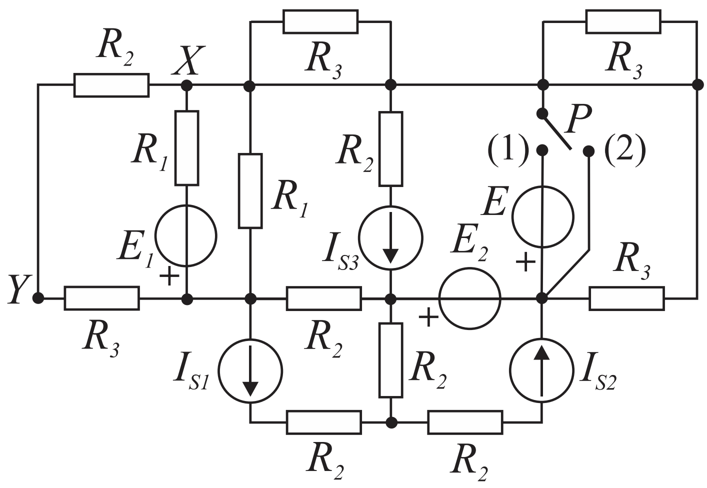
16.06.2021.

ZADACI

Zadatak 1. U mreži vremenski konstantnih struja sa slike 1 preklopnik P prebaci se iz položaja (2) u položaj (1).

a) Primenjujući teoremu superpozicije, odrediti promenu napona U_{XY} između tačaka X i Y, prilikom prebacivanja preklopnika. Kolo rešavati metodom potencijala čvorova.
b) Da li će se prilikom prebacivanja preklopnika napon U_{XY} povećati ili smanjiti?
c) Izračunati snagu naponskog genetora E, kada se preklopnik nalazi u položaju (2), kao i u kolu u kom deluje samo naponski generator E.

Brojni podaci su: $R_1 = 3\ k\Omega$, $R_2 = 1\ k\Omega$, $R_3 = 2\ k\Omega$, $E = 12\ V$, $E_1 = 7\ V$, $E_2 = 3\ V$, $I_{S1} = I_{S3} = 1\ mA$, $I_{S2} = -1\ mA$.



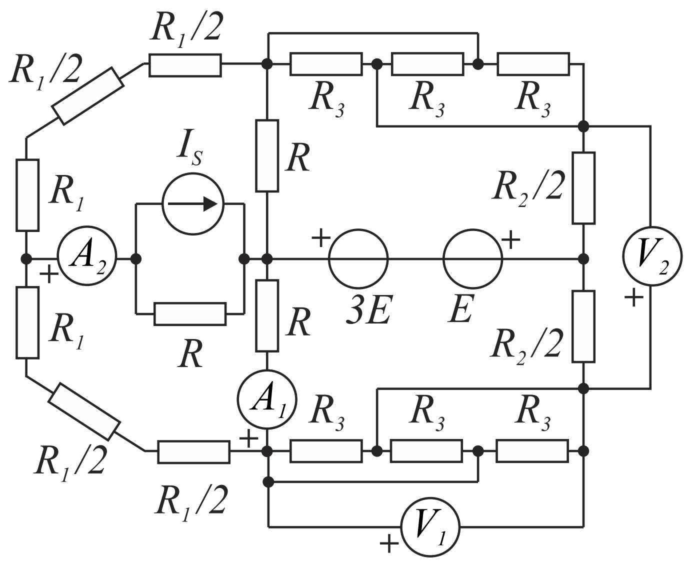
Slika 1.

Zadatak 2. U simetričnoj mreži vremenski konstantnih struja sa slike 2:

a) izračunati pokazivanja idealnih mernih instrumenata i
b) pokazati da je teorema o održanju snage zadovoljena.

Prilikom rešavanja zadatka koristiti simetriju mreže i metodu konturnih struja.

Brojni podaci su:
$R = 20\ \Omega$, $R_1 = 25\ \Omega$, $R_2 = 80\ \Omega$, $R_3 = 180\ \Omega$, $E = 10\ V$, $I_S = 0{,}7\ A$.



Slika 2.

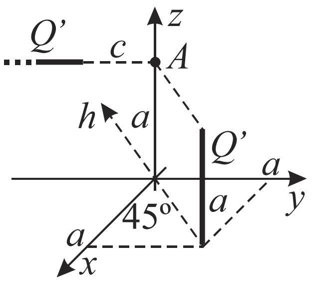
PRAVILA POLAGANJA

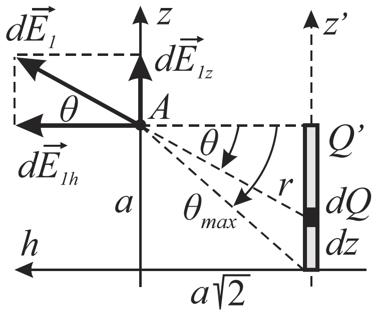
Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

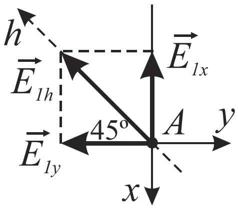
<u>Ispit iz OET1, 16.06.2021.</u>

I-1

a)







$$dE_1 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2} \qquad dE_{1h} = dE_1\cos\theta \qquad \left(dz' = \frac{r\,d\theta}{\cos\theta}, \qquad r = \frac{a\sqrt{2}}{\cos\theta} \right)$$

$$dE_{1z} = dE_1 \sin\theta$$

$$E_{1z} = \int\limits_{\text{duž štapa}} dE_{1z} = \int \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2}\sin\theta = \int \frac{Q'\dfrac{r\,d\theta}{\cos\theta}}{4\pi\varepsilon_0 r^2}\sin\theta = \int \frac{Q'\dfrac{d\theta}{\cos\theta}}{4\pi\varepsilon_0 \dfrac{a\sqrt{2}}{\cos\theta}}\sin\theta = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\int_0^{\theta_{\max}} \sin\theta\, d\theta$$

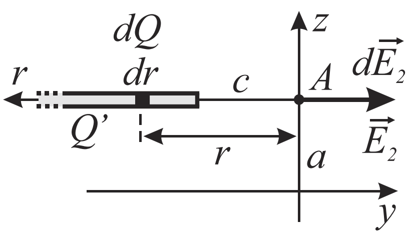
$$E_{1z} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\cos\theta_{\max}\right) = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\frac{a\sqrt{2}}{\sqrt{a^2+2a^2}}\right) \qquad \boxed{\vec{E}_{1z} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\frac{\sqrt{2}}{\sqrt{3}}\right)\cdot\vec{i}_z}$$

$$E_{1h} = \int\limits_{\text{duž štapa}} dE_{1h} = \int \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2}\cos\theta = \int \frac{Q'\dfrac{r\,d\theta}{\cos\theta}}{4\pi\varepsilon_0 r^2}\cos\theta = \int \frac{Q'\,d\theta}{4\pi\varepsilon_0 \dfrac{a\sqrt{2}}{\cos\theta}} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\int_0^{\theta_{\max}} \cos\theta\, d\theta$$

$$E_{1h} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(\sin\theta_{\max}-0\right) = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{a}{\sqrt{a^2+2a^2}} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}$$

$$E_{1x} = E_{1h}\sin 45^\circ = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}\frac{\sqrt{2}}{2} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} \qquad \boxed{\vec{E}_{1x} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}\cdot\left(-\vec{i}_x\right)}$$

$$E_{1y} = E_{1h}\cos 45^\circ = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}\frac{\sqrt{2}}{2} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} \qquad \boxed{\vec{E}_{1y} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}\cdot\left(-\vec{i}_y\right)}$$



$$dE_2 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dr}{4\pi\varepsilon_0 r^2}$$

$$E_2 = \int\limits_{\text{duž štapa}} dE_2 = \frac{Q'}{4\pi\varepsilon_0}\int_c^{\infty}\frac{dr}{r^2} = \frac{Q'}{4\pi\varepsilon_0}\left(\frac{1}{c}-\frac{1}{\infty}\right) = \frac{Q'}{4\pi\varepsilon_0 c} \qquad \boxed{\vec{E}_2 = \frac{Q'}{4\pi\varepsilon_0 c}\cdot\vec{i}_y}$$

$$\boxed{\vec{E}_A = \vec{E}_{1x}+\vec{E}_{1y}+\vec{E}_{1z}+\vec{E}_2 = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}\cdot\left(-\vec{i}_x\right) + \left(\frac{Q'}{4\pi\varepsilon_0 c}-\frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}\right)\cdot\vec{i}_y + \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\frac{\sqrt{2}}{\sqrt{3}}\right)\cdot\vec{i}_z}$$

b)

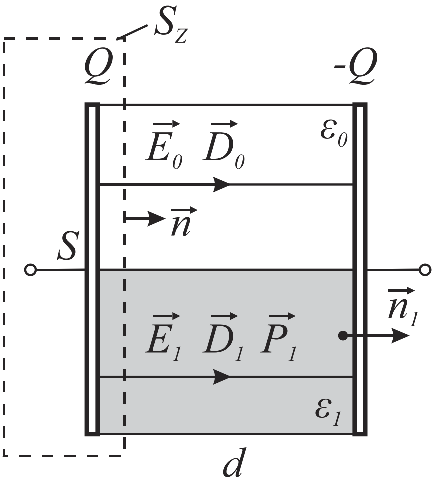
$$\vec{E}_{Ay} = 0 \qquad \Rightarrow \qquad E_{1y} = E_2 \qquad \Rightarrow \qquad \frac{Q'}{4\pi\varepsilon_0 c} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}$$

$$c = 2\sqrt{3}\,a = 2\sqrt{3}\cdot 5\cdot 10^{-2} = 17{,}32\cdot 10^{-2}\ m \qquad \boxed{c = 17{,}32\ cm}$$

Ispit iz OET1, 16.06.2021.

I-2

a)



Granični uslov:

$$E_{t0} = E_{t1}, \qquad E_0 = E_1 = E \qquad (D_{n0} = D_{n1} = 0)$$

$$\oint_{S_Z} \vec{D} \cdot \vec{ds} = Q_{u\,S_Z}$$

$$\int_{S_0} D_0\, ds + \int_{S_1} D_1\, ds = Q \qquad \left(S_0 = S_1 = \frac{S}{2}\right)$$

$$D_0 \frac{S}{2} + D_1 \frac{S}{2} = Q \qquad D_0 = \varepsilon_0 E \qquad D_1 = \varepsilon_1 E$$

$$\boxed{E = \frac{Q}{\varepsilon_0 \frac{S}{2} + \varepsilon_1 \frac{S}{2}}}$$

$$U = \int_+^- \vec{E} \cdot \vec{dl} = \int_+^- E\, dl = E\, d = \frac{Q}{\varepsilon_0 \frac{S}{2} + \varepsilon_1 \frac{S}{2}} d \qquad \boxed{C = \frac{Q}{U} = \frac{\varepsilon_0 \frac{S}{2} + \varepsilon_1 \frac{S}{2}}{d}}$$

b)

$$P_1 = D_1 - \varepsilon_0 E = (\varepsilon_1 - \varepsilon_0) E = \varepsilon_0 (\varepsilon_{r1} - 1) E$$

$$\sigma_{VD}{}^{PRE} = \vec{P}_1 \cdot \vec{n}_1 = P_1 = \varepsilon_0 (\varepsilon_{r1} - 1) E = \varepsilon_0 (\varepsilon_{r1} - 1) \frac{Q}{\varepsilon_0 \frac{S}{2} + \varepsilon_1 \frac{S}{2}}$$

$$Q_{VD}{}^{PRE} = \sigma_{VD}{}^{PRE} \frac{S}{2} = \varepsilon_0 (\varepsilon_{r1} - 1) \frac{Q}{\varepsilon_0 + \varepsilon_1} = (\varepsilon_{r1} - 1) \frac{Q}{1 + \varepsilon_{r1}}$$

$$\sigma_{VD}{}^{POSLE} = \vec{P}_1 \cdot \vec{n}_1 = P_1 = \varepsilon_0 (\varepsilon_{r1} - 1) E^{POSLE} = \varepsilon_0 (\varepsilon_{r1} - 1) \frac{Q}{\varepsilon_0 \frac{3S}{4} + \varepsilon_1 \frac{S}{4}}$$

$$Q_{VD}{}^{POSLE} = \sigma_{VD}{}^{POSLE} \frac{S}{4} = \varepsilon_0 (\varepsilon_{r1} - 1) \frac{Q}{3\varepsilon_0 + \varepsilon_1} = (\varepsilon_{r1} - 1) \frac{Q}{3 + \varepsilon_{r1}}$$

$$Q_{VD}{}^{POSLE} = \frac{2}{3} Q_{VD}{}^{PRE}$$

$$(\varepsilon_{r1} - 1) \frac{Q}{3 + \varepsilon_{r1}} = \frac{2}{3} \cdot (\varepsilon_{r1} - 1) \frac{Q}{1 + \varepsilon_{r1}}. \qquad \frac{1}{3 + \varepsilon_{r1}} = \frac{2}{3 + 3\varepsilon_{r1}}, \qquad 6 + 2\varepsilon_{r1} = 3 + 3\varepsilon_{r1}$$

$$\boxed{\varepsilon_{r1} = 3}$$

c)

$$E_{\max} = \frac{Q_{\max}}{\varepsilon_0 \frac{S}{2} + \varepsilon_1 \frac{S}{2}} \le \min\{E_{\check{C}0}, E_{\check{C}1}\} = E_{\check{C}0} \quad \Rightarrow \quad Q_{\max} = E_{\check{C}0} \left(\varepsilon_0 \frac{S}{2} + \varepsilon_1 \frac{S}{2} \right)$$

$$U_{\max} = \frac{Q_{\max}}{C} = \frac{E_{\check{C}0} \left(\varepsilon_0 \frac{S}{2} + \varepsilon_1 \frac{S}{2} \right)}{\frac{\varepsilon_0 \frac{S}{2} + \varepsilon_1 \frac{S}{2}}{d}} = E_{\check{C}0}\, d \qquad \boxed{U_{\max} = 30\, kV}$$

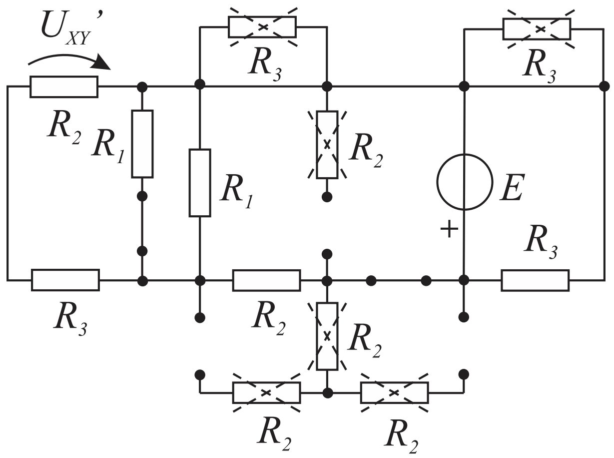
Ispit iz OET1, 16.06.2021.

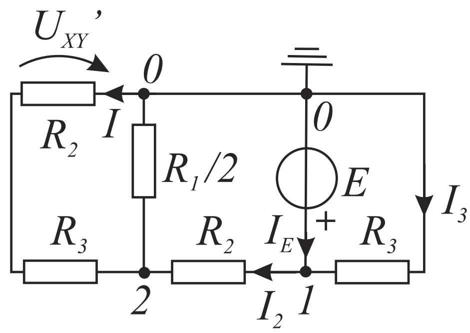
II-1

a)

$$\underset{(1)}{\boxed{\begin{matrix}\textit{Svi}\\ \textit{generatori}\end{matrix}}} = \underset{(2)}{\boxed{\begin{matrix}\textit{Svi}\\ \textit{sem } E\end{matrix}}} + \boxed{\begin{matrix}\textit{Samo}\\ E\end{matrix}}$$

$$U_{XY}^{(1)} = U_{XY}^{(2)} + U_{XY}' \qquad \Rightarrow \qquad \Delta U_{XY} = U_{XY}^{(1)} - U_{XY}^{(2)} = U_{XY}'$$





$V_0 = 0\,V, \qquad V_1 = E = 12\,V$

$$V_2\left(\frac{1}{R_2+R_3}+\frac{2}{R_1}+\frac{1}{R_2}\right)-V_1\left(\frac{1}{R_2}\right)=0$$

$$V_2\left(\frac{1}{1k+2k}+\frac{2}{3k}+\frac{1}{1k}\right)-12\cdot\left(\frac{1}{1k}\right)=0$$

$$V_2\left(\frac{1}{3k}+\frac{2}{3k}+\frac{1}{1k}\right)=12\cdot\frac{1}{1k} \qquad /\cdot 3k$$

$$V_2\left(1+2+3\right)=12\cdot 3$$

$$6\,V_2 = 36 \qquad \Rightarrow \qquad V_2 = 6\,V$$

$$I = \frac{V_0 - V_2}{R_2+R_3} = \frac{0-6}{1k+2k} = \frac{-6}{3k} = -2\,mA$$

$$U_{\mathrm{XY}}' = R_2 I = 1k\cdot(-2m)$$

$$U_{\mathrm{XY}}' = -2\,V$$

$$\boxed{\Delta U_{\mathrm{XY}} = U_{\mathrm{XY}}' = -2\,V}$$

b)

Prilikom prebacivanja prekidača napon će se smanjiti za 2V.

c)

Kada je prekidač u položaju (2): $\boxed{P_E^{(2)} = 0\,W}$

Kada u kolu deluje samo naponski generator E:

$$I_2 = \frac{V_1 - V_2}{R_2} = \frac{12-6}{1k} = \frac{6}{1k} = 6\,mA$$

$$I_3 = \frac{V_0 - V_1}{R_3} = \frac{0-12}{2k} = \frac{-12}{2k} = -6\,mA$$

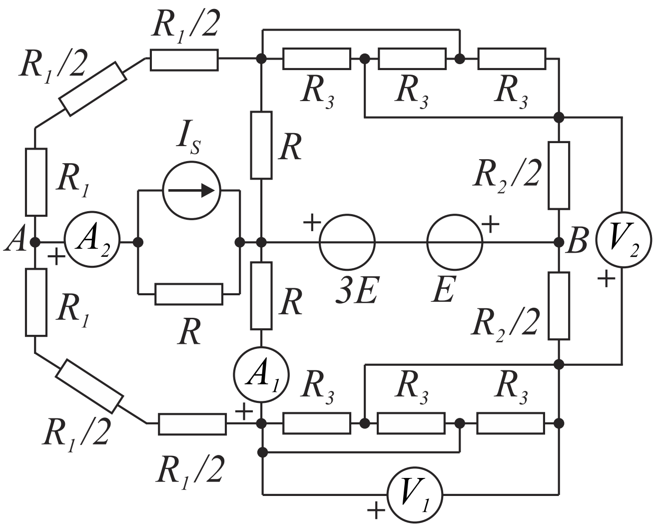
$$I_E = I_2 - I_3 = 6\,mA - (-6\,mA) = 12\,mA$$

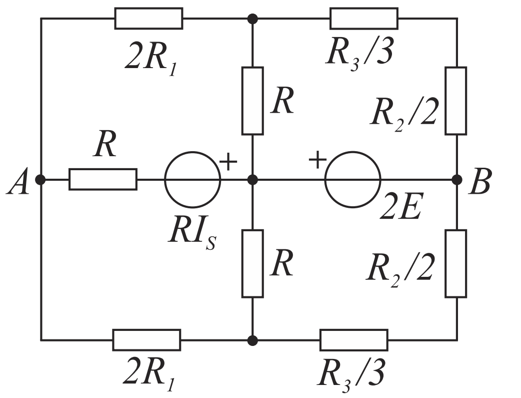
$$P_E' = E\,I_E = 12\cdot 12\,m$$

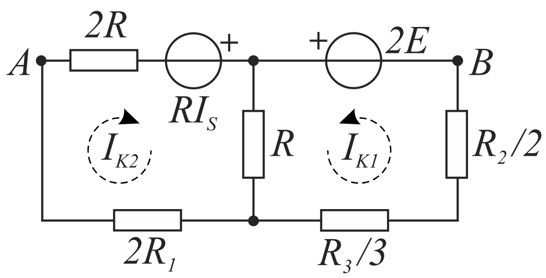
$$\boxed{P_E' = 144\,mW}$$

II-2

a) Kolo je simetrično u odnosu na tačke A i B.







$$MKS:\ n_g-(n_c-1)-n_{s.g}=3-(2-1)-0=2$$

$$I_{K1}\left(\frac{R_3}{3}+\frac{R_2}{2}+R\right)+I_{K2}\,R=2E$$

$$I_{K2}\left(R+2R+2R_1\right)+I_{K1}\,R=RI_S$$

$$I_{K1}\left(60+40+20\right)+I_{K2}\cdot 20=20$$

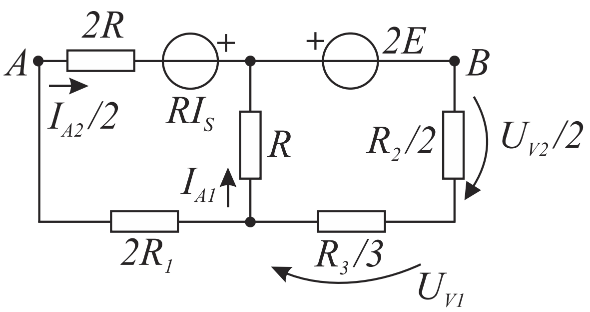
$$I_{K2}\left(20+40+50\right)+I_{K1}\cdot 20=14$$

$$120\,I_{K1}+20\,I_{K2}=20$$

$$20\,I_{K1}+110\,I_{K2}=14$$

$$I_{K1}=0{,}15\ A$$

$$I_{K2}=0{,}1\ A$$



$$I_{A1}=-I_{K1}-I_{K2}=-0{,}25\ A \qquad \boxed{I_{A1}=-0{,}25\ A}$$

$$I_{A2}=2I_{K2}=0{,}2\ A \qquad \boxed{I_{A2}=0{,}2\ A}$$

$$U_{V1}=\frac{R_3}{3}I_{K1}=60\cdot 0{,}15=9\,V \qquad \boxed{U_{V1}=9\,V}$$

$$U_{V2}=2\cdot\frac{R_2}{2}I_{K1}=80\cdot 0{,}15=12\,V \qquad \boxed{U_{V2}=12\,V}$$

b)

$$P_E=2E\,I_{K1}=20\cdot 0{,}15=3W$$

$$P_{RIs}=RI_S\,I_{K2}=20\cdot 0{,}7\cdot 0{,}1=1{,}4\,W$$

$$P_{gen}=P_E+P_{RIs}=4{,}4W$$

$$P_R=\left(2R+2R_1\right)I_{K2}^2+R\,I_{A1}^2+\left(\frac{R_2}{2}+\frac{R_3}{3}\right)I_{K1}^2$$

$$P_R=90\cdot 0{,}01+20\cdot 0{,}0625+100\cdot 0{,}0225$$

$$P_R=4{,}4W \qquad \boxed{P_{gen}=P_R=4{,}4W}$$