**STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE**

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

# Osnovi elektrotehnike 1 (I kolokvijum)

**K1**

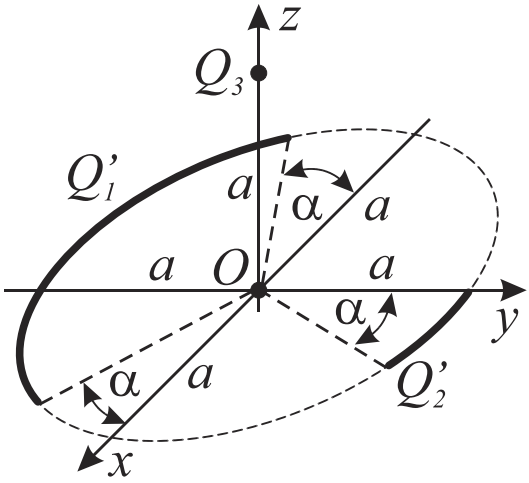
**23.06.2022.**

## ZADACI

**Zadatak 1.** Dva tanka štapa, od izolacionog materijala, savijena kao na slici 1, naelektrisani su ravnomerno podužnim gustinama naelektrisanja $Q_1' > 0$ i $Q_2' > 0$. Štapovi su savijeni u obliku lukova, poluprečnika $a$, i leže u $x$-$y$ ravni Dekartovog koordinatnog sistema. Tačkasto naelektrisanje, naelektrisano sa $Q_3$, nalazi se na $z$ osi, na udaljenosti $a$ od centra koordinatnog sistema (tačka $O$).

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja u tački $O$, koji potiče od štapova i tačkastog naelektrisanja.
b) Odrediti podužne gustine naelektrisanja štapova, $Q_1'$ i $Q_2'$, tako da sve tri komponente ($x$, $y$ i $z$) vektora jačine električnog polja u tački $O$ budu jednake.

Brojni podaci su: $a = 1\ cm$, $Q_3 = 1\ nC$, $\alpha = \pi/6$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.

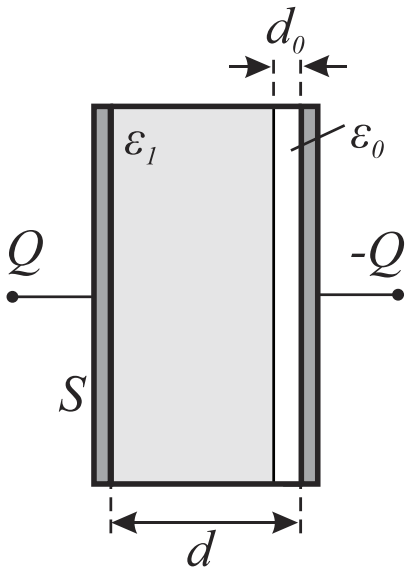



Slika 1.

**Zadatak 2.** Pločasti kondenzator sa čvrstim dielektrikom permitivnosti $\varepsilon_1 = 6 \cdot \varepsilon_0$, prikazan je na slici 2. Dielektrik ne naleže idealno na desnu oblogu kondenzatora, tako da postoji sloj vazduha nepoznate debljine $d_0$ između dielektrika i desne obloge kondenzatora.

a) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora.
b) Izračunati debljinu vazdušnog sloja, $d_0$, ukoliko se zna da je kapacitivnost kondenzatora, kada bi dielektrik idealno nalegao na obe obloge, za 20% veća u odnosu na kapacitivnost kondenzatora kada postoji vazdušni sloj.
c) Proveriti da li će doći do proboja u kondenzatoru sa vazdušnim slojem, ukoliko se kondenzator priključi na izvor napona $U = 53\ kV$.

Ostali brojni podaci: $E_{Č0} = 30\ kV/cm$, $E_{Č1} = 65\ kV/cm$, $d = 10\ cm$, $S = 50\ cm^2$.




Slika 2.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

**STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE**

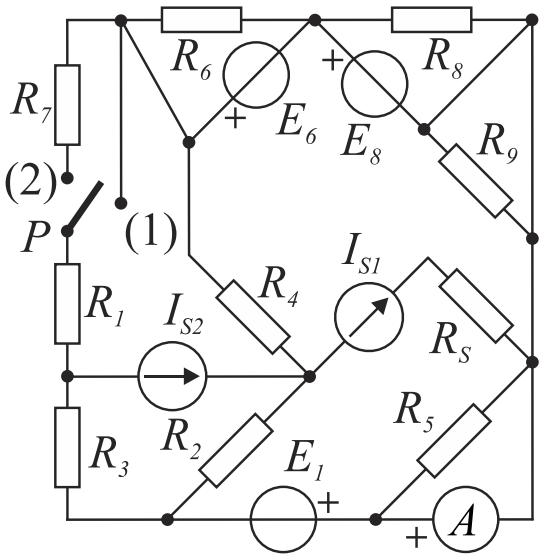
| K T E T<br>Katedra za teorijsku elektrotehniku<br>www.ktet.ftn.uns.ac.rs | Osnovi elektrotehnike 1<br>(II kolokvijum) | **K2**<br>**23.06.2022.** |
|---|---|---|

## ZADACI

**Zadatak 1.** U kolu vremenski konstatnih struja sa slike 1, odrediti:

a) položaj prekidača $P$, ako se zna da se na otporniku otpornosti $R_3$ razvija najveća moguća snaga,
b) pokazivanje idealnog ampermetra,
c) snagu strujnog generatora $I_{S1}$.

Brojni podaci su: $R_1 = R_3 = R_S = 50\ \Omega$, $R_4 = R_5 = R_7 = 100\ \Omega$, $R_2 = R_6 = R_8 = 200\ \Omega$, $R_9 = 300\ \Omega$, $E_1 = 50\ V$, $E_6 = 110\ V$, $E_8 = 100\ V$, $I_{S1} = 1\ A$, $I_{S2} = 0{,}5\ A$.

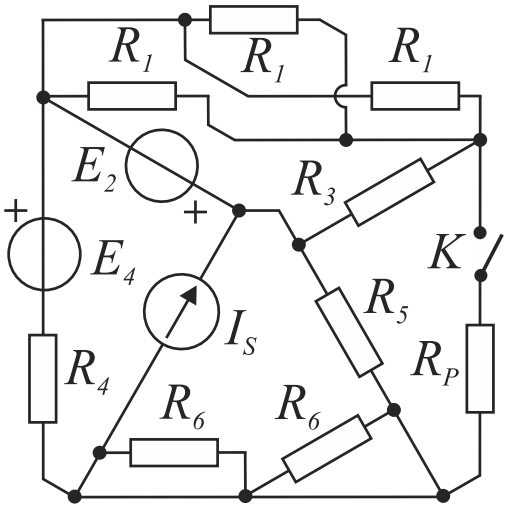



Slika 1.

**Zadatak 2.** U električnom kolu, prikazanom na slici 2, zatvaranjem prekidača $K$, otpornik otpornosti $R_P = 250\ \Omega$, čija je maksimalna snaga $P_{max} = 2{,}5\ mW$, priključuje se u mrežu.

a) Proveriti da li će otpornik $R_P$ pregoreti nakon zatvaranja prekidača $K$. Prilikom računanja *ems* Tevenenovog generatora, mrežu rešavati primenom metode potencijala čvorova.
b) Ukoliko će otpornik $R_P$ pregoreti, odrediti otpornost zaštitnog otpornika $R_Z$, koji treba vezati na red sa njim, kako bi se otpornik $R_P$ zaštitio od pregorevanja.

Brojni podaci su: $E_2 = 5\ V$, $E_4 = 12\ V$, $I_S = 4\ mA$, $R_1 = R_3 = 3\ k\Omega$, $R_4 = 300\ \Omega$, $R_5 = 600\ \Omega$, $R_6 = 2{,}5\ k\Omega$.




Slika 2.

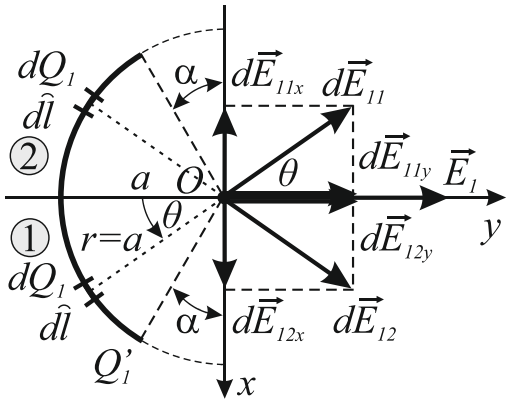
## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

# Ispit iz OET1, 23.06.2022.

**I-1**

a)




$\alpha = \frac{\pi}{6}$

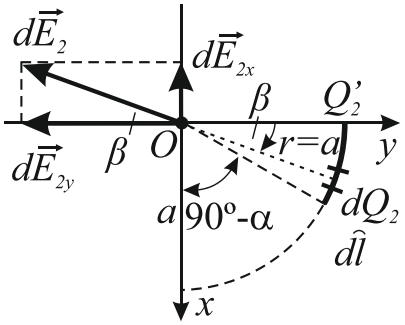
Zbog simetrije: $\vec{E}_{11x} + \vec{E}_{12x} = 0$ $\quad \boxed{\vec{E}_{1x} = 0}$

$$dE_{11y} = dE_{11}\cos\theta = \frac{dQ_1}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q_1'\,dl}{4\pi\varepsilon_0 a^2}\cos\theta = \frac{Q_1'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\cos\theta$$

$$E_1 = 2\int_{\substack{po\ pola\\ luka\ 1}} dE_{11y} = 2\frac{Q_1'}{4\pi\varepsilon_0 a}\int_0^{\frac{\pi}{2}-\alpha}\cos\theta\,d\theta = \frac{Q_1'}{2\pi\varepsilon_0 a}\left(\sin\frac{\pi}{3} - \sin 0\right)$$

$$E_1 = \frac{Q_1'}{2\pi\varepsilon_0 a}\left(\frac{\sqrt{3}}{2} - 0\right)$$

$$\boxed{E_1 = \frac{Q_1'}{2\pi\varepsilon_0 a}\frac{\sqrt{3}}{2}} \qquad \boxed{\vec{E}_1 = E_1\cdot\vec{i}_y}$$




$\alpha = \frac{\pi}{6}$

$$dE_{2x} = dE_2\sin\beta = \frac{dQ_2}{4\pi\varepsilon_0 r^2}\sin\beta = \frac{Q_2'\,dl}{4\pi\varepsilon_0 a^2}\sin\beta = \frac{Q_2'\,a\,d\beta}{4\pi\varepsilon_0 a^2}\sin\beta$$

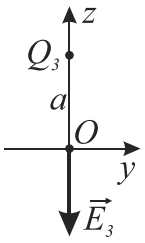
$$E_{2x} = \int_{\substack{po\\ luku\ 2}} dE_{2x} = \frac{Q_2'}{4\pi\varepsilon_0 a}\int_0^{\alpha}\sin\beta\,d\beta = \frac{Q_2'}{4\pi\varepsilon_0 a}\left(\cos 0 - \cos\frac{\pi}{6}\right)$$

$$\boxed{E_{2x} = \frac{Q_2'}{4\pi\varepsilon_0 a}\left(1 - \frac{\sqrt{3}}{2}\right)} \qquad \boxed{\vec{E}_{2x} = E_{2x}\cdot\left(-\vec{i}_x\right)}$$

$$dE_{2y} = dE_2\cos\beta = \frac{dQ_2}{4\pi\varepsilon_0 r^2}\cos\beta = \frac{Q_2'\,dl}{4\pi\varepsilon_0 a^2}\cos\beta = \frac{Q_2'\,a\,d\beta}{4\pi\varepsilon_0 a^2}\cos\beta$$

$$E_{2y} = \int_{\substack{po\\ luku\ 2}} dE_{2y} = \frac{Q_2'}{4\pi\varepsilon_0 a}\int_0^{\alpha}\cos\beta\,d\beta = \frac{Q_2'}{4\pi\varepsilon_0 a}\left(\sin\frac{\pi}{6} - \sin 0\right) = \frac{Q_2'}{4\pi\varepsilon_0 a}\frac{1}{2}$$

$$\boxed{E_{2y} = \frac{Q_2'}{8\pi\varepsilon_0 a}} \qquad \boxed{\vec{E}_{2y} = E_{2y}\cdot\left(-\vec{i}_y\right)}$$




$$\boxed{E_3 = \frac{Q_3}{4\pi\varepsilon_0 a^2}} \qquad \boxed{\vec{E}_3 = E_3\cdot\left(-\vec{i}_z\right)}$$

$$\boxed{\vec{E}_O = \vec{E}_1 + \vec{E}_{2x} + \vec{E}_{2y} + \vec{E}_3 = \frac{Q_2'}{4\pi\varepsilon_0 a}\left(1 - \frac{\sqrt{3}}{2}\right)\cdot\left(-\vec{i}_x\right) + \left(\frac{Q_1'}{2\pi\varepsilon_0 a}\frac{\sqrt{3}}{2} - \frac{Q_2'}{8\pi\varepsilon_0 a}\right)\cdot\vec{i}_y + \frac{Q_3}{4\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_z\right)}$$

b)

$$E_{Ox} = E_{Oy} = E_{Oz} \quad \Rightarrow \quad \frac{Q_2'}{4\pi\varepsilon_0 a}\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{Q_1'}{2\pi\varepsilon_0 a}\frac{\sqrt{3}}{2} - \frac{Q_2'}{8\pi\varepsilon_0 a} = \frac{Q_3}{4\pi\varepsilon_0 a^2}$$

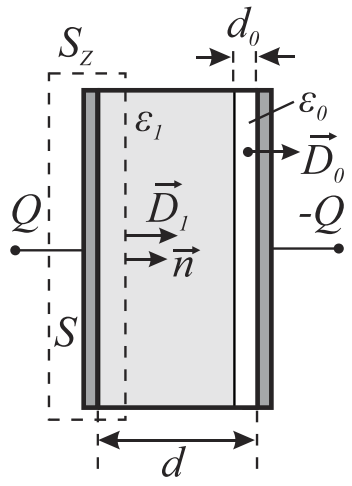
$$\frac{Q_2'}{4\pi\varepsilon_0 a}\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{Q_3}{4\pi\varepsilon_0 a^2} \quad \Rightarrow \quad \boxed{Q_2' = 746{,}44\ nC/m}$$

$$\frac{Q_1'}{2\pi\varepsilon_0 a}\frac{\sqrt{3}}{2} = \frac{Q_2'}{8\pi\varepsilon_0 a} + \frac{Q_2'}{4\pi\varepsilon_0 a}\left(1 - \frac{\sqrt{3}}{2}\right) \quad \Rightarrow \quad \boxed{Q_1' = 273{,}2\ nC/m}$$

Ispit iz OET1, 23.06.2022.

**I-2**

a)




Granični uslov:

$D_{1n} = D_{0n} \qquad D_1 = D_0 = D$

$E_{1t} = E_{0t} = 0$

$\oint_{S_Z} \vec{D} \cdot \vec{ds} = Q_{u\,S_Z}$

$\int_{S_Z} D\, ds = Q$

$D\, S = Q$

$D = \frac{Q}{S}$

$E_1 = \frac{Q}{\varepsilon_1 S} \qquad E_0 = \frac{Q}{\varepsilon_0 S}$

$$U = \int_{-}^{+} \vec{E} \cdot \vec{dl} = \int_0^{d-d_0} E_1\, dl + \int_0^{d_0} E_0\, dl = E_1(d-d_0) + E_0\, d_0 = \frac{Q}{\varepsilon_1 S}(d-d_0) + \frac{Q}{\varepsilon_0 S} d_0$$

$C = \frac{Q}{U} \qquad \boxed{C = \frac{1}{\frac{d-d_0}{\varepsilon_1 S} + \frac{d_0}{\varepsilon_0 S}}}$

b)

$$C = \frac{1}{\frac{d-d_0}{\varepsilon_1 S} + \frac{d_0}{\varepsilon_0 S}} = \frac{1}{\frac{d-d_0}{6\varepsilon_0 S} + \frac{d_0}{\varepsilon_0 S}} = \frac{6\varepsilon_0 S}{d+5d_0}, \qquad C_{idealno} = \varepsilon_1 \frac{S}{d} = 6\varepsilon_0 \frac{S}{d}$$

$C_{idealno} = 1{,}2\, C \qquad \frac{C_{idealno}}{C} = 1{,}2$

$$\frac{C_{idealno}}{C} = \frac{6\varepsilon_0 \frac{S}{d}}{\frac{6\varepsilon_0 S}{d+5d_0}} = \frac{d+5d_0}{d} = 1 + 5\frac{d_0}{d} = 1{,}2 \quad \Rightarrow \quad d_0 = \frac{0{,}2\, d}{5} = \frac{0{,}2 \cdot 0{,}1}{5} = 0{,}004\, m$$

$\boxed{d_0 = 4\ mm}$

c)

$$\left.\begin{aligned} E_{1\max} = \frac{Q_{1\max}}{\varepsilon_1 S} = E_{\check{C}1} \quad &\Rightarrow \quad Q_{1\max} = E_{\check{C}1}\, \varepsilon_1\, S = 1725{,}75\ nC \\ E_{0\max} = \frac{Q_{0\max}}{\varepsilon_0 S} = E_{\check{C}0} \quad &\Rightarrow \quad Q_{0\max} = E_{\check{C}0}\, \varepsilon_0\, S = 132{,}75\ nC \end{aligned}\right\} \quad \Rightarrow \quad Q_{\max} = \min\{Q_{1\max}, Q_{0\max}\} = Q_{0\max}$$

$$U_{\max} = \frac{Q_{\max}}{6\varepsilon_0 S}(d-d_0) + \frac{Q_{\max}}{\varepsilon_0 S} d_0 = \frac{Q_{\max}}{\varepsilon_0 S}\left(\frac{d-d_0}{6} + d_0\right) = \frac{Q_{\max}}{\varepsilon_0 S}\frac{d+5d_0}{6} = \frac{E_{\check{C}0}\, \varepsilon_0\, S}{\varepsilon_0 S}\frac{d+5d_0}{6} = E_{\check{C}0}\frac{d+5d_0}{6} = 60\ kV$$

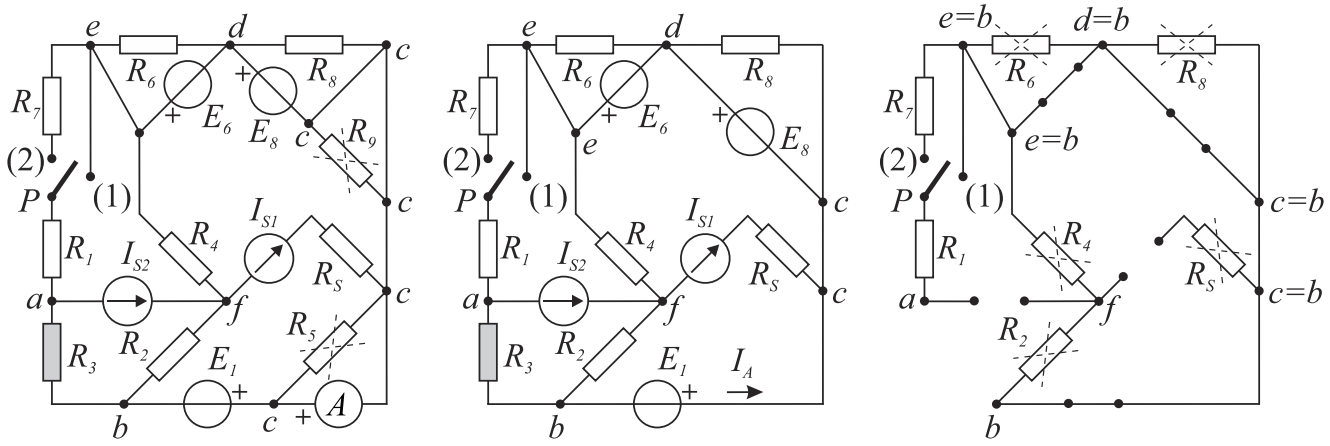
$U = 53\, kV$

$U < U_{\max} \quad \Rightarrow \quad$ Neće doći do proboja.

Ispit iz OET1, 23.06.2022.

**II-1**

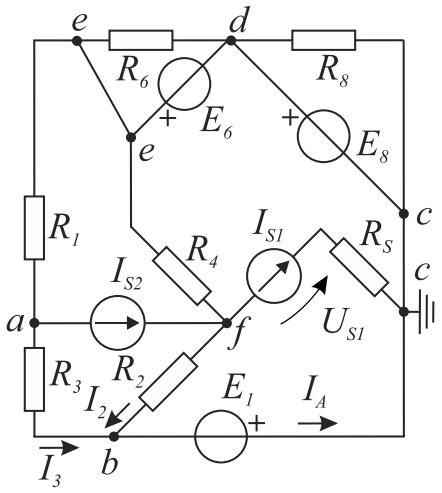
a)




$$R_3 = R_T = 50\,\Omega$$

$$R_T = R_{ab} = \begin{cases} R_1 = 50\,\Omega & (1) \\ R_1 + R_7 = 150\,\Omega & (2) \end{cases} \quad \Rightarrow \quad \boxed{P\ u\ položaju\ (1)}$$

b)




$$V_c = 0\,V, \qquad V_b = -E_1 = -50\,V,$$
$$V_d = E_8 = 100\,V, \qquad V_e = E_8 + E_6 = 210\,V$$

$$V_a\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{\infty}\right) - V_b\left(\frac{1}{R_3}\right) - V_e\left(\frac{1}{R_1}\right) - V_f\left(\frac{1}{\infty}\right) = -I_{S2}$$

$$V_f\left(\frac{1}{\infty}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{\infty+R_S}\right) - V_a\left(\frac{1}{\infty}\right) - V_b\left(\frac{1}{R_2}\right) - V_e\left(\frac{1}{R_4}\right) = -I_{S1}+I_{S2}$$

$$V_a\left(\frac{1}{50}+\frac{1}{50}\right) + 50\cdot\left(\frac{1}{50}\right) - 210\cdot\left(\frac{1}{50}\right) = -0,5$$

$$V_f\left(\frac{1}{200}+\frac{1}{100}\right) + 50\cdot\left(\frac{1}{200}\right) - 210\cdot\left(\frac{1}{100}\right) = -1+0,5$$

$$2V_a = 135 \quad \Rightarrow \quad V_a = 67,5\,V$$
$$3V_f = 270 \quad \Rightarrow \quad V_f = 90\,V$$

$$I_2 = \frac{V_f - V_b}{R_2} = \frac{90+50}{200} = 0,7\ A$$

$$I_3 = \frac{V_a - V_b}{R_3} = \frac{67,5+50}{50} = 2,35\ A$$

$$I_A = I_2 + I_3 = 0,7 + 2,35$$

$$\boxed{I_A = 3,05\ A}$$

c)

$$U_{S1} = R_S I_{S1} + U_{cf} = R_S I_{S1} + \left(V_c - V_f\right) = 50\cdot 1 + \left(0-90\right) = -40\,V$$

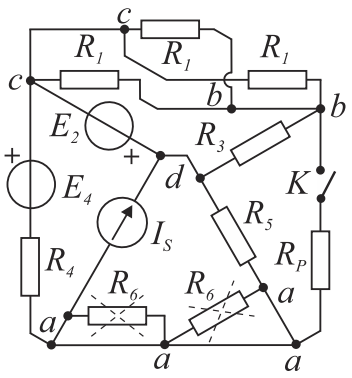
$$P_{S1} = U_{S1} I_{S1} = -40\cdot 1$$

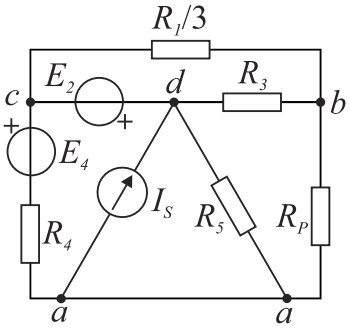
$$\boxed{P_{S1} = -40\,W}$$

Ispit iz OET1, 23.06.2022.

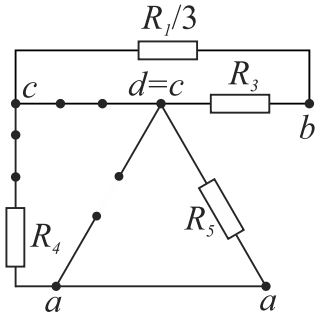
**II-2**

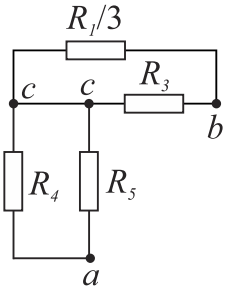
a)




$R_T$:



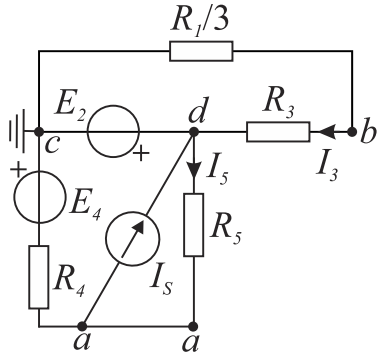




$$R_T = R_{ab} = \left(R_4 \,||\, R_5\right) + \left(\frac{R_1}{3} \,||\, R_3\right)$$

$$R_T = \left(0{,}3k \,||\, 0{,}6k\right) + \left(1k \,||\, 3k\right) = 0{,}2k + 0{,}75k$$

$$\boxed{R_T = 0{,}95\,k\Omega}$$

$E_T$:




$$V_c = 0\,V, \qquad V_d = E_2 = 5\,V$$

$$V_a\left(\frac{1}{R_5} + \frac{1}{\infty} + \frac{1}{R_4 + 0}\right) - V_d\left(\frac{1}{R_5} + \frac{1}{\infty}\right) = -I_S - \frac{E_4}{R_4}$$

$$V_a\left(\frac{1}{600} + \frac{1}{300}\right) - 5\cdot\left(\frac{1}{600}\right) = -4\,m - \frac{12}{300} \qquad /\cdot 600$$

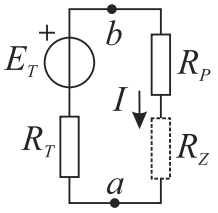
$$3V_a = -21{,}4$$

$$V_a = -7{,}13\,V$$

$$I_3 = \frac{V_c - V_d}{\frac{R_1}{3} + R_3} = \frac{0-5}{1k+3k} = -1{,}25\,mA$$

$$I_5 = \frac{V_d - V_a}{R_5} = \frac{5-\left(-7{,}13\right)}{600} = 20{,}22\,mA$$

$$E_T = U_{ba} = I_3 R_3 + I_5 R_5 = -1{,}25\,m\cdot 3k + 20{,}22\,m\cdot 600 \qquad \boxed{E_T = 8{,}38\,V}$$




$$I_{\max} = \sqrt{\frac{P_{\max}}{R_P}} = \sqrt{\frac{2{,}5\,m}{250}} = 3{,}16\,mA$$

$$I = \frac{E_T}{R_T + R_P} = \frac{8{,}38}{0{,}95k + 0{,}25k} = 6{,}98\,mA > 3{,}16\,mA$$

Otpornik $R_P$ će pregoreti. Treba dodatni zaštitni otpornik $R_Z$.

b)

$$I' = \frac{E_T}{R_T + R_P + R_Z} \le I_{\max}$$

$$R_Z \ge \frac{E_T}{I_{\max}} - R_T - R_P = \frac{8{,}38}{3{,}16\,m} - 0{,}95k - 0{,}25k = 1{,}45k\Omega \qquad \boxed{R_Z \ge 1{,}45k\Omega}$$